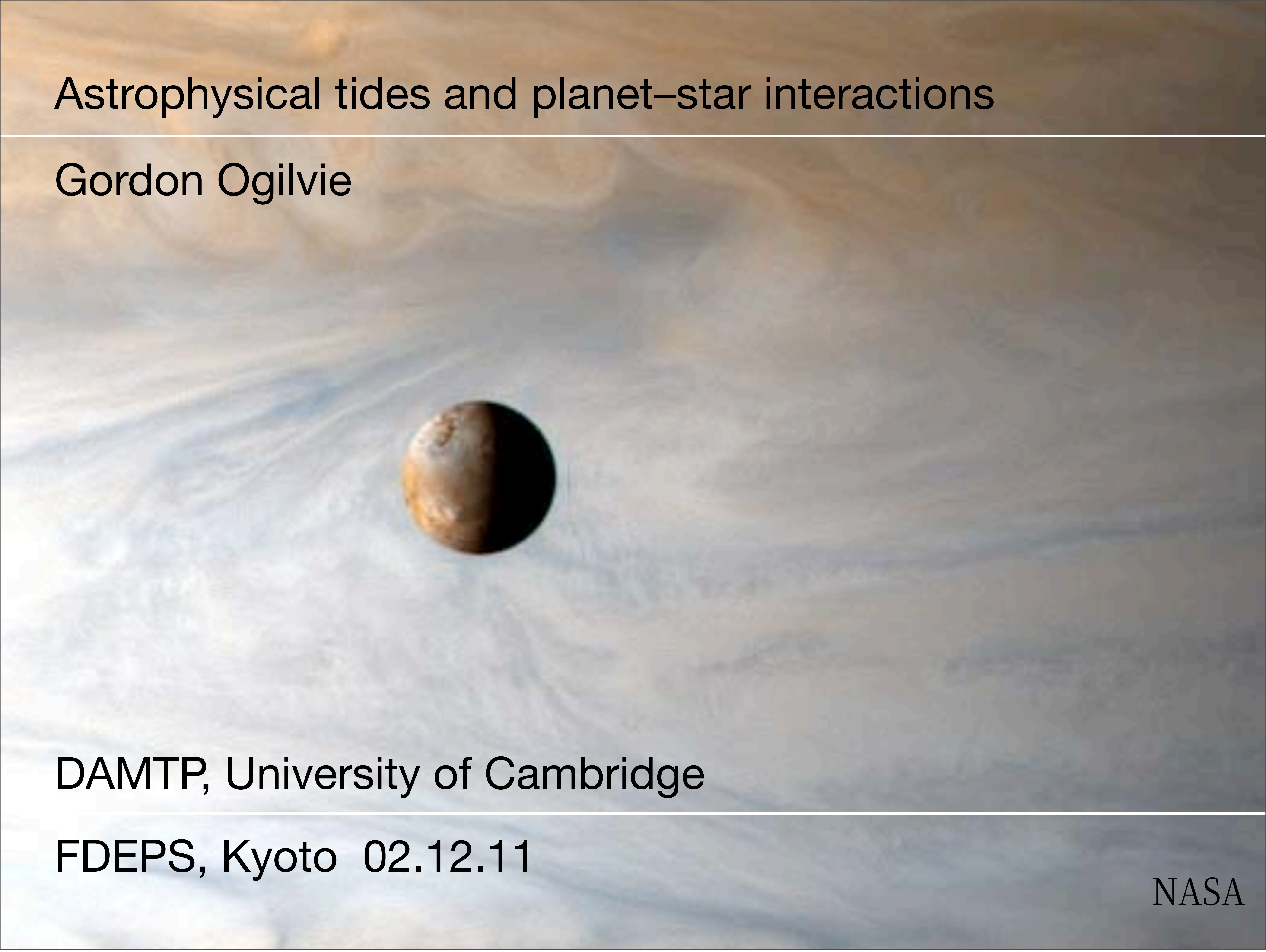
Astrophysical tides and planet–star interactions

Gordon Ogilvie

DAMTP, University of Cambridge

FDEPS, Kyoto 02.12.11



NASA

Outline

- Introduction
- Linear tides in uniformly rotating unstratified fluids

  (with Doug Lin, UC Santa Cruz / KIAA, Beijing)
- Breaking internal gravity waves at the centre of a star

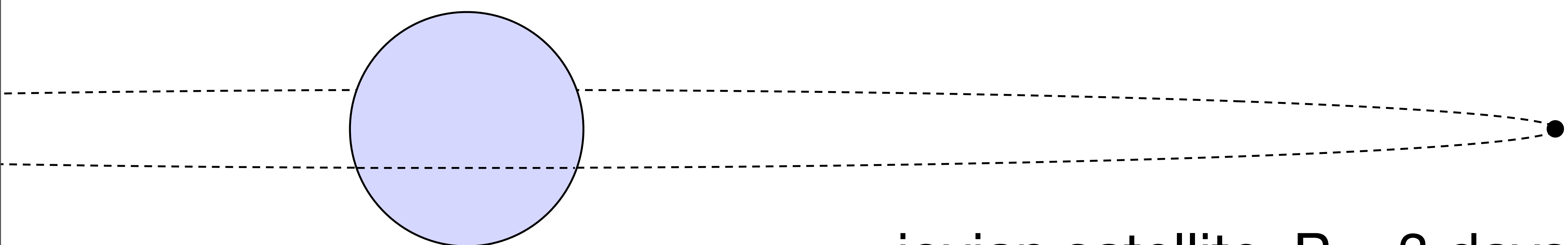
  (with Adrian Barker, DAMTP / Northwestern)
- Effective viscosity of turbulent convection and other flows

  (with Geoffroy Lesur, DAMTP / IPAG, Grenoble)
- Conclusions

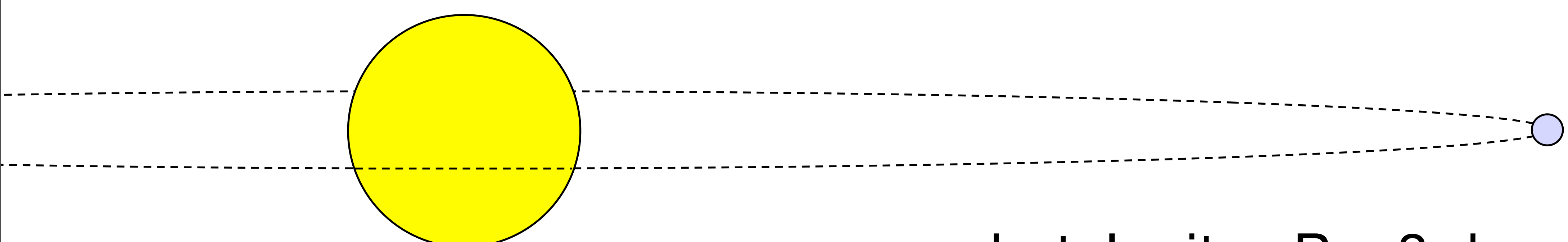
Introduction

Tidal interactions

solar-type binary star, P ≈ 10 days

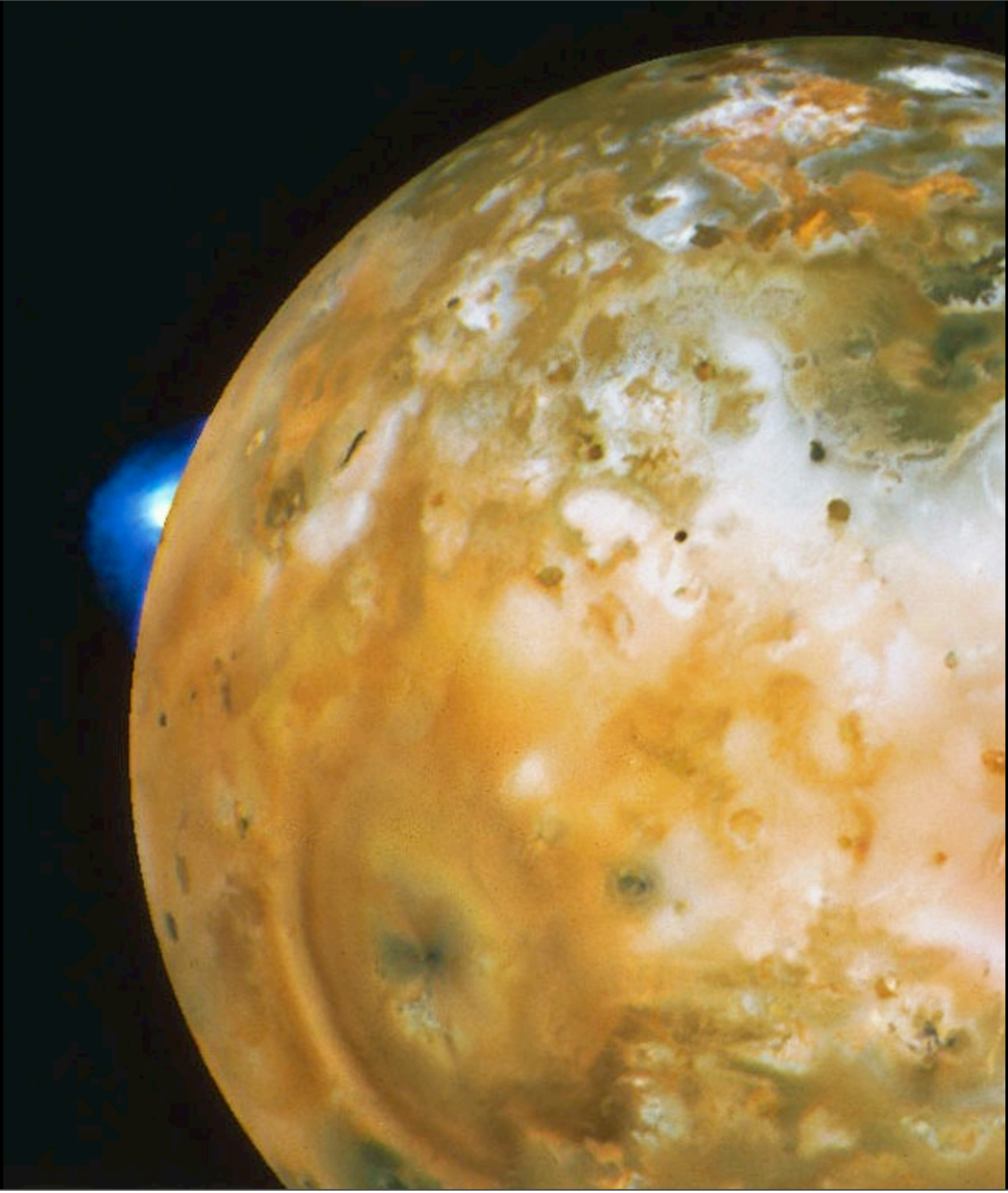


jovian satellite, P ≈ 3 days

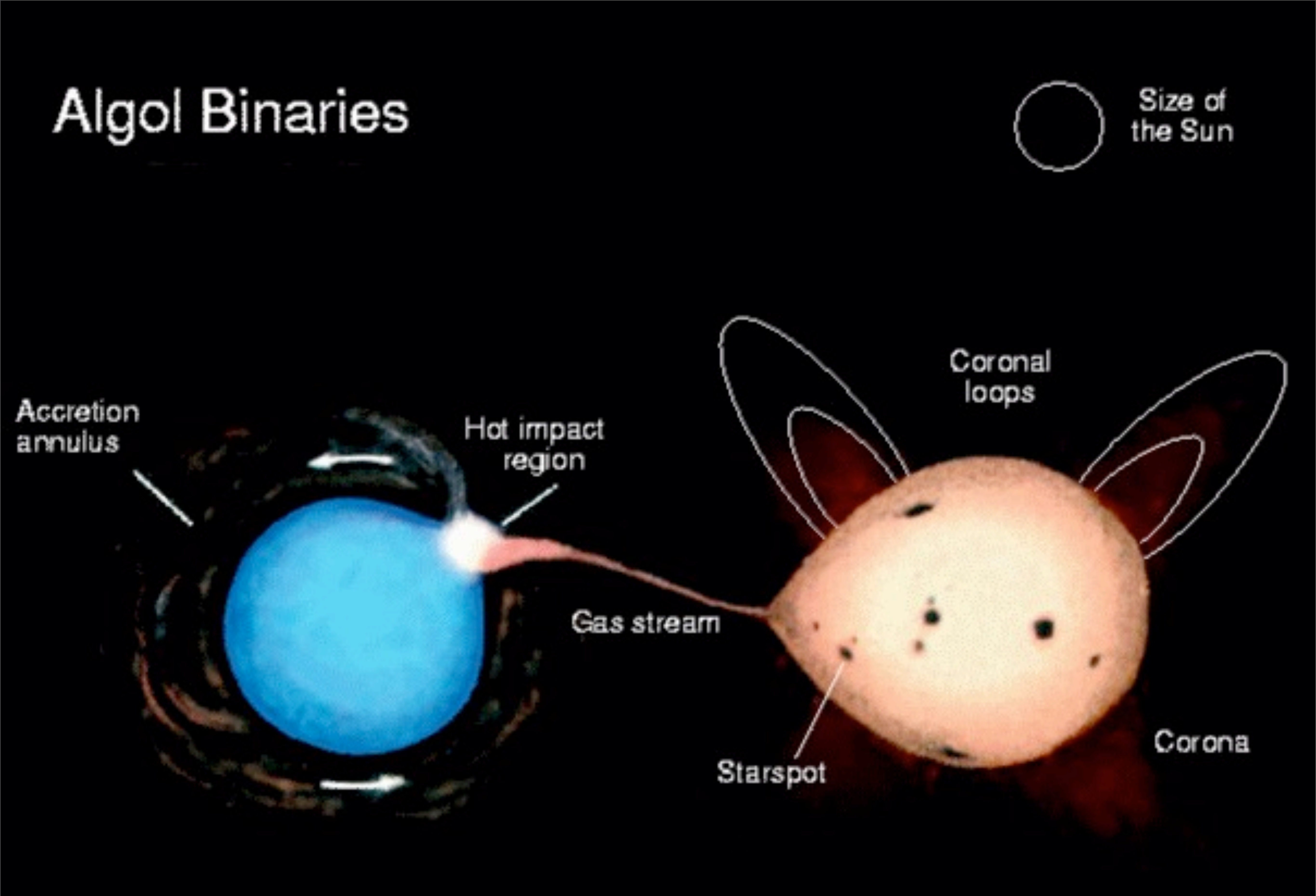


hot Jupiter, P ