## Lecture 4: Stratospheric Transport

- (i) Quantifying transport rates: Effective diffusivity
- (ii) Quantifying transport rates: Age
- (iii) Stratospheric trace gases: Global structure and tracer-tracer relationships

FDEPS 2010 Alan Plumb, MIT Nov 2010



### stirring

#### diabatic motion

Plumb et al (2007)

(i) Quantifying transport rates: Effective diffusivity "Effective diffusivity" [Nakamura, *J Atmos Sci*, 1996]

$$\begin{split} \frac{\partial q}{\partial t} + \mathbf{u} \cdot \nabla q &= \kappa \nabla^2 q \\ \widehat{X} &= \oint X \frac{dl}{|\nabla q|} / \oint \frac{dl}{|\nabla q|} \\ \widehat{\frac{\partial q}{\partial t}} &= \left(\frac{\partial Q}{\partial t}\right)_A \\ \widehat{\mathbf{u} \cdot \nabla q} &= 0 \\ \widehat{\kappa \nabla^2 q} &= \left(\frac{\partial A}{\partial Q}\right)^{-1} \frac{\partial}{\partial Q} \left[\kappa \frac{\partial A}{\partial Q} \left(\widehat{|\nabla q|^2}\right)\right] \\ \frac{\partial Q}{\partial t} &= \frac{1}{a^2 \cos \phi_e} \frac{\partial}{\partial \phi_e} \left[K_{eff} \cos \phi_e \frac{\partial Q}{\partial \phi_e}\right] \\ K_{eff} &= \kappa \frac{L_e^2}{4\pi^2 a^2 \cos^2 \phi_e} = \kappa \frac{L_e^2}{L^2(\phi_e)} \\ L_e(Q) &= \sqrt{\left(\oint |\nabla q| \ dl\right) \left(\oint \frac{dl}{|\nabla q|}\right)} \end{split}$$

diffusion equation

A(Q,t)

effective diffusivity

equiivalent length

"Effective diffusivity" [Nakamura, *J Atmos Sci*, 1996]

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A(Q,t) diffusion equation

effective diffusivity

equiivalent length



[Haynes & Shuckburgh, J Atmos Sci,2002]



Plumb et al (2007)



Haynes & Shuckburgh, J Atmos Sci, 2002



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#### Transport rates

In "surf zone",

$$K_{eff} \sim 3 \times 10^6 \mathrm{m}^2 \mathrm{s}^{-1}$$

mixing time across L = 3000km:

$$\tau_{mix} \sim \frac{L^2}{K_{eff}} \sim \frac{(3 \times 10^6)^2}{3 \times 10^6} \sim 3 \times 10^6 \,\mathrm{s} \sim 35 \,\mathrm{days}$$

Time scale for residual advection across surf zone ( $\bar{v}_* \sim 0.1 \text{ms}^{-1}$ ):

$$\tau_{adv} \sim \frac{L}{\bar{\nu}_*} \sim 3 \times 10^7 \text{s} \sim 1 \text{ year}$$



Stirring and mixing is the dominant poleward transport process



Plumb et al (2007)



Plumb et al (2007)



Plumb et al (2007)

## (ii) Quantifying transport rates: Age

a stratospheric clock

Plumb et al (2007)



θ

Theoretical (ideal) age:

 $\frac{\partial \Gamma}{\partial t} + \mathcal{T}(\Gamma) = 1$ steady equilibrium:  $\mathcal{T}(\Gamma) = 1 \rightarrow \Gamma(\varphi, z) = \mathcal{T}^{-1}(1); \quad \Gamma_0 = 0$ 

Age from observed tracers:



linearly growing tracer 
$$q_0(t) = Q_0 + \Lambda t$$
  
n equilibrium  $q(\varphi, z, t) = Q(\varphi, z) + \Lambda t$   
 $\frac{\partial q}{\partial t} + T(q) = \Lambda + T(Q) = 0$   
 $\rightarrow Q(\varphi, z) = -T^{-1}(\Lambda) ; \qquad Q(\varphi_0, z_0) = Q_0$   
 $\rightarrow Q - Q_0 = -\Lambda\Gamma$   
 $\rightarrow q(\varphi, z, t) = Q_0 + \Lambda(t - \Gamma)$   
 $= q_0(t - \Gamma)$ 





 $CO_2$  – derived age (yr)

# [Engel et al, *Nature Geoscience*, 2008]

#### Figure 2 | Vertical profiles of mean age derived from the $CO_2$ data shown in Fig. 3. The mean age is derived in the same way from the $CO_2$ observations, as explained in Engel *et al.*<sup>20</sup> using the reference tropospheric data set as discussed in the text. The colour code shows the (northern) latitude of the measurements. The red line shows the 24 km level, which corresponds to the 30 hPa level chosen as the lower pressure altitude limit of data included in this analysis. The uncertainty due to analytical error is of the order of 0.1 years. Systematic uncertainties are discussed in Supplementary Information.

#### Global flux of age

$$\frac{\partial \Gamma}{\partial t} + \mathcal{T}(\Gamma) = \frac{\partial \Gamma}{\partial t} + \frac{1}{\rho} \nabla \cdot \mathbf{F}_{\Gamma} = 1$$
  
in equilibrium  $\frac{1}{\rho} \nabla \cdot \mathbf{F}_{\Gamma} = 1$ 

Integrate over volume above surface  $\theta = \Theta$ :



 $\rightarrow$  so we know the net flux of age through any surface if we know the mass above that surface (which we do if we know p along the surface)

$$\rightarrow \iint_{\theta=\Theta} \mathbf{F}_{\Gamma} \cdot \mathbf{n} \, dA = M(\Theta)$$

If diabatic diffusion is negligible ( $K_{zz}$  weak in stratosphere), flux through  $\theta$  surface is purely advective:

$$\mathbf{F}_{\Gamma} = \rho w_{d} \Gamma$$

$$\rightarrow \iint_{\theta=\Theta} \mathbf{F}_{\Gamma} \cdot \mathbf{n} \, dA = \iint_{\theta=\Theta} \rho w_{d} \Gamma dA$$

$$= \iint_{up} \rho w_{d} \Gamma dA + \iint_{down} \rho w_{d} \Gamma dA$$

$$= -\mathcal{M} \Big[ \langle \Gamma \rangle_{down} - \langle \Gamma \rangle_{up} \Big]$$



where

$$\mathcal{M} = \iint_{up} \rho w_d dA = -\iint_{down} \rho w_d dA$$
 overturning mass flux  
$$\langle \Gamma \rangle_{down} = -\frac{1}{\mathcal{M}} \iint_{down} \rho w_d \Gamma dA \; ; \; \langle \Gamma \rangle_{up} = \frac{1}{\mathcal{M}} \iint_{up} \rho w_d \Gamma dA$$
$$\int_{up} \Delta \Gamma(\Theta) = \langle \Gamma \rangle_{down} - \langle \Gamma \rangle_{up} = \frac{M(\Theta)}{\mathcal{M}(\Theta)}$$

#### Age trends from WACCM



FIG. 1. Evolution of the age of air near 10 hPa averaged over  $\pm 22^{\circ}$  [months (10 yr)<sup>-1</sup>] for three-member ensemble simulations of the climate of the twentieth century (REF1; black curve); the climate of the twenty-first century under increasing loading of GHG (REF2; red); and the climate of the twenty-first century with GHG held constant at 1995 values (NCC; blue). See text for details.

[Garcia & Randel, J Atmos Sci, 2008]



[Engel et al, Nature Geoscience, 2008]

(iii) Stratospheric trace gases: Global structure and tracer-tracer relationships

### HALOE data

#### [Russell et al, J Geophys Res, 1993]



CH<sub>4</sub> : HF comparison (HALOE data)



→ Similar isopleth shapes, despite different locations of sources and sinks

*In situ* data

(SOLVE experiment 2000)

Ertel PV, 480K





Plumb et al (2002)

Different relationships in different regions

#### In situ balloon data (LACE, Elkins/Moore) CFC-11 : N<sub>2</sub>O



[Plumb, Rev Geophys, 2007]

Different relationships in different regions



[Plumb, Rev Geophys, 2007]

Different relationships in different regions



Michelsen et al, J Geophys Res, 1998]



Michelsen et al, J Geophys Res, 1998]



[Plumb, Rev Geophys, 2007]

Michelsen et al, J Geophys Res, 1998]

N<sub>2</sub>O mixing ratio (ppb)

°S

°S

°S

300

from chemical transport model [Plumb et al., *J Geophys Res*, 2002]

N Hem S Hem CFC-11 (ppt) 0120 0120 Tropics global CFC-11 (ppt) 0120 100 midlatitudes vortex tropics ) 200 N<sub>2</sub>O (ppb) N<sub>2</sub>O (ppb) 

P(N<sub>2</sub>O,CFC-11) 22 Jan 2000

Theory:  $\tau_{mix} \ll \tau_{adv}$  $\frac{\partial \chi}{\partial t} + \mathbf{u} \cdot \nabla \chi - \kappa \nabla^2 \chi = S$ tracer mixing ratio sources and sinks [Plumb, Rev Geophys, 2007]







Theory: 
$$\tau_{mix} \ll \tau_{adv}$$

Plumb (2007)

$$\frac{\partial \chi}{\partial t} + \mathbf{u}_d \cdot \nabla \chi + \dot{\theta} \frac{\partial \chi}{\partial \theta} + \mathcal{H}(\chi) = S$$



Properties of  $\mathcal{H}$  :

• only acts on isentropic gradients:  $\mathcal{H}[f(\theta, t)] = 0$ 

• it is linear: 
$$\mathcal{H}(\chi + \phi) = \mathcal{H}(\chi) + \mathcal{H}(\phi)$$
 and  $\mathcal{H}[f(\theta, t)\chi] = f(\theta, t)\mathcal{H}(\chi)$ 

• redistribution operator (does not create or destroy tracer)

$$\iint \sigma \mathcal{H}(\chi) \, dA = \text{boundary fluxes}$$

it is uniquely invertible; solution to

$$\mathcal{H}(\chi) = X$$

subject to zero net boundary flux, has solution

$$\chi = \mathcal{H}^{-1}(X)$$

(solvability condition:

$$\bar{X} = \left[\iint \sigma \, dA\right]^{-1} \iint \sigma X \, dA = 0$$

Theory:  $au_{mix} \ll au_{adv}$ 



$$\begin{split} \dot{\theta} &= \varepsilon (\dot{\theta}_0 + \varepsilon \dot{\theta}_1) \ ,\\ \frac{\partial}{\partial t} &= \varepsilon \left( \frac{\partial}{\partial t} + \varepsilon \frac{\partial}{\partial \tau} \right) \\ S &= \varepsilon^2 S_0 \end{split}$$

$$\chi = \chi_0 + \varepsilon \chi_1 + \varepsilon^2 \chi_2 + \dots$$

At leading order  $\varepsilon^0$ ,

$$\frac{\partial \chi}{\partial t} + \mathbf{u}_d \cdot \nabla \chi + \dot{\theta} \frac{\partial \chi}{\partial \theta} + \mathcal{H}(\chi) = S$$

 $\mathcal{H}(\chi_0) = 0$  $\rightarrow \chi_0 = \chi_0(\theta, t)$ 

 $\rightarrow$  complete isentropic homogenization



 $\chi_0^{(n)} = \chi_0^{(n)}(\theta, t) \rightarrow f(\chi_0^{(1)}, \chi_0^{(2)}, t) = 0$ , trivially

At order  $\epsilon^1$ ,

$$\frac{\partial \chi}{\partial t} + \mathbf{u}_{d} \cdot \nabla \chi + \dot{\theta} \frac{\partial \chi}{\partial \theta} + \mathcal{H}(\chi) = S$$

$$\mathcal{H}(\chi_{1}) = S - \frac{\partial \chi_{0}}{\partial t} - \mathbf{u}_{d} \cdot \nabla \chi_{0} - \dot{\theta} \frac{\partial \chi_{0}}{\partial \theta}$$
solvability condition  $\longrightarrow$ 
advection by *average* vertical motion
mixing ratio of entrained tropical air
$$\frac{\partial \chi_{0}}{\partial t} + \ddot{\theta} \frac{\partial \chi_{0}}{\partial \theta} = \bar{S} + \frac{1}{\tau_{e}} (\chi_{T} - \chi_{0}),$$
where  $\tau_{e} = \iint \sigma \, dA \, / \oint \sigma V \, dl$ 

time for entrained air to fill surf zone

entrainment velocity

At order  $\varepsilon^1$ ,

$$\frac{\partial \chi}{\partial t} + \mathbf{u}_d \cdot \nabla \chi + \dot{\theta} \frac{\partial \chi}{\partial \theta} + \mathcal{H}(\chi) = S$$
$$\mathcal{H}(\chi_1) = S - \frac{\partial \chi_0}{\partial t} - \mathbf{u}_d \cdot \nabla \chi_0 - \dot{\theta} \frac{\partial \chi_0}{\partial \theta}$$

if  $\chi_0$  steady, and  $\overline{S}$  negligible,

$$\chi_{1}^{(n)} = -\zeta \frac{\partial \chi_{0}^{(n)}}{\partial \theta} , \text{ where}$$
$$\zeta = \mathcal{H}^{-1}(\dot{\theta}') - \overline{\dot{\theta}}\tau_{e}\phi\{\sigma V\}$$

 $\zeta(\lambda, \varphi, t)$  is the vertical displacement of tracer isopleths it is *purely kinematic*: same for all tracers

 $\rightarrow$  the  $O(\varepsilon)$  correction lies *along* the canonical curve, so

$$f(\chi^{(1)},\chi^{(2)},t) = 0$$

remains valid at this order: non-trivial compact relationships

#### Creation of tropical relationships



2 tropospheric source gases, destroyed in tropical stratosphere tracer 2 has shorter lifetime than tracer 1  $\rightarrow$  tracer-tracer relation in tropics is curved as shown (T)

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2 tropospheric source gases, destroyed in tropical stratosphere tracer 2 has shorter lifetime than tracer 1  $\rightarrow$  tracer-tracer relation in tropics is curved as shown (T)

#### Creation of midlatitude relationships



#### Creation of midlatitude relationships



midlatitude curve lies on concave side of tropical curve



vortex curve lies on concave side of midlatitude curve

from chemical transport model [Plumb et al., J Geophys Res, 2002]

N Hem S Hem 

Μ



P(N<sub>2</sub>O,CFC-11) 22 Jan 2000



[Plumb, Rev Geophys, 2007]



Michelsen et al, J Geophys Res, 1998]

### HALOE data

#### [Russell et al, J Geophys Res, 1993]



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#### [Russell et al, J Geophys Res, 1993]



stratospheric sink

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