Geophysical Fluid Dynamics: from the Lab, up and down!

Henri-Claude Nataf
Univ Grenoble Alpes / CNRS
Grenoble, France

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Lecture 5

The formation of planets

FDEPS

Kyoto, November 29, 2018

Pot-pourri or final bouquet?

5. The formation of planets

  5.1. The Sun and helioseismology

  5.2. The formation of the solar system

  5.3. The formation of the Earth

5.1. The Sun and helioseismology

The Sun

Lsmpascal images

Helioseismology

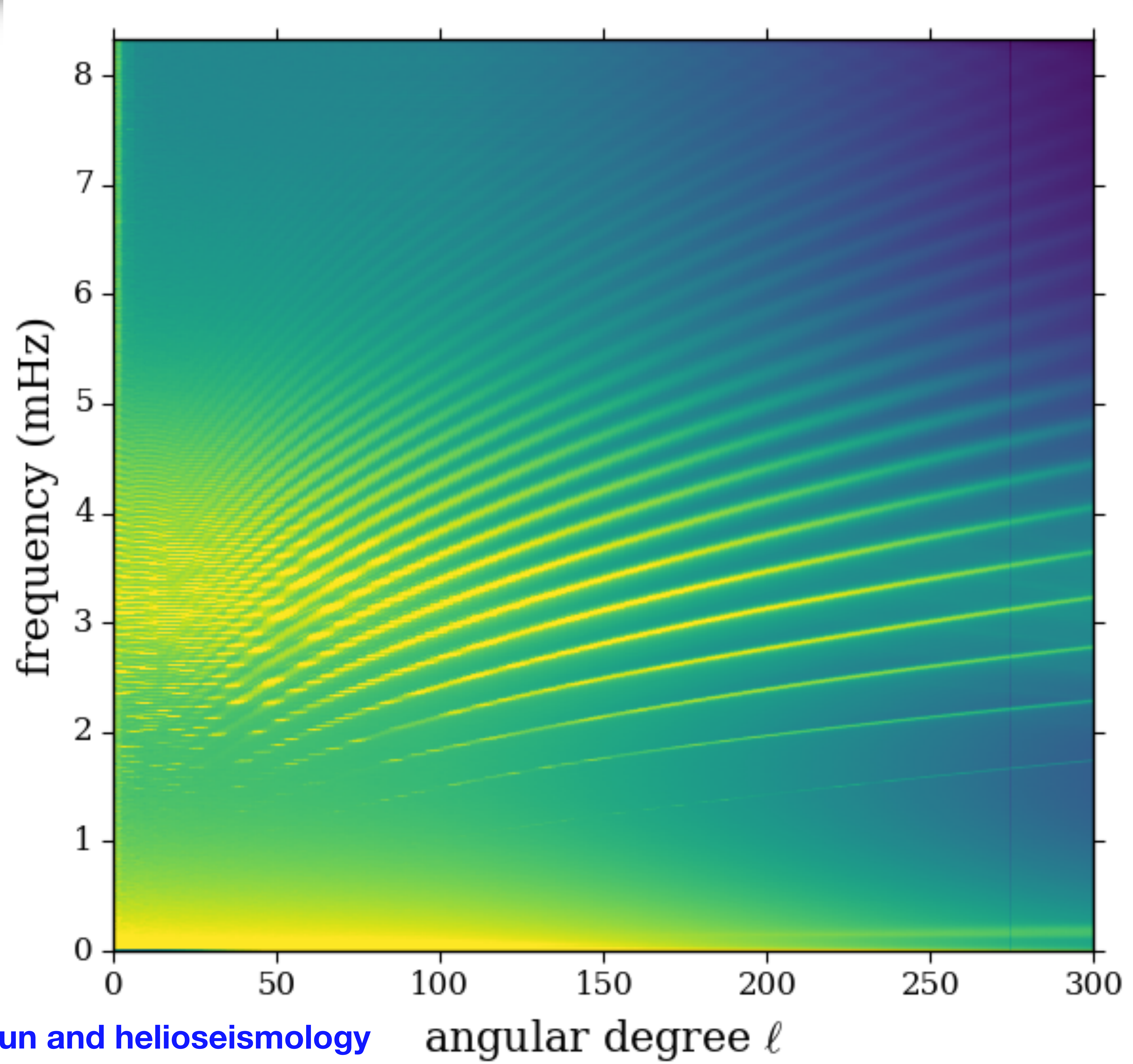
- The Sun is a rather turbulent guy! Its perpetual agitation in the **convective zone** excites waves, which interfere to create **normal modes** (or *free oscillations)* similar to the normal modes of the Earth.

- By observing the **Doppler shift** of some luminous spectral lines at the surface of the Sun, astrophysicists have detected and identified almost a **million of such modes**. As on Earth, the modes are identified by their **radial dependance** (*n* number), **horizontal wave number** (*l* number or *degree*) and **azimuthal dependance** (*m* number or *order*). We write them:

$$ {}_{n}S_{l}^{m} $$

- If the Sun were spherically symmetric and non-rotating, the **frequency** of such a mode should **not depend upon *m***: it should be **degenerate**.

The dispersion diagram of solar normal modes

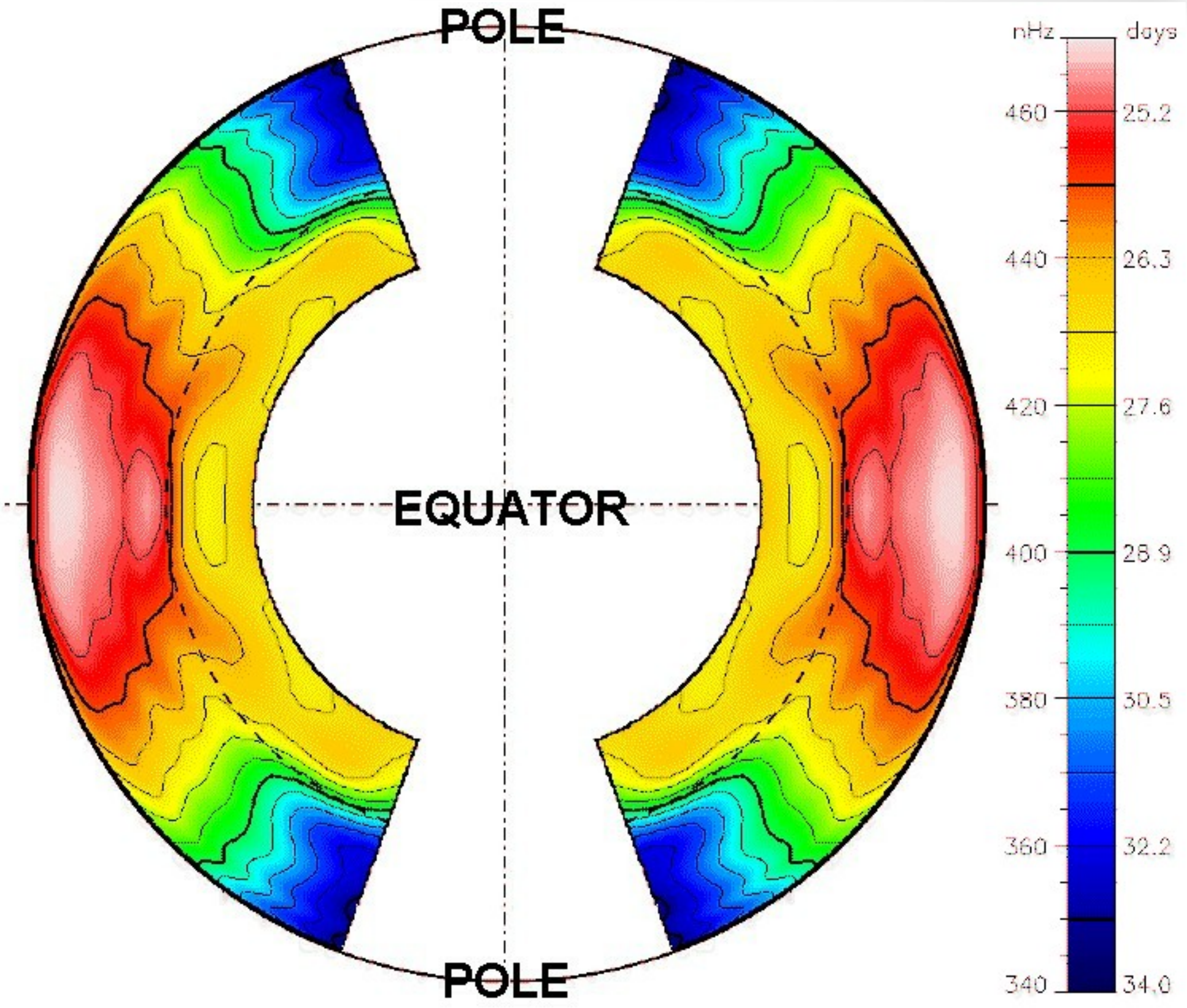


- Several dedicated missions from the **ground** (such as GONG and BISON) and in **space** (with SOHO) have permitted the retrieval of several decades of observations.
- It is difficult to overestimate the impact of these observations on solar physics and beyond!

The tachocline

- One **key discovery** (but there were many others) was the **radial** and **latitudinal** variation of the **angular velocity** in the Sun.

- Equatorial regions are spinning much faster than polar regions. There is an abrupt variation of angular velocity with depth, defining a **strong shear** region called the **tachocline**, near the base of the convective region.

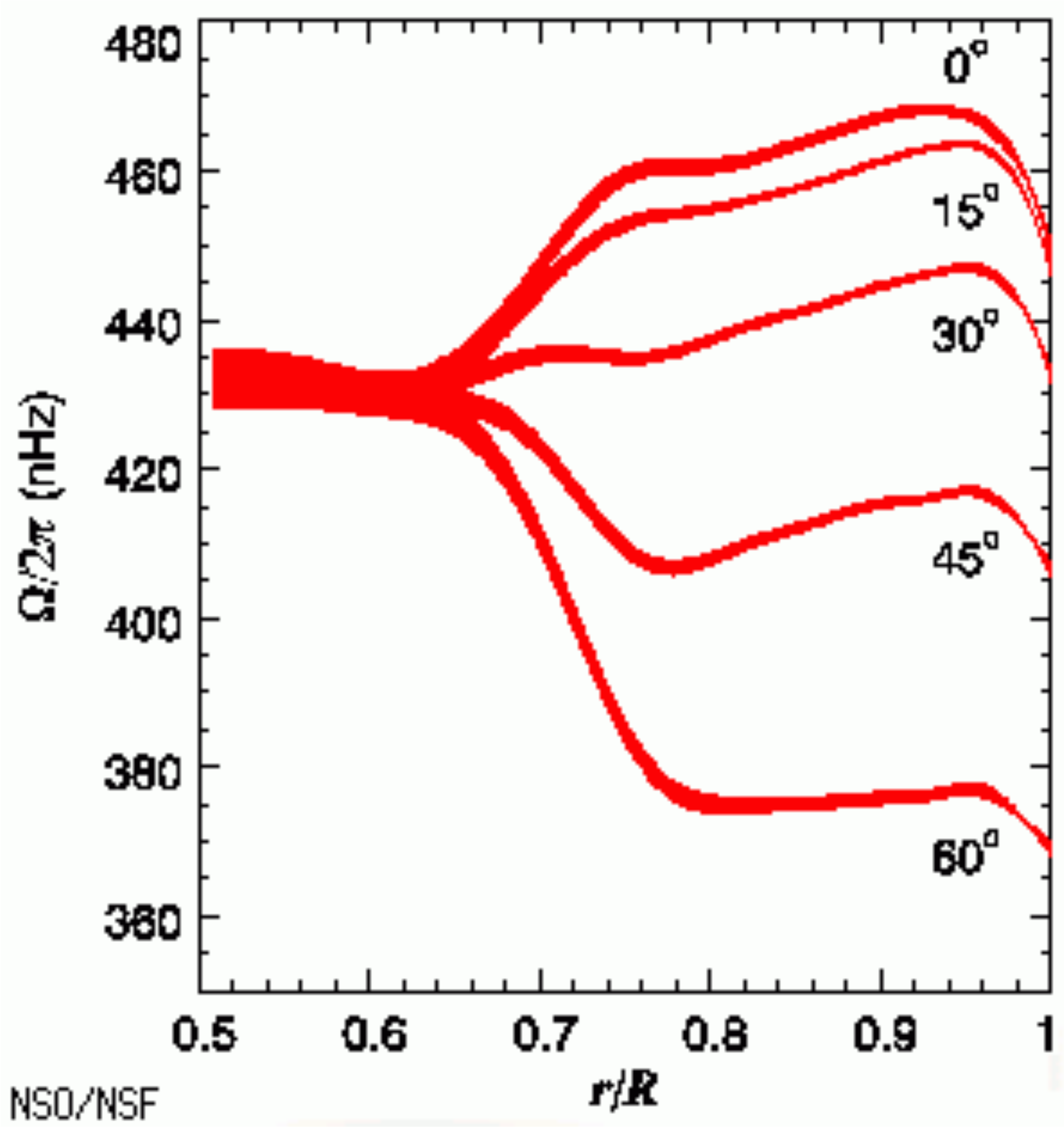


Ageostrophic zonal flow

- The tachocline is thought to play a major role in the generation of the **magnetic field** of the Sun.

- Note that the differential rotation varies strongly with latitude: it is **not geostrophic**. One reason for that is that the **Rossby number** is of order 1 in the Sun.

- Presenting all the discoveries brought by helioseismology and **asteroseismology** would fill a complete course, and I would not be the right person to give it at all!



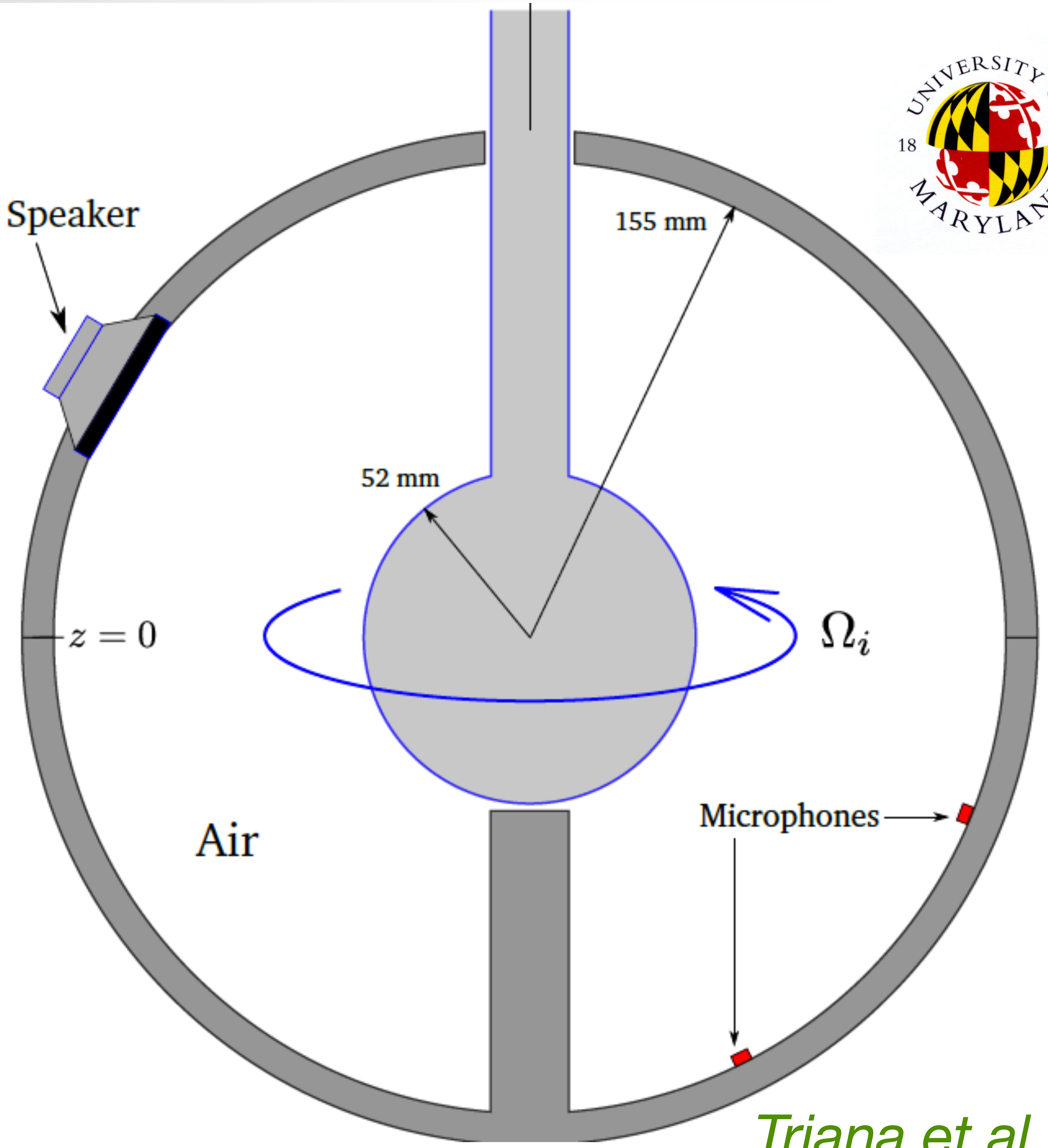
Helioseismology in a bottle!

- Instead, keeping up with my ‘laboratory experiments’ red thread, I will present a fun transposition of helioseismology to the study of **flow in a rotating sphere**.

- The idea is to excite **acoustic waves** in a spherical cavity filled with **gas**, to record pressure signals, and identify normal modes in their frequency spectrum, with the goal of maping **zonal flows** in a rotating sphere.

- A proof of concept of the method was provided by Triana et al (2014).

Spherical Couette flow set-up



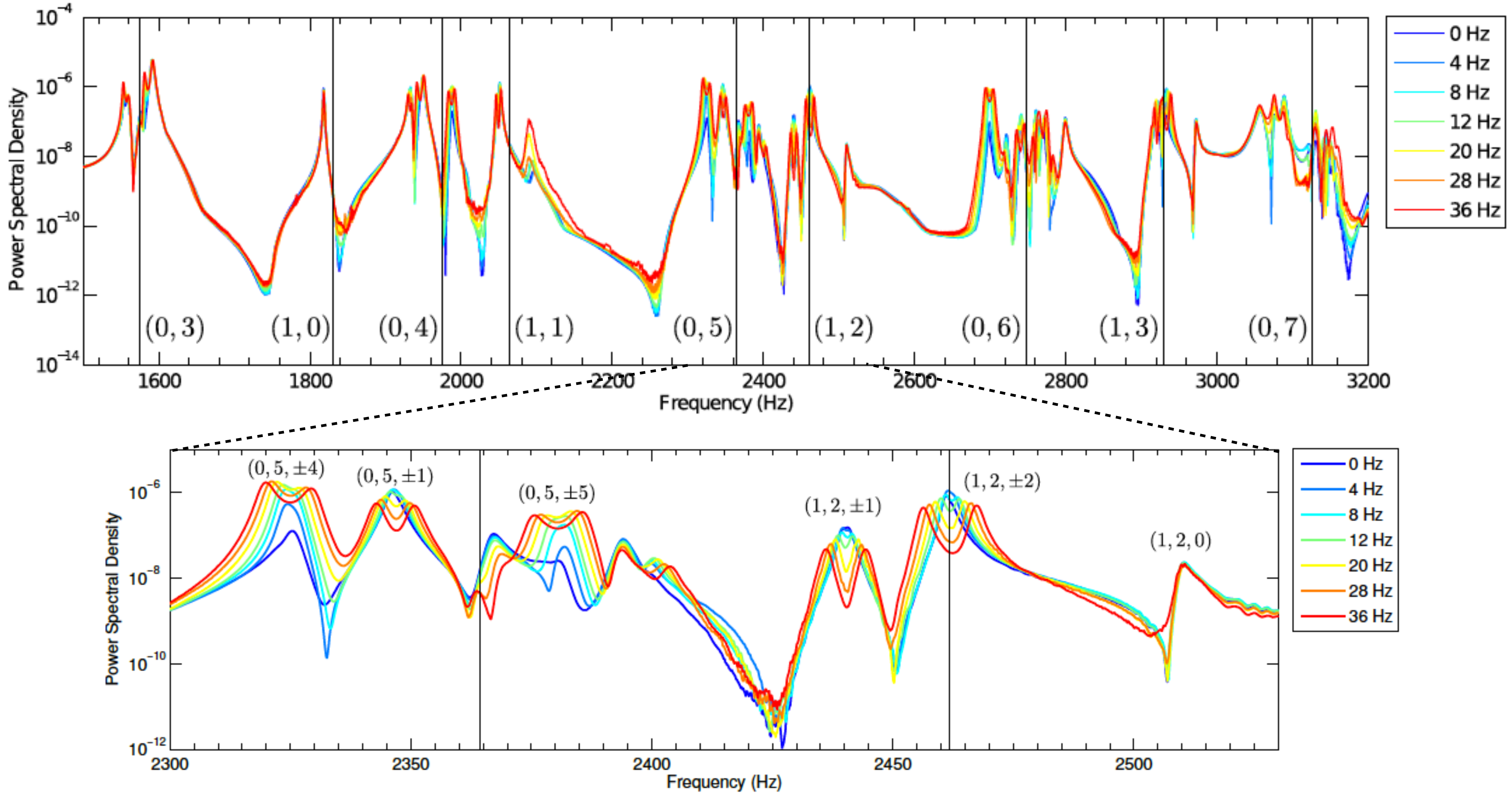
Triana et al, 2014

- The set-up of the experiment, performed at the University of Maryland, is displayed here. The outer shell is at rest. The fluid (**air**) is entrained by **spinning the inner sphere** at rotation rates up to 36 Hz.

- A loudspeaker excites waves at sweeping frequencies. Microphones record the waves and their resonances.

- The following slide shows the **typical frequency spectra** we record.

Frequency spectra modified by the flow



Triana et al, 2014

Theory of rotational splitting of acoustic normal modes

- In this simple geometry, and assuming homogeneous and constant physical parameters, pressure obeys **Helmholtz equation**:

$$(\nabla^2 + k^2)p = 0$$

where $p = p(r,\theta,\phi,\omega)$ is the acoustic pressure in the frequency domain.

- This comes from combining the momentum equation and the thermodynamics relation between pressure and acoustic velocity $\mathbf{u}$.

$$\nabla p = -\rho\frac{\partial \mathbf{u}}{\partial t} = -i\rho\omega\mathbf{u} \qquad \nabla \cdot \mathbf{u} = -\frac{1}{\rho c^2}\frac{\partial p}{\partial t}$$

where c is the sound speed, yielding the wavenumber $k = \omega/c$.

Theory of rotational splitting of acoustic normal modes

- The solutions of the Helmholtz equation are the product of an *R(r)* function by spherical harmonics $Y_l^m(\theta, \varphi)$, yielding:

$$p(r, \theta, \varphi, \omega) = R(r) Y_l^m(\theta, \varphi) e^{i\omega t}$$

- The radial function satisfies the equation: $\left[\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} + k^2 - \frac{l(l+1)}{r^2} \right] R(r) = 0$

whose solutions are combinaisons of spherical Bessel functions $j_l(kr)$ of the first kind and $y_l(kr)$ of the second kind: $R(r) = a_{nl} j_l(kr) + b_{nl} y_l(kr)$

where n is an index describing the number of zeroes of the radial function, the a_{nl}/b_{nl} ratio being determined by the boundary conditions at the inner and outer boundary (vanishing radial velocity).

Theory of rotational splitting of acoustic normal modes

- This determines the frequency f_{nl} of each mode defined by its three indices: *n, l, m*. For a **spherically symmetric** fluid and container, the **frequency does not depend upon the order *m***: the mode is **degenerate**.

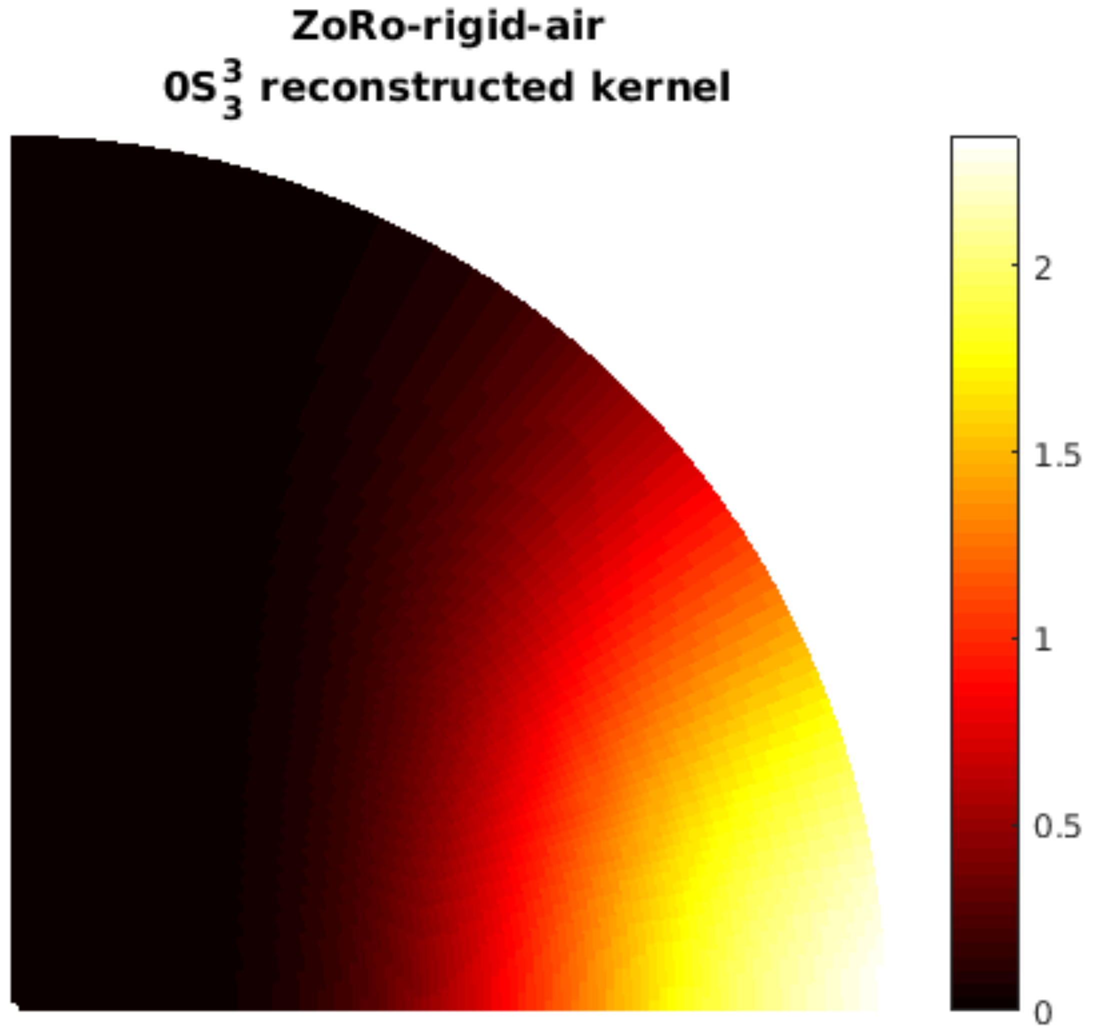
- The *m*-degeneracy is **lifted** by **global rotation** (through the Coriolis force), **differential fluid rotation**, and **ellipticity**, in particular. When these effects are small, a **linear perturbation** of the spherically symmetric modes provides the **frequency splitting**.

- The *m*-splitting due to a differential fluid rotation $\Omega(r,\theta)$ can thus be written:

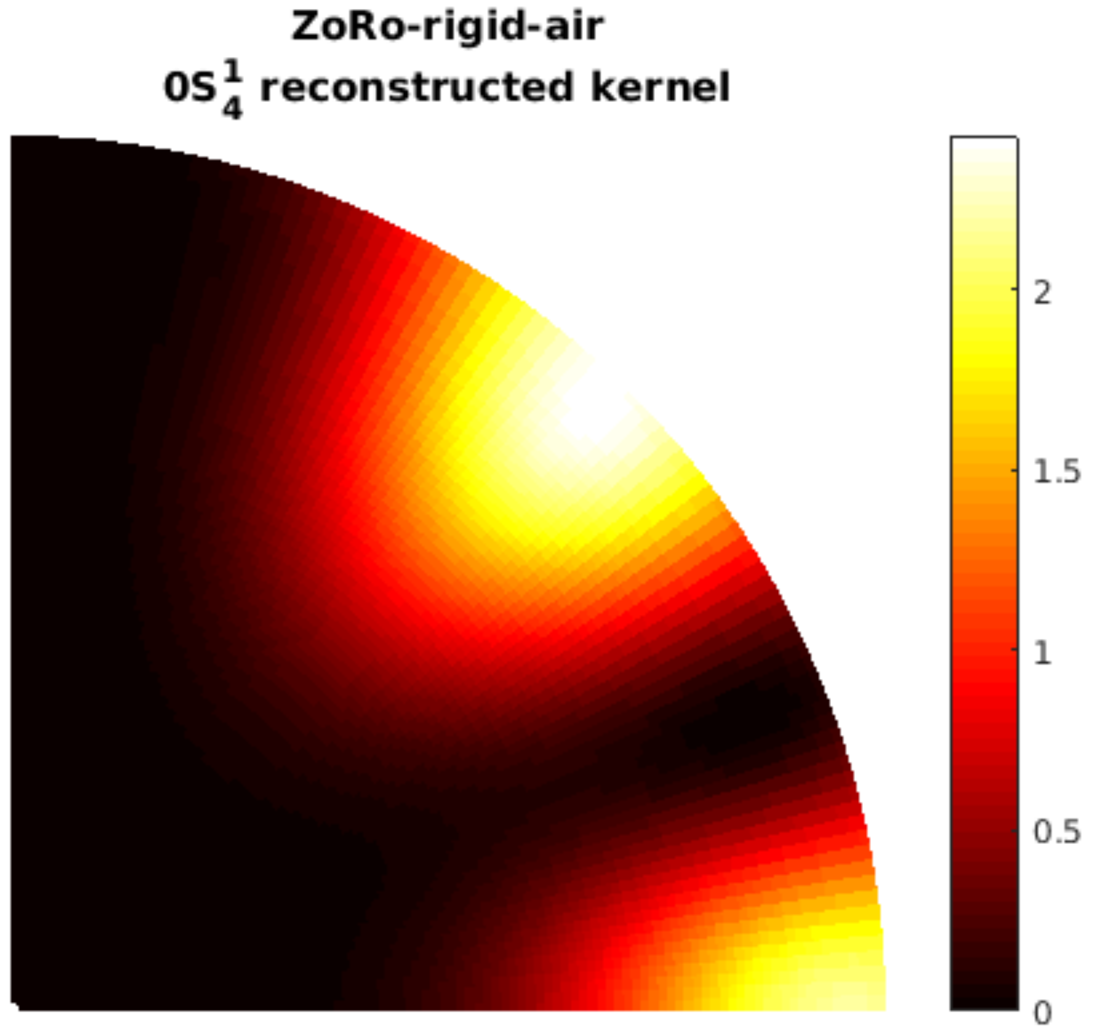
$$\delta_{nl}^{m} = m \int_{r_i}^{r_o} \int_{0}^{\pi} K_{nl}^{m}(r,\theta)\Omega(r,\theta)\, r\, dr\, d\theta$$

Rotational splitting kernels of acoustic normal modes

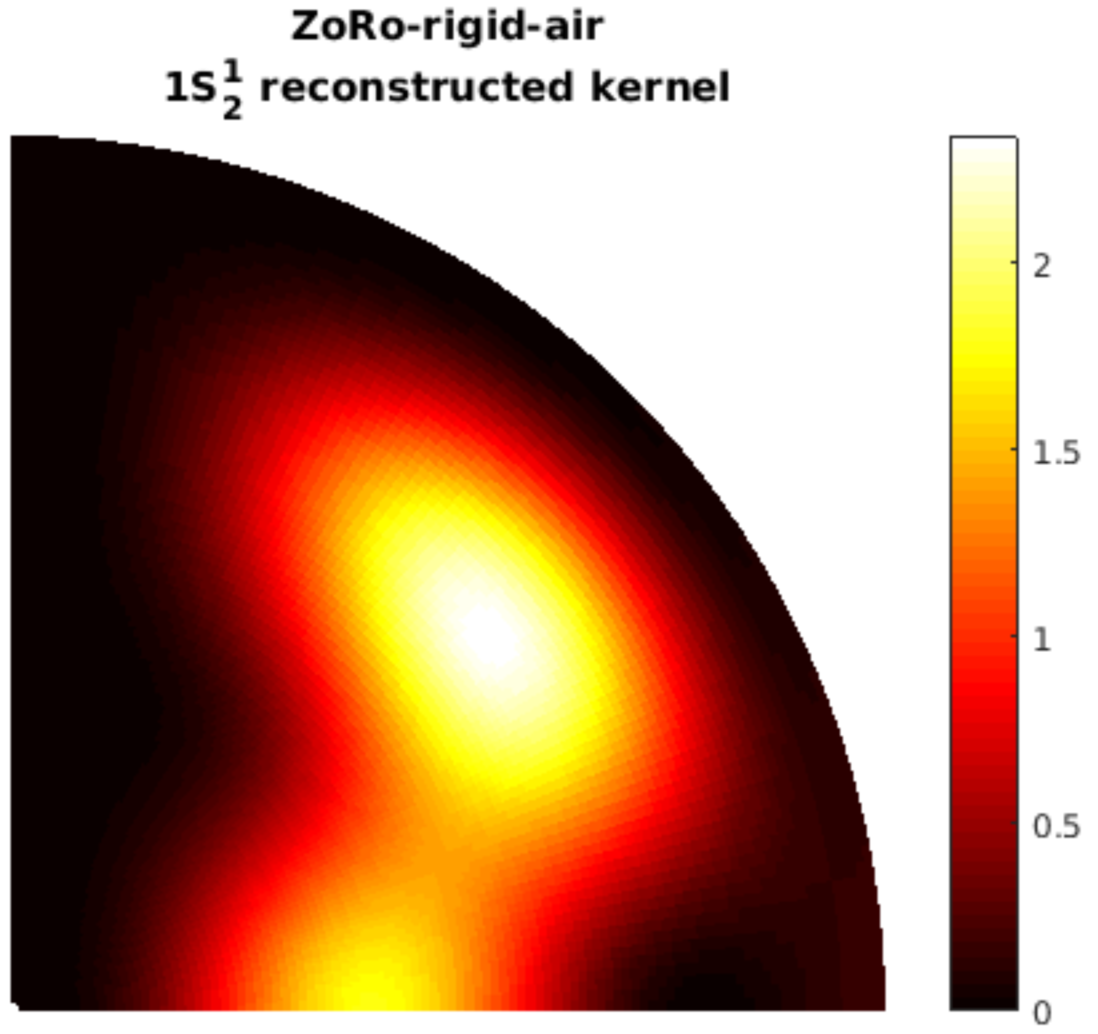
- The kernel *K* is calculated from the displacement functions of the unperturbed mode. Here are a few kernels for a full sphere:


ZoRo-rigid-air
$0S_3^3$ reconstructed kernel

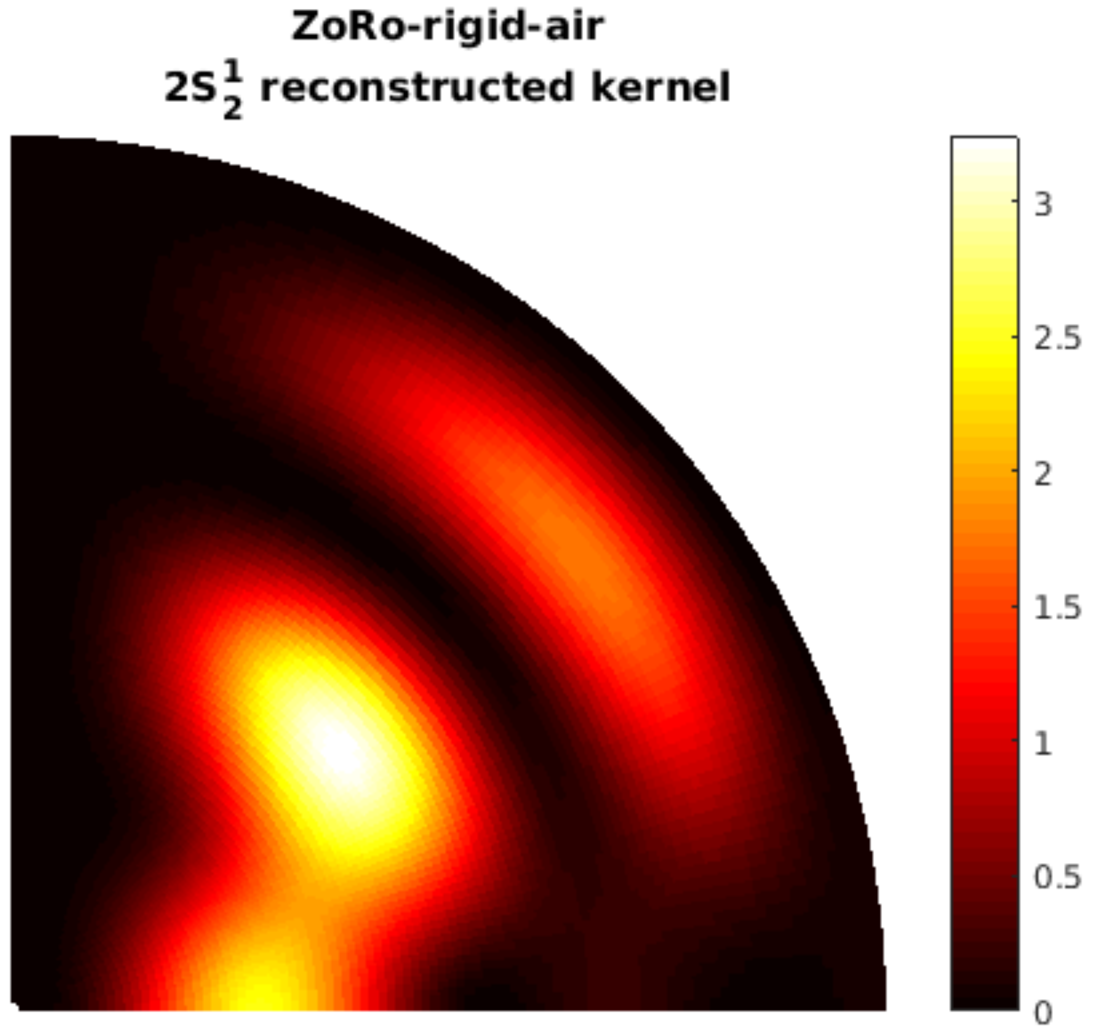
$${}_0S_3^3$$


ZoRo-rigid-air
$0S_4^1$ reconstructed kernel

$${}_0S_4^1$$


ZoRo-rigid-air
$1S_2^1$ reconstructed kernel

$${}_1S_2^1$$

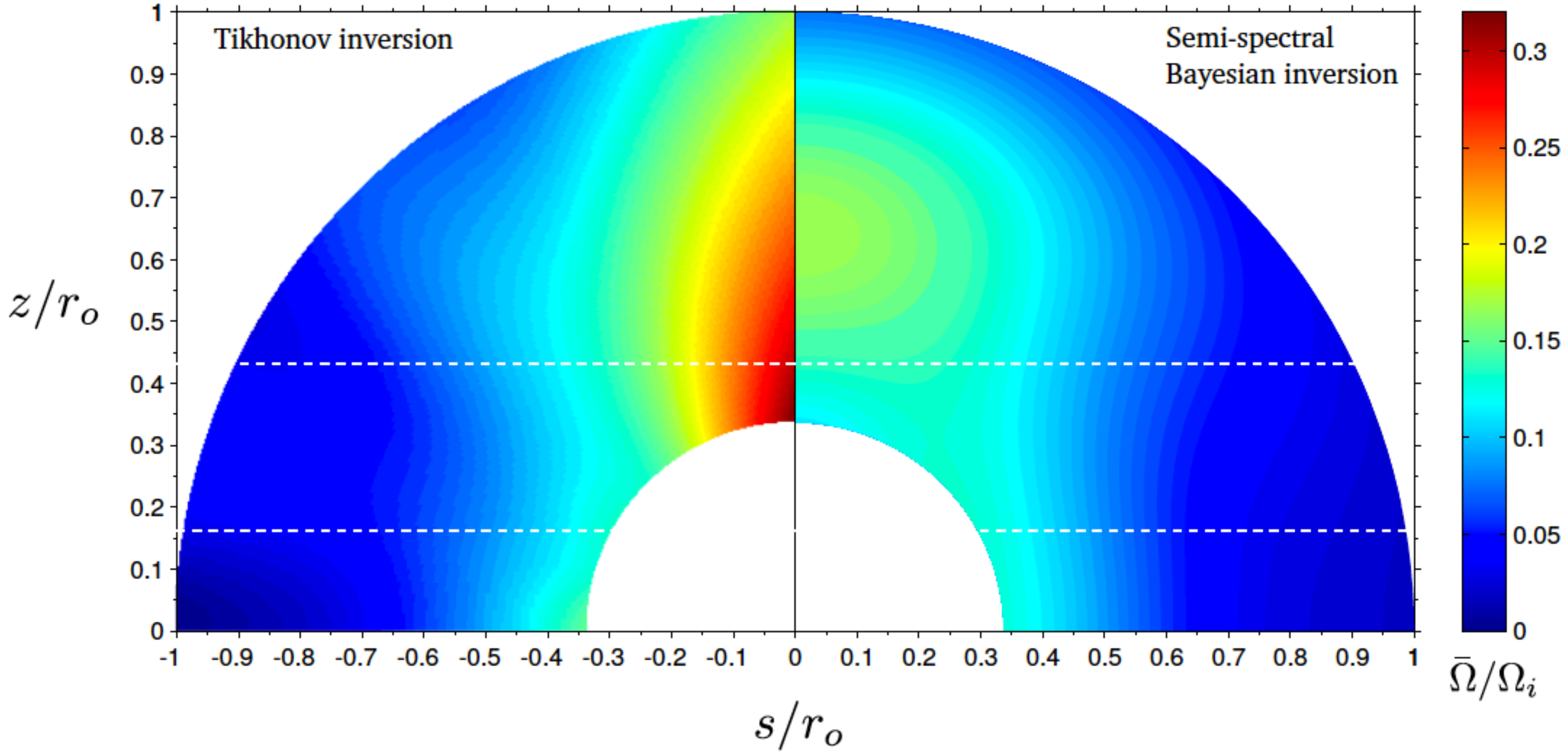

ZoRo-rigid-air
$2S_2^1$ reconstructed kernel

$${}_2S_2^1$$

Inversion

- We now have all the ingredients to try and **invert** the measurements to obtain models of the fluid angular velocity.

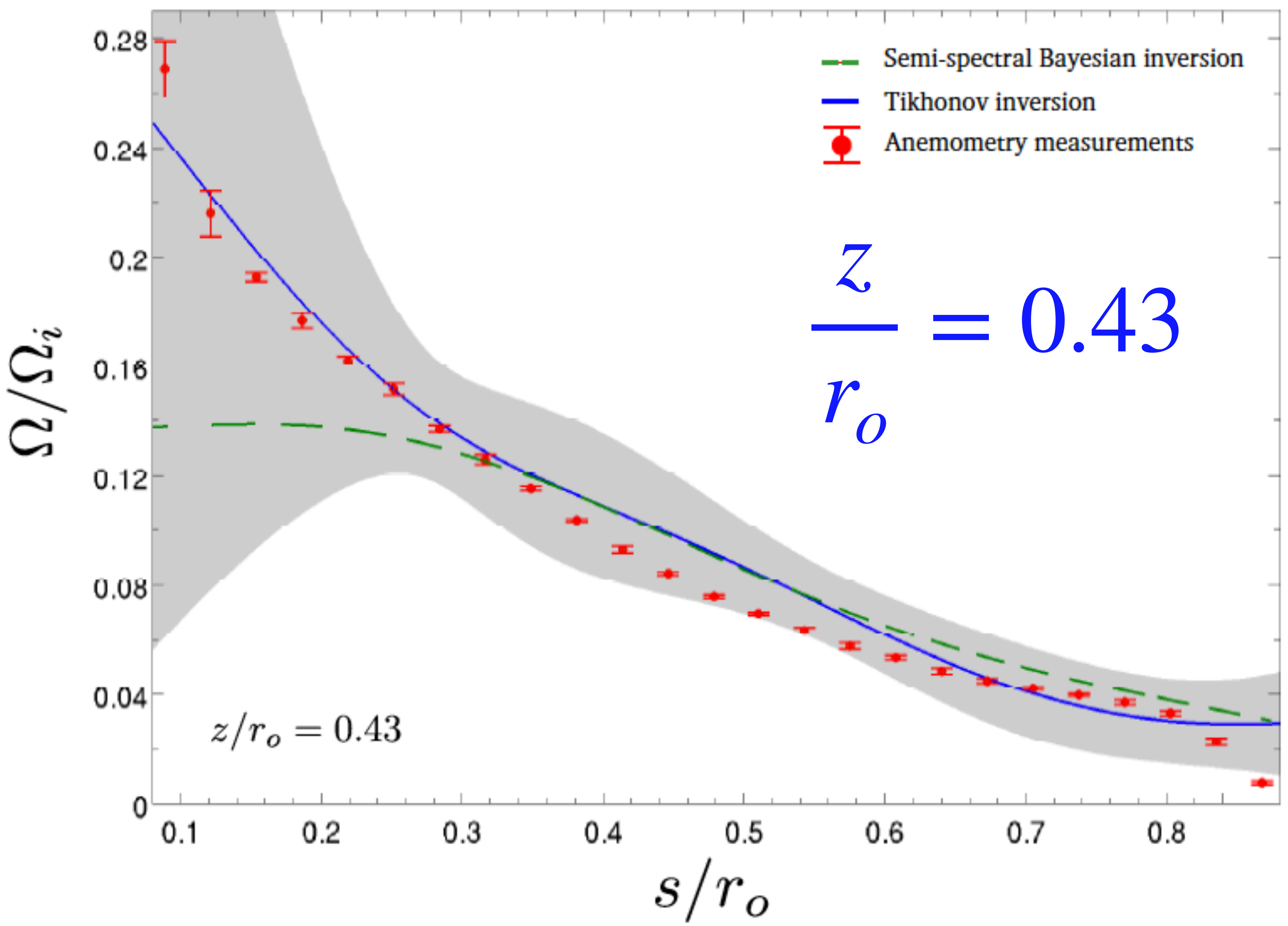
Two inversion models for angular velocity



Meridional maps of the angular velocity from the inversion of the measured splitting of **26 *(n, l, ±m)* acoustic normal modes**, by **two different inversion** methods (Tikhonov *vs* semi-spectral Bayesian).

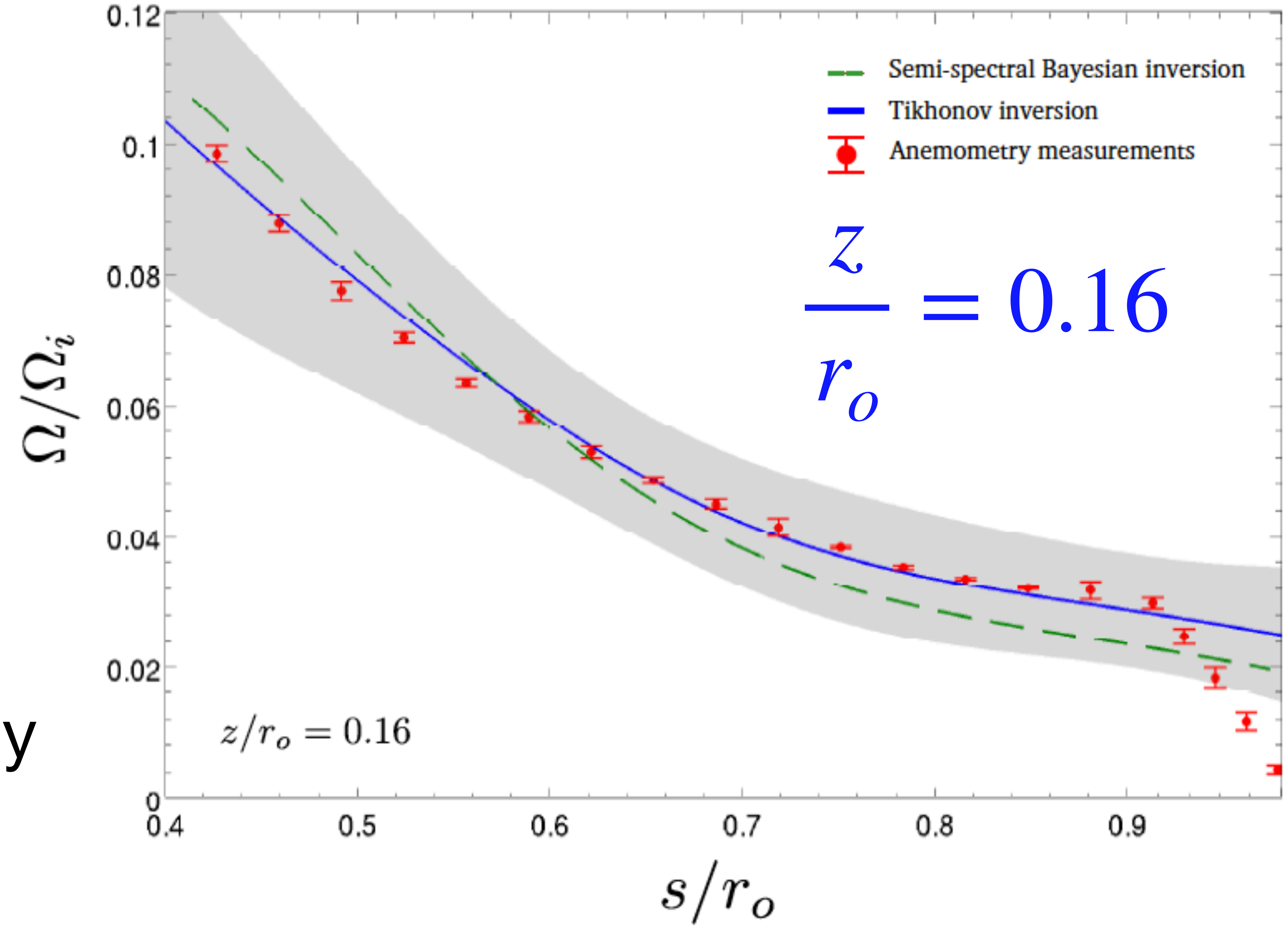
Triana et al, 2014

Comparison with direct anemometry measurements



$$\frac{z}{r_o} = 0.43$$

The inverted angular velocity models **match well** direct measurements obtained with an **anemometric** probe…

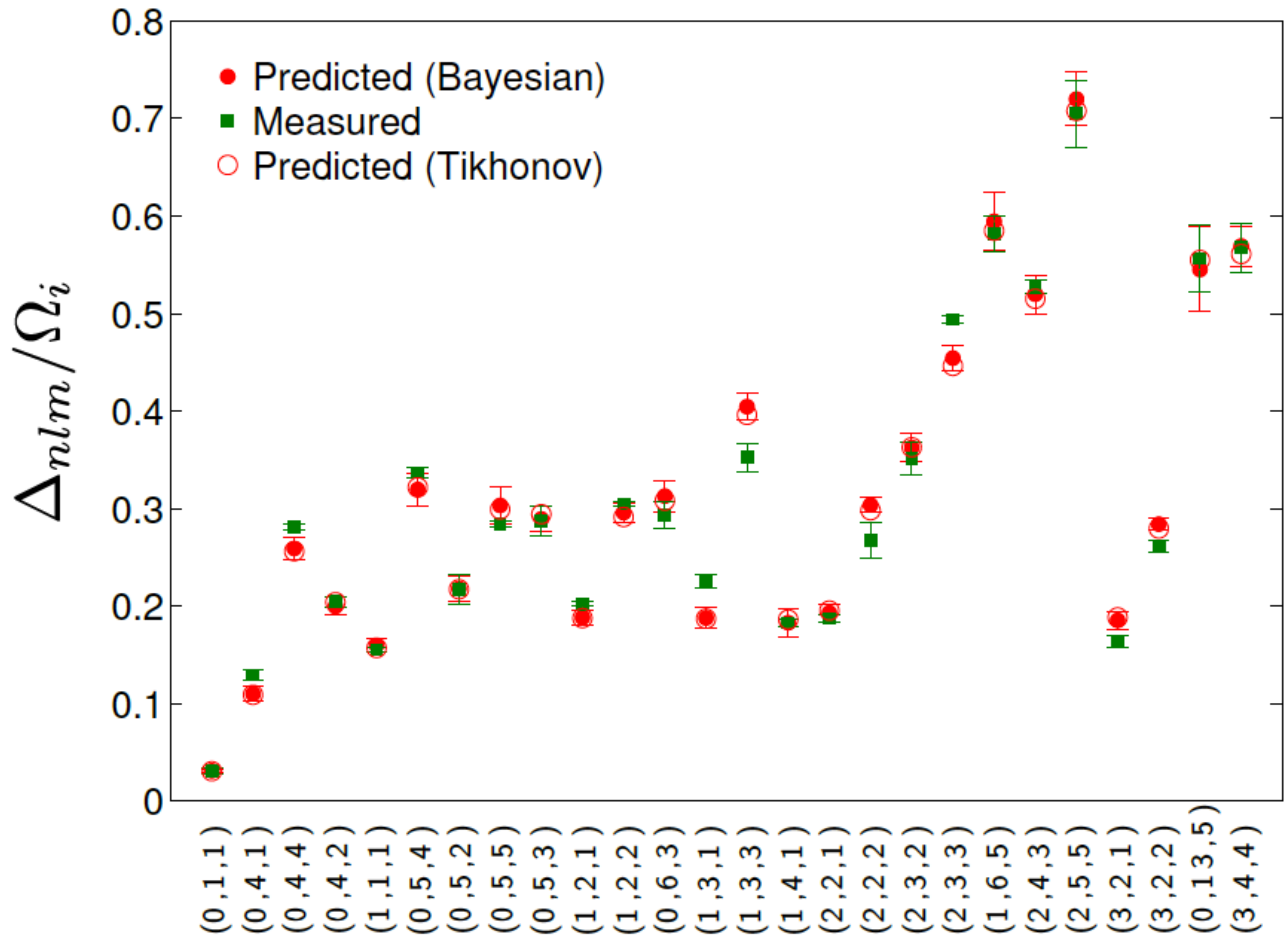


$$\frac{z}{r_o} = 0.16$$

…except in regions in which acoustic modes **lack resolution**, as indicated by the large model error (grey).

Triana et al, 2014

Fit of the measured splittings



- Most of the splittings of the 26 identified *(n, l, ±m)* modes are **fit** within their error bars **by both models**.
- Despite their differences, the two models yield almost the same splittings, illustrating **resolution issues**.

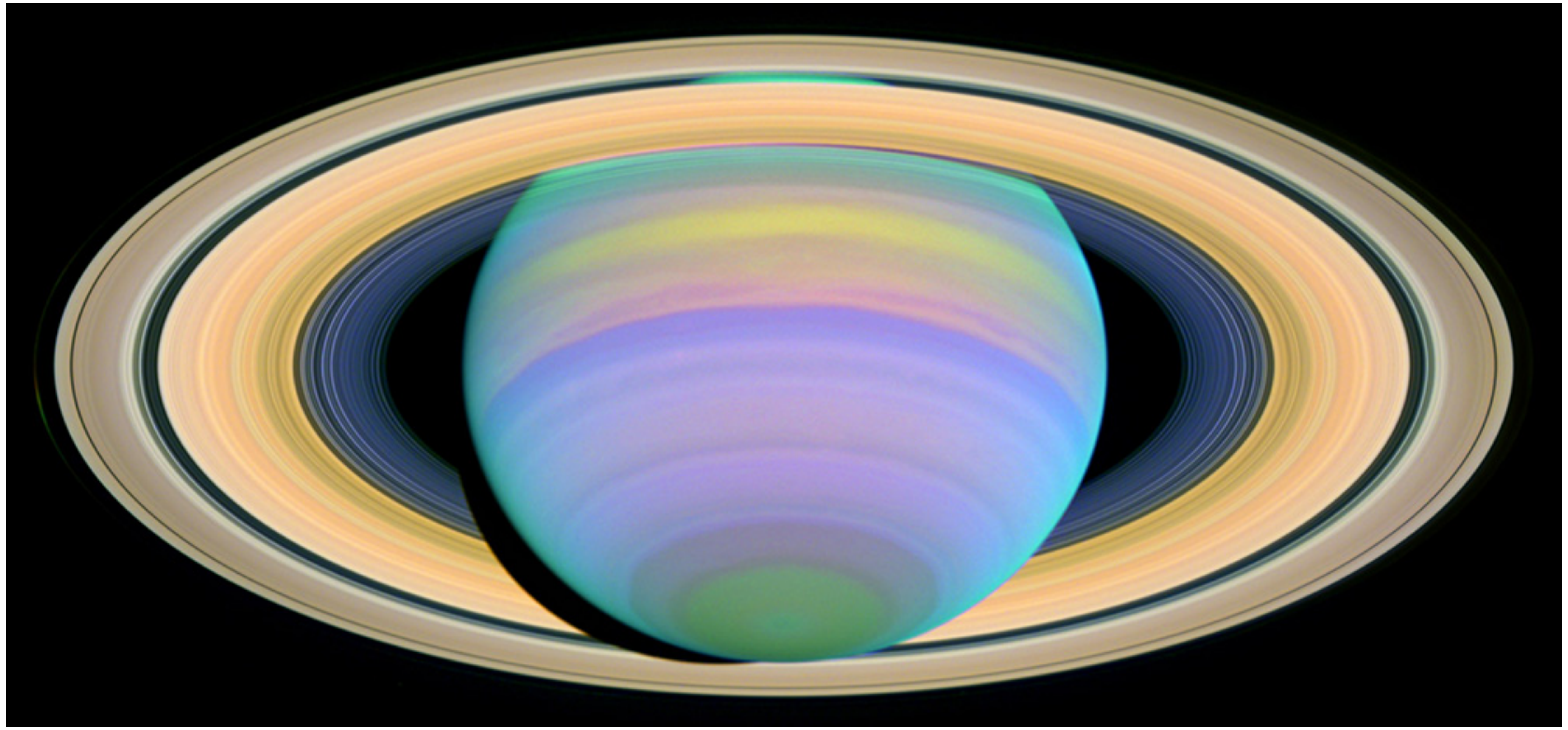
Triana et al, 2014

Advantages of the 'modal acoustic velocimetry' method

- The ***modal acoustic velocimetry*** method can be used in opaque fluids (such as sodium).
- In contrast to other velocimetry techniques, it does not rely on scattering (of light or sound) by **particles** entrained by the fluid. The use of such particles is problematic in rapidly rotating experiments because they tend to migrate inwards or outwards and **disappear**, especially in long-lasting experiments, because of the **centrifugal force**.
- In Grenoble, we are currently implementing this technique in the ***ZoRo*** experiment designed for the study of **zonal flows** in **rotating thermal convection** (Su et al, 2018).

Normal modes of giant planets

- Before leaving this part, I want to mention recent amazing observations: the most fundamental modes of **Jupiter** has been identified from ground observations (Gaulme et al, 2011). It is hoped that the coming JUICE mission could pursue this goal.



- Even more amazing: several normal modes of **Saturn** have been detected by the signal they imprint in the 5m-thick **C-rings** through **gravitational coupling** (Rosen, 1991; Hedman & Nicholson, 2013; Marley, 1990, 2014)!

5.2. The formation of the solar system

The classical scenario

- Until 1995, the scenario for the formation of the solar system was rather simple and well established, and was supposed to be fairly universal:
  - planets form in the disc (gaz and dust) surrounding their accreting star.
  - the temperature in the disc controls which elements can condensate to form planetesimals: refractory elements close to the star, more volatile elements farther away. Telluric planets form in the inner solar system; giant gas and ice planets form in the outer solar system.
  - all planets stay on the orbits where they formed 4.568 billion years ago.

A planet that could not exist!

- In 1995, Mayor and Queloz discovered the **first planet orbiting around a solar-type star**: *51 Pegasi b*. Its mass is at least **half Jupiter's mass**, and its **orbit is smaller than Mercury's orbit**! Such a planet could just ***not exist*** in the framework of the classical scenario.

- Note that the first exoplanet discovery is due to polish radio-astronomer Aleksander Wolszczan in 1992, around pulsar *PSR 1257*.

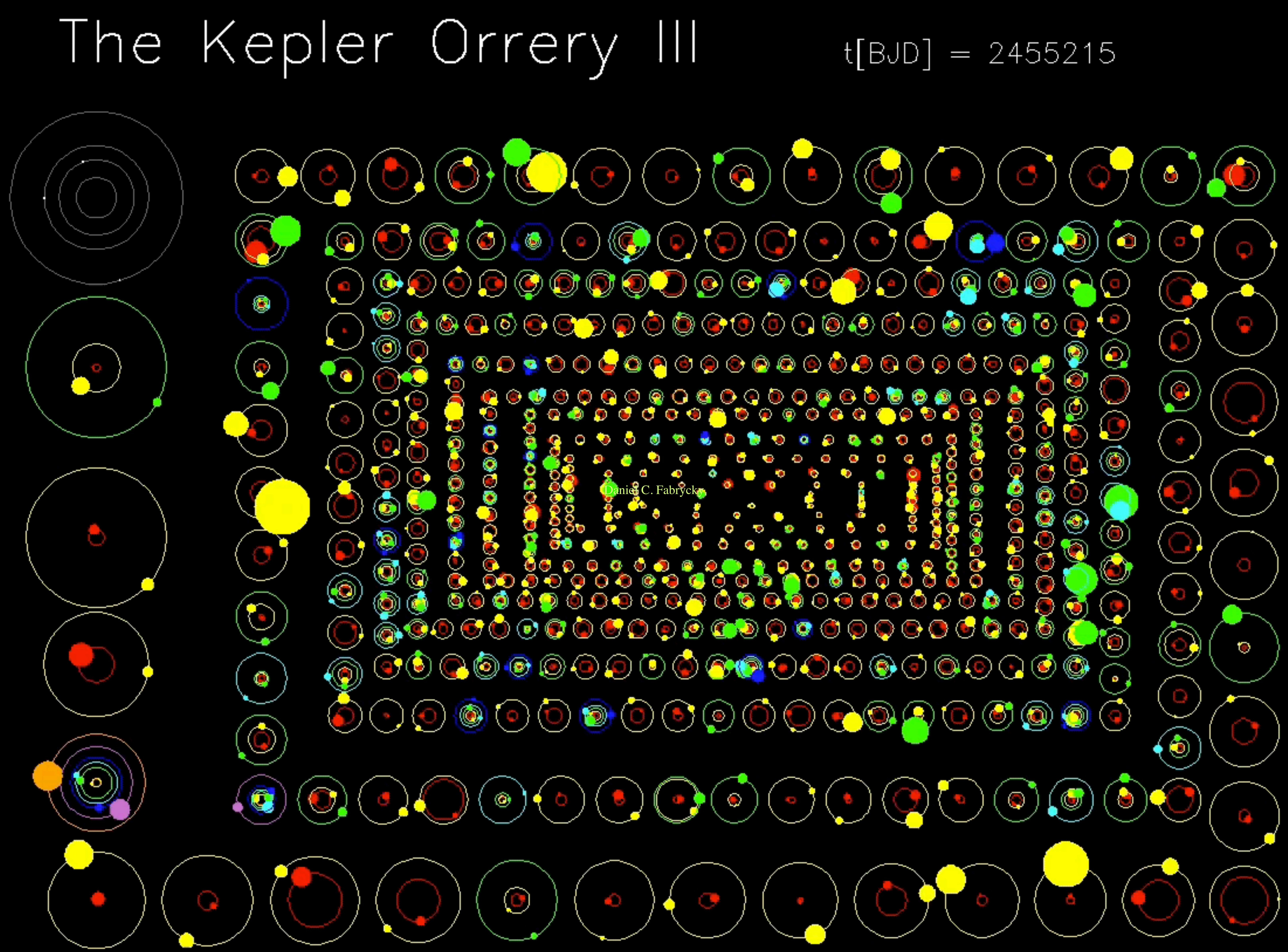
New scenarii for stellar system formation

- As more and more **‘hot Jupiter’** were discovered, new scenarii were needed and were produced. In these new scenarii, temperature of the disc does control the planets’ birthplace, but giant gas planets **migrate** inwards or outwards, transforming completely the initial plan.

- This did not happen for Jupiter in the solar system: it seems that Saturn held it back of doing so…

- Astronomers have now discovered **thousands of exoplanets**, which show an **incredible variety** of stellar systems. Let’s enjoy this ‘orrery’ created by Daniel Fabrycky from satellite Kepler’s discoveries…

Exoplanets



http://astro.uchicago.edu/~fabrycky/kepler/

© Daniel Fabrycky

Accretion discs

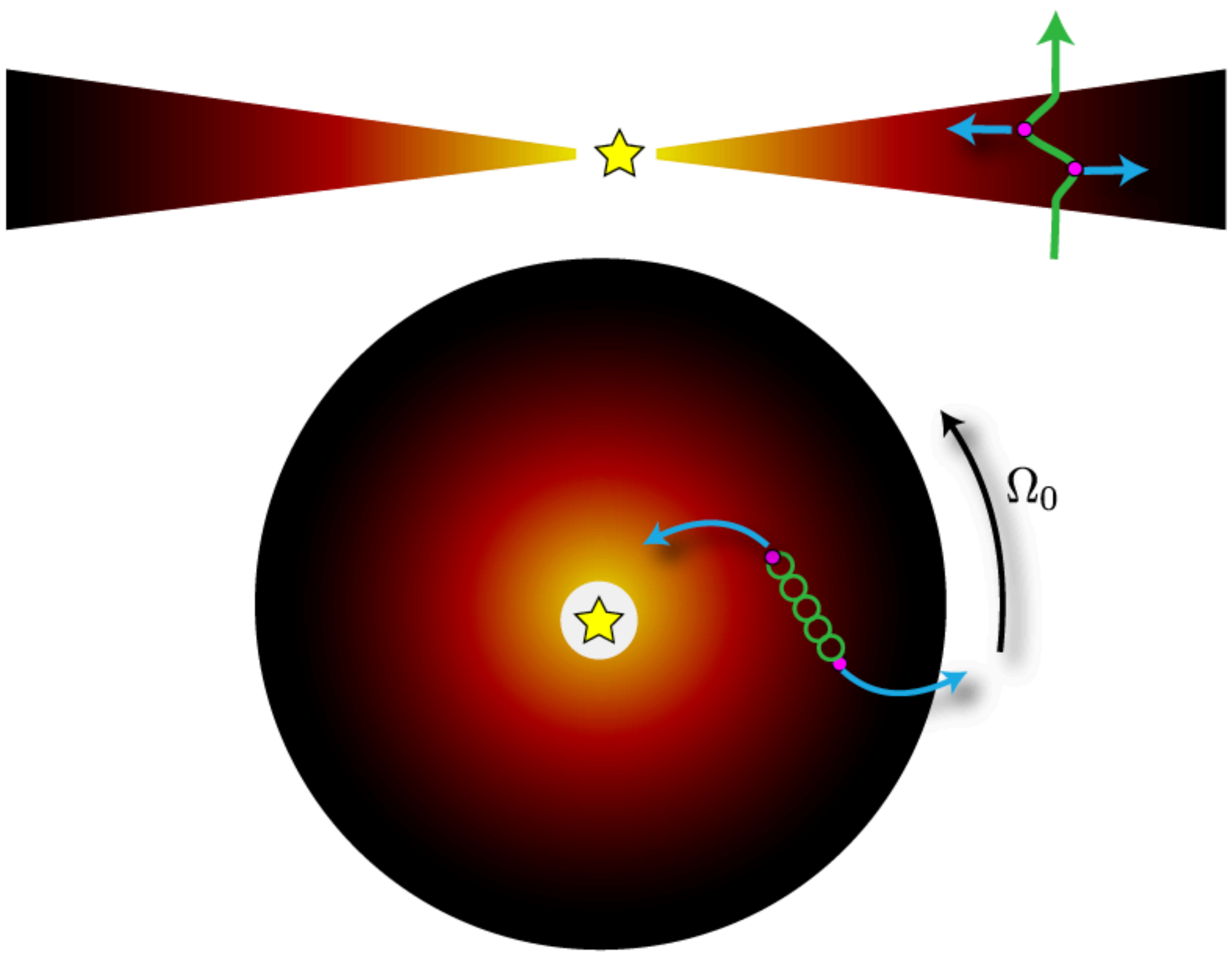
- Before planets form, there is a phase when the disc, around the accreting star, only contains gas and dust. Matter in that disc rotate in a **Keplerian** fashion ($\Omega(r) \sim r^{-3/2}$), much like in Saturn's rings today.

- This raises an important problem: how does matter fall down on the star? **Viscous friction** can do it, but observations show that it would be orders of magnitude **too slow**.

- Hydrodynamic instabilities could appear and enhance the transport. However, **no hydrodynamic** instabilities are expected in **Keplerian discs**.

- In 1991, Balbus & Hawley showed that, if a magnetic field is present , instabilities could occur through the **magneto-rotational instability** (MRI).

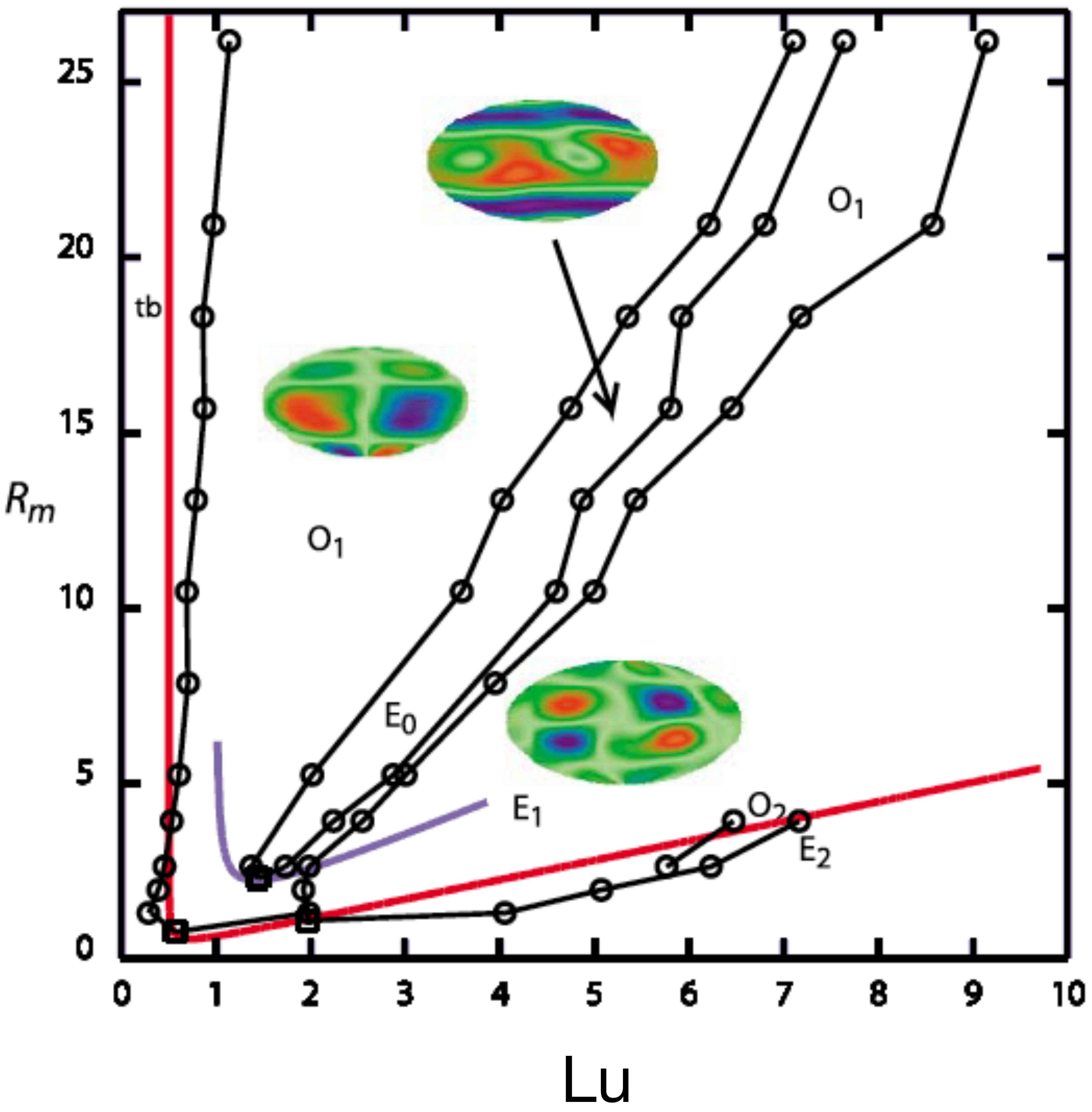
Ideal Magneto-Rotational Instability



- « Consider 2 fluid particles attached to a vertical field line and assume we slightly move these particles radially. At first, they start an epicyclic motion and drift azimuthally. As they drift away, the azimuthal magnetic tension acts as a spring bringing back the particles together, slowing down the inner particle and accelerating the outer particle. This results in a loss of angular momentum for the inner particle, which falls further down, and reversely for the outer particle. »

cited from Lesur, 2018

Experimental MRI



Sisan et al, 2004

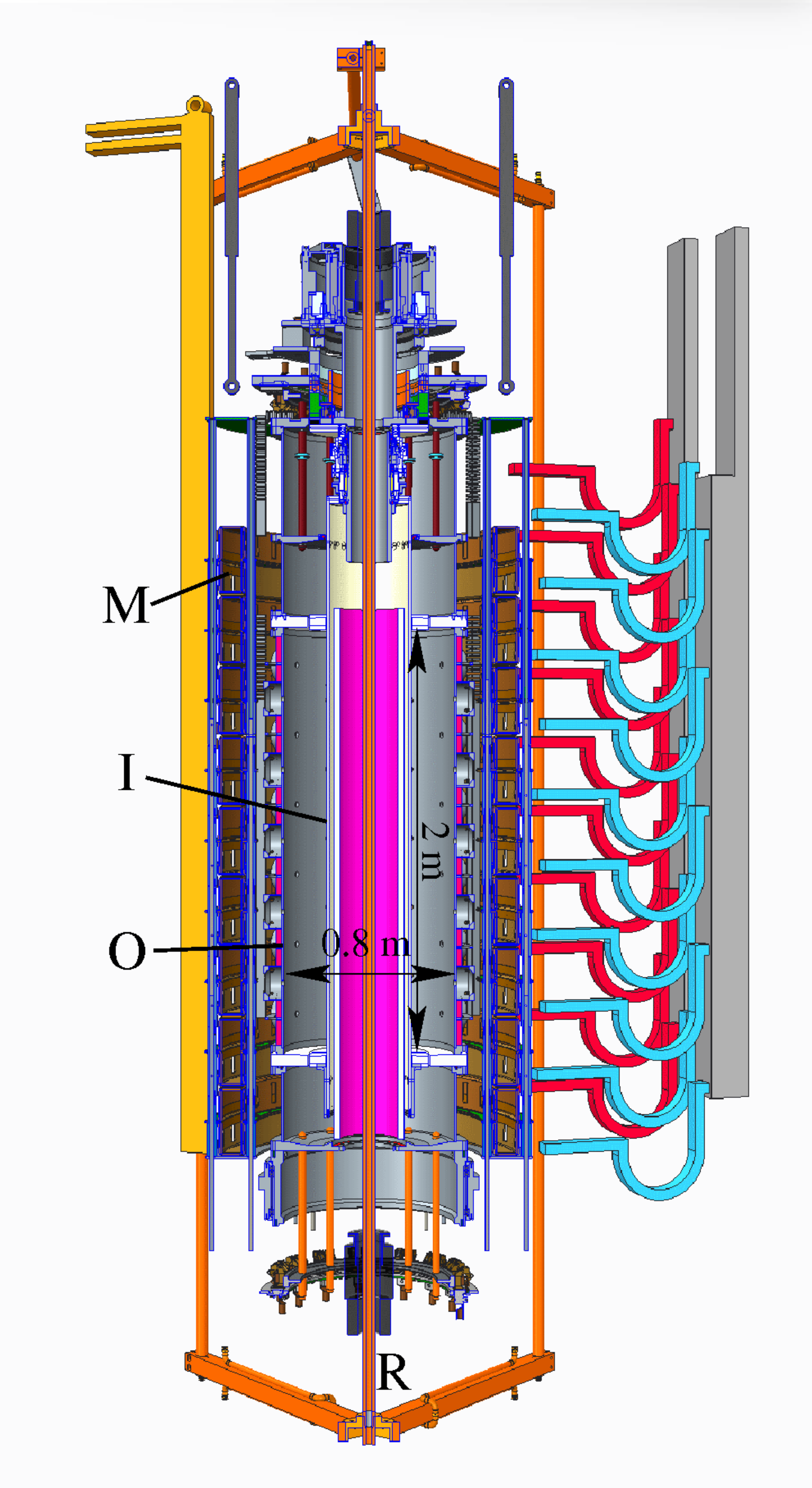
- There has been several experimental attempts at observing the MRI in the Lab, starting with the discovery of hydromagnetic instabilities in magnetized spherical Couette flow by Sisan et al (2004).

- The interpretation in terms of the MRI was debated because the base flow was already very turbulent, event though the shear profile was close to Keplerian.

Experimental MRI

- Further evidence was brought by the group of Dresden, who focused on non-ideal variants of the MRI with an added azimuthal magnetic field. The resulting MRI threshold is then governed by the hydrodynamic Reynolds number rather than by the magnetic Reynolds number.

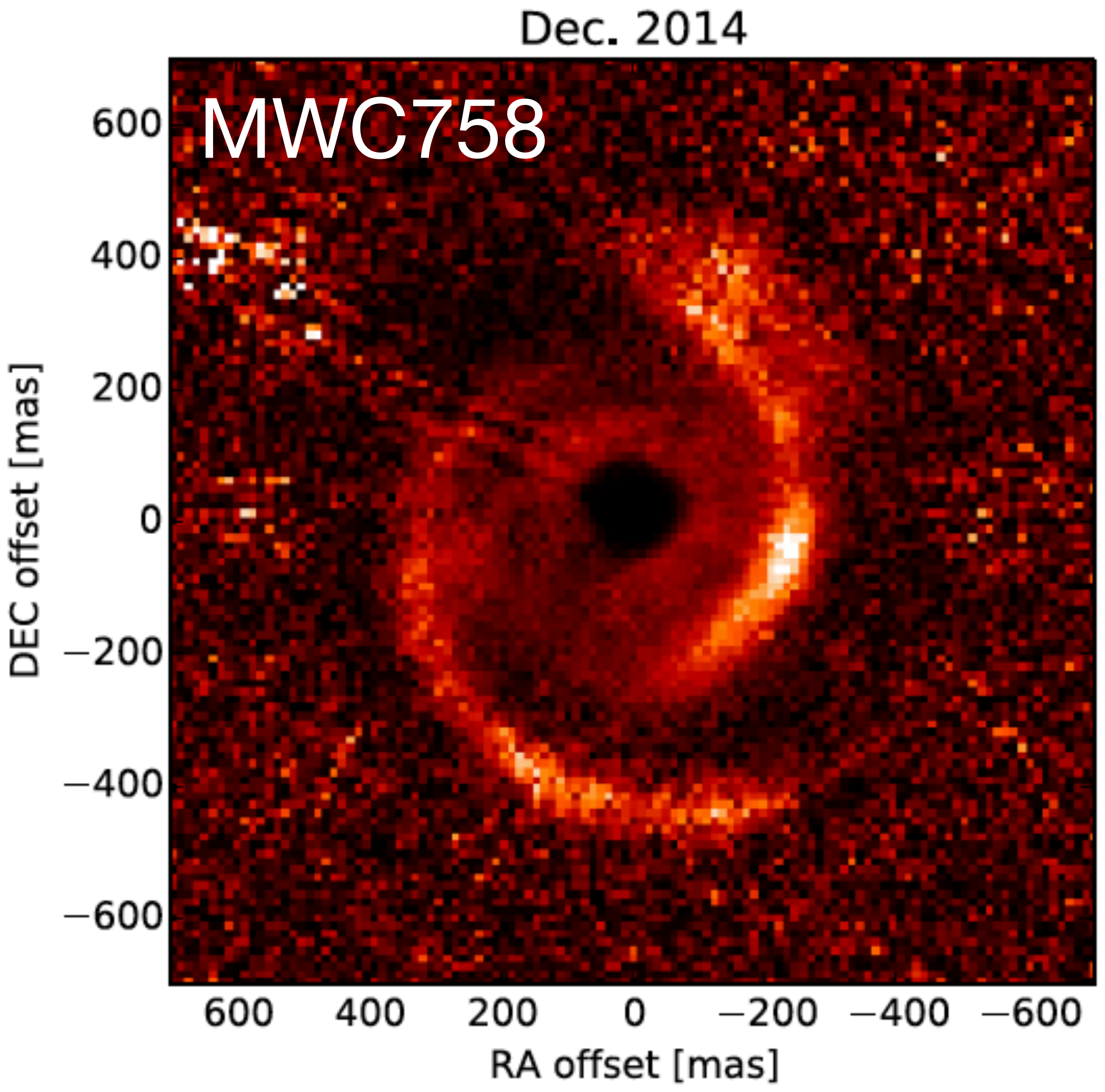
- The DRESDYN facility will host a huge liquid sodium set-up to further study this instability.



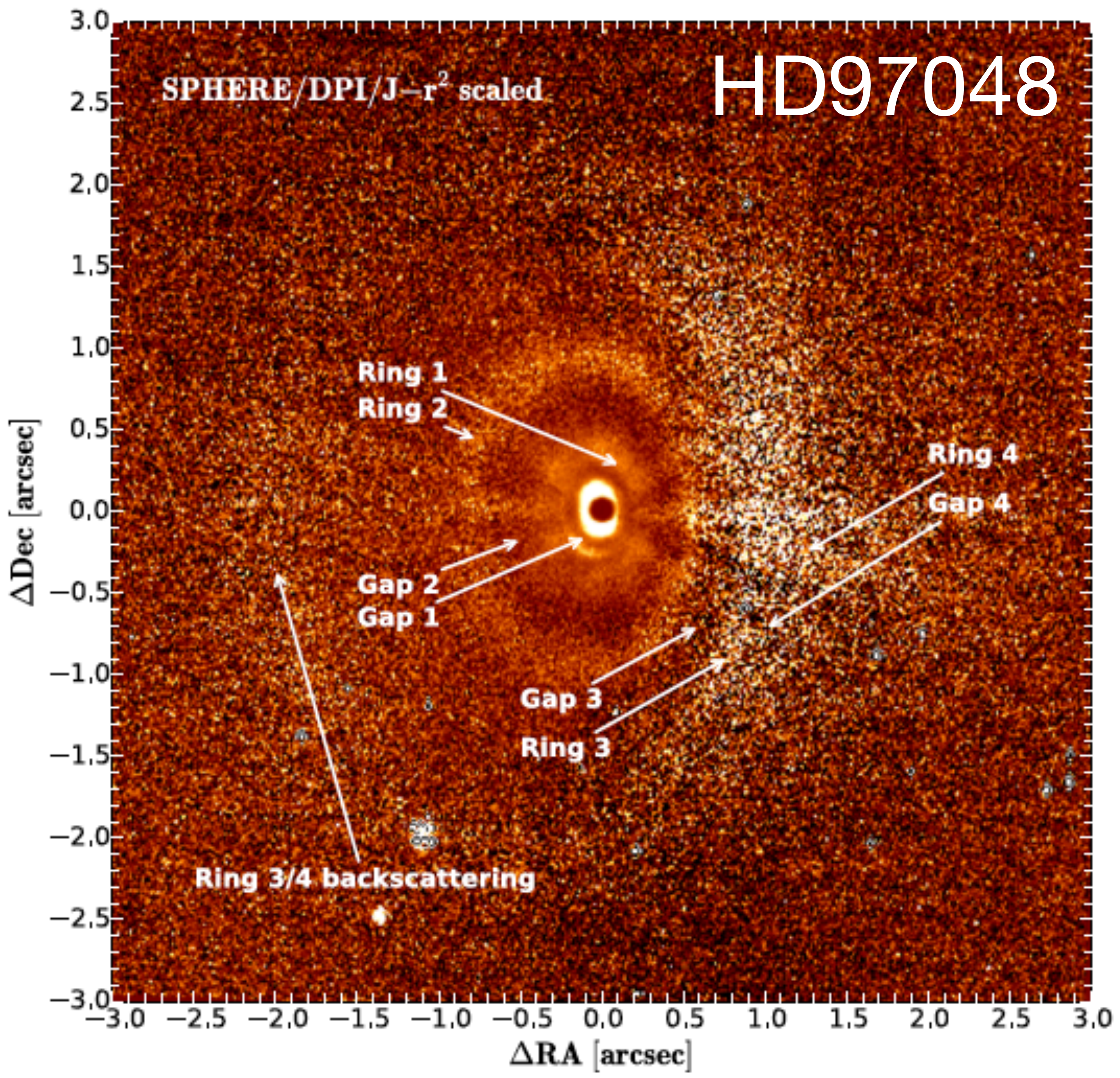
Direct images of discs

- In the mean time, new observational evidence have weakened the relevance of the MRI model, in particular the expected weakness of ionization (and hence electrical conductivity) in most of the disc, and the observations of gaps, bands and spirals in young discs.

Near infrared image using polarimetric differential imaging with SPHERE at VLT.



Benisty et al, 2015

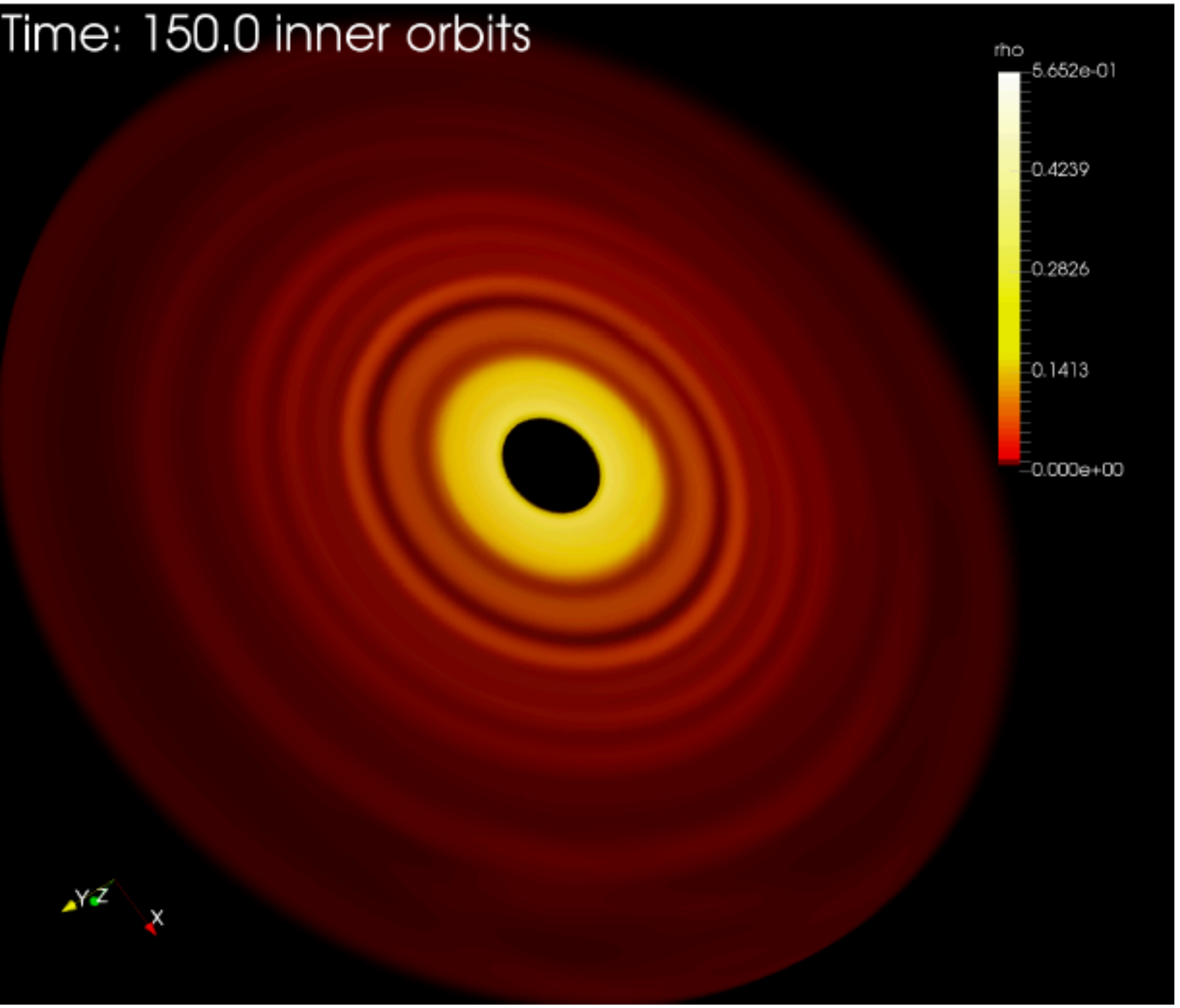


Ginski et al, 2016

Near infrared image with the extreme adaptative optic imager SPHERE at VLT.

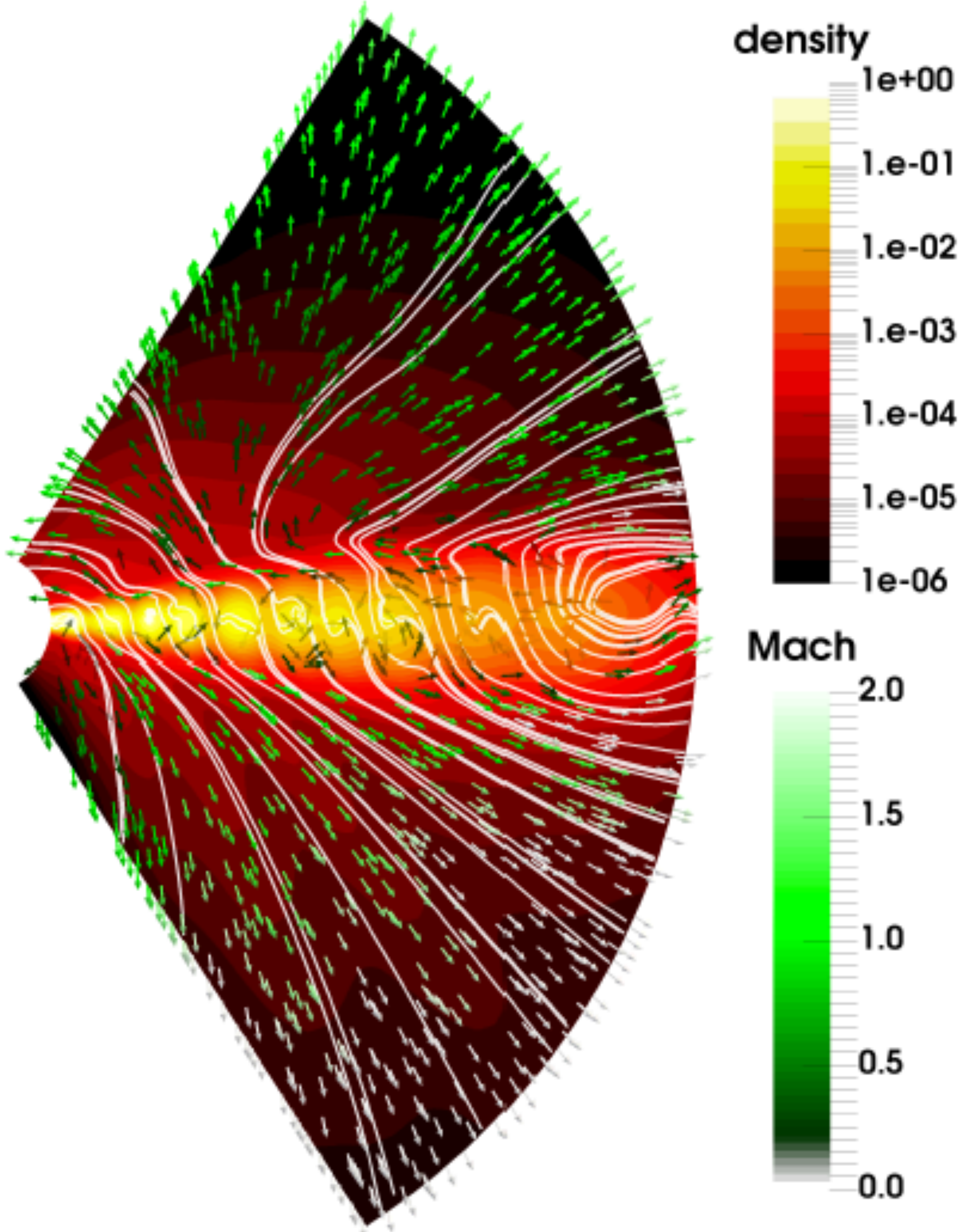
from Lesur, 2018

Thermo-magnetic dynamics in global disc models



Lesur, 2018

- Recent models incorporate more physics and show that thermal effects also play an important role in the dynamics of discs.
- They also reveal the spontaneous formation of gaps, which certainly lead to new interpretations.



Béthune et al, 2017

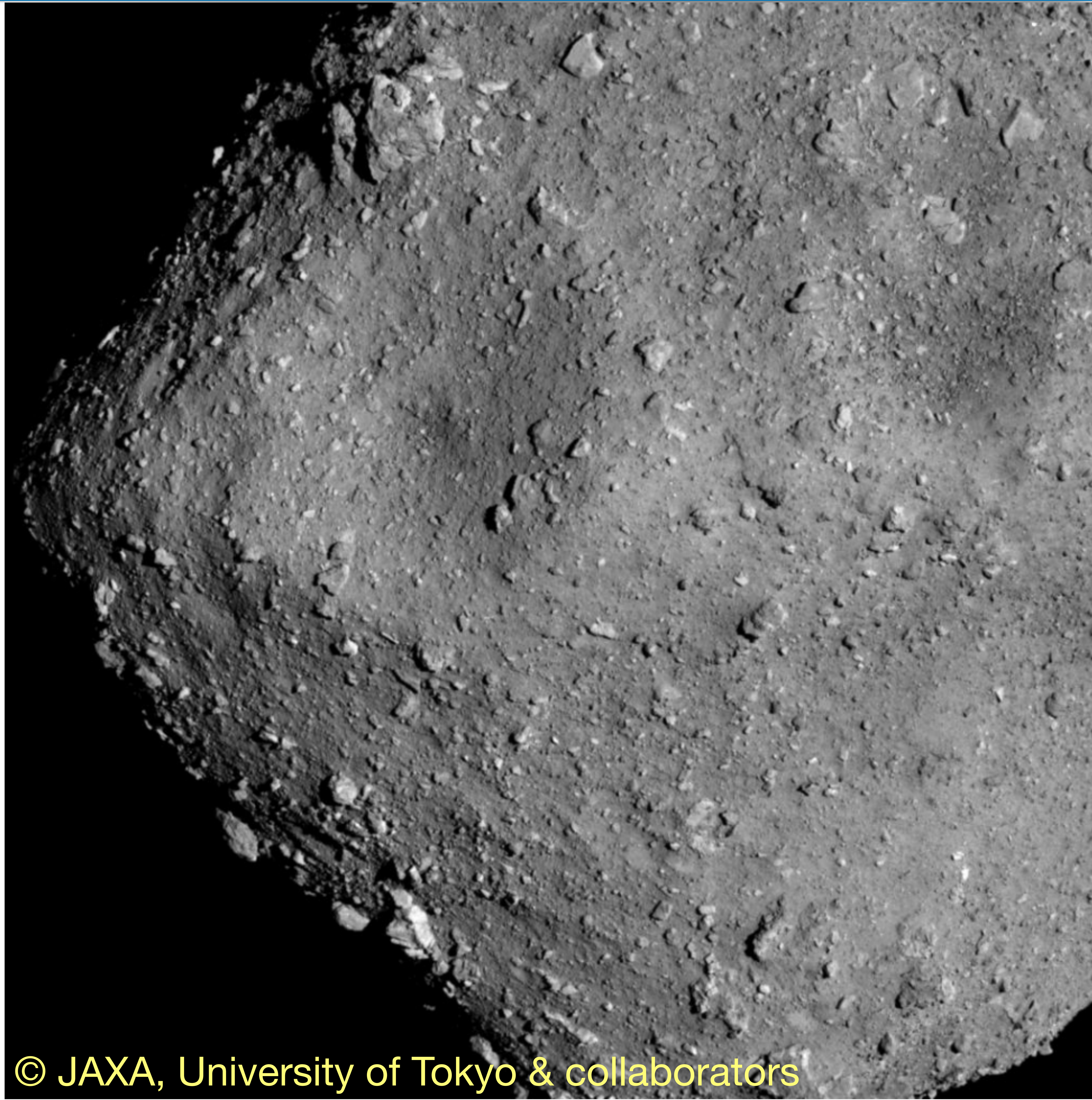
Updating views on planetary compositions?

- The MRI accretion disc idea of Balbus & Hawley (1991) **predates** the great revolution in planetary system formation.

- So does the ‘composition of the Earth’ model of McDonough & Sun (1995), which is our reference, but rests on cosmochemical arguments linked to the formation of the solar system.

Sample return space missions

- These are good reasons for trying to get more constraints from observations, with in particular the fantastic **sample return mission Hayabusa 2** which just touched down on **Ryugu**, a 900m-diameter asteroid 300 million kilometers away from us!



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5.3. The formation of the Earth

A turbulent youth!

- The formation of the Earth took place rather violently: planetesimals of all sizes impacted the growing Earth with large kinetic energies. Both were probably differentiated into an iron core and a silicate mantle. After a number of such impacts, the proto-earth's mantle was largely molten. The dense metallic part of the impactors would **splash** into this magma ocean and sink down to join the forming core of the Earth.

- A key question concerning the core composition and the present-day core-mantle interaction is: how did elements **fractionate** between core and mantle during that phase? Had core-forming diapirs time to **equilibrate** with the silicate magma ocean? All of them or only the smallest ones?

Fragmentation experiments

- We have no definite answer yet, but experiments help exploring this problem.

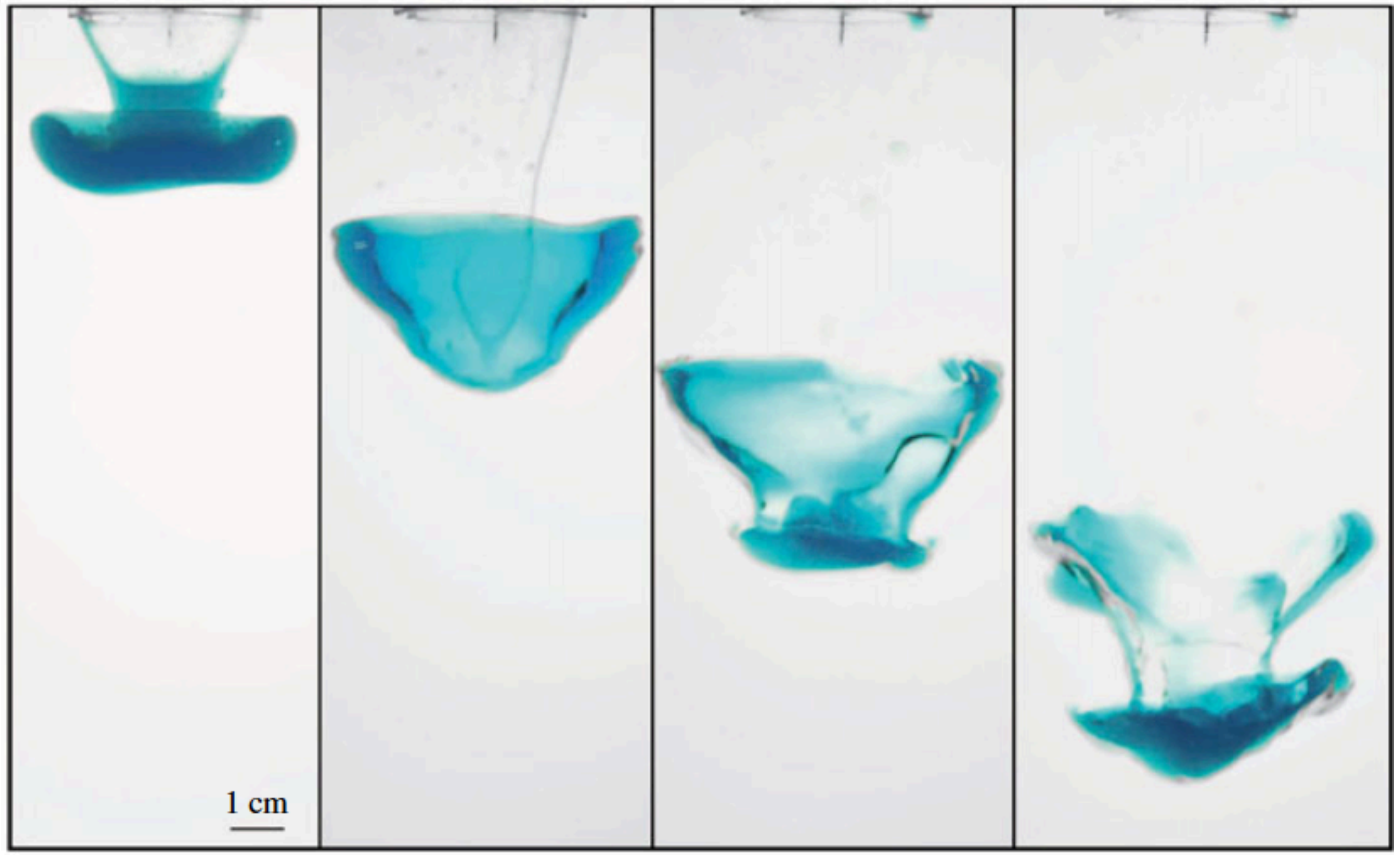
- In Landeau et al (2014), a dense liquid is suddenly released in an immiscible less dense liquid. Several different regimes are observed, yielding different fragmentation levels, hence different entrainment and equilibration.

- The main control parameter appears to be the Weber number:

$$We = \frac{\rho_r U^2 R}{\sigma} \qquad P = \frac{\rho_r - \rho_a}{\rho_a}$$

- where ρ_r and ρ_a are the density of the released and ambient liquid, respectively, U and R the diapir velocity and radius, and σ surface tension.

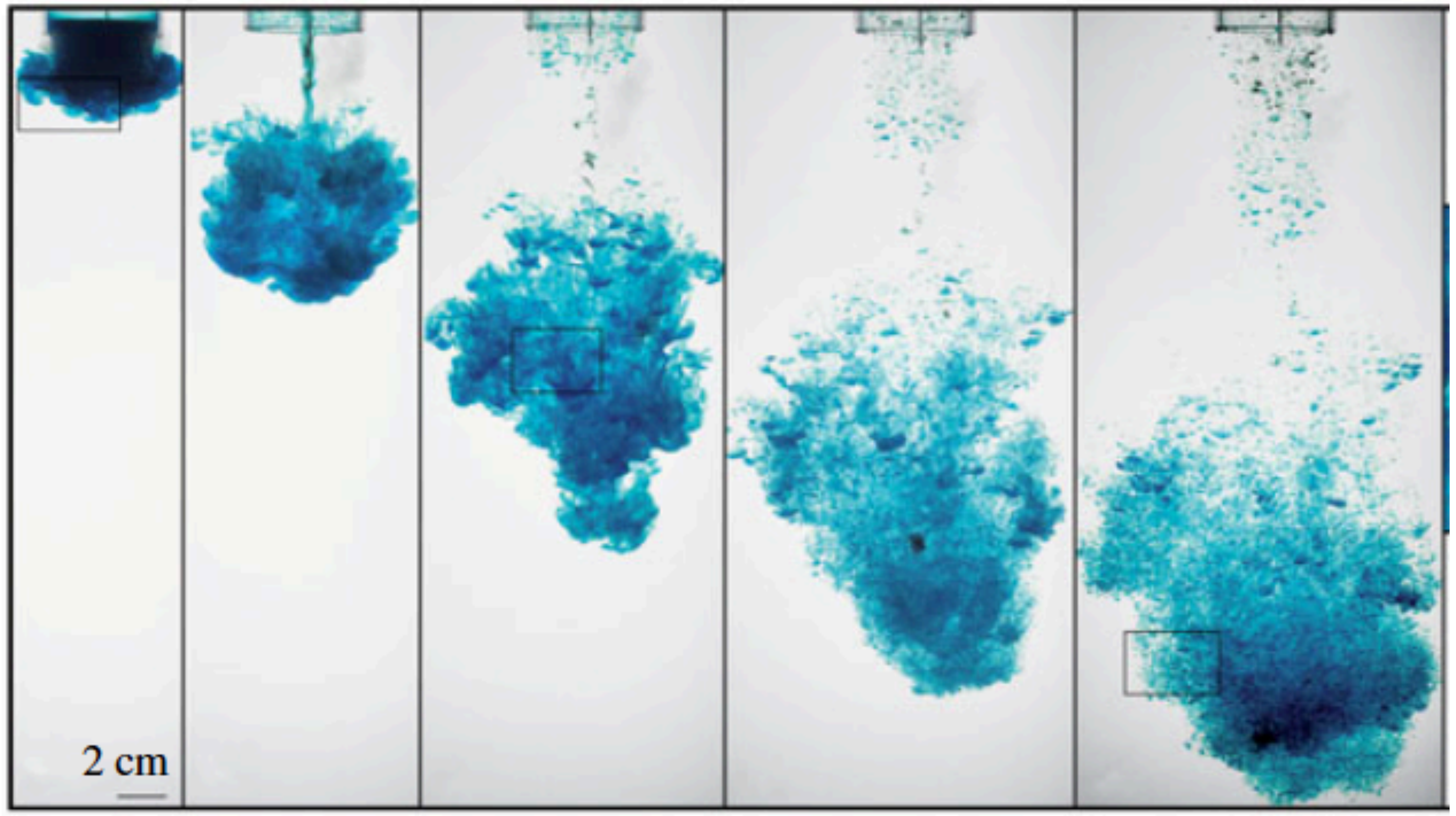
Different fragmentation regimes



$$We \simeq 24$$
$$P \simeq 0.22$$

Landeau et al, 2014

$$We \simeq 1000$$
$$P \simeq 0.92$$



Impactor liquid experiments

- More recently, Maylis Landeau added the effect of the **impact** of the released liquid. This enhances equilibration by about a factor 4. Applied to the Earth, these scalings predict **full metal-silicate equilibratio**n for impactors much **smaller** than the Earth, but **partial equilibration** for **giant impacts**.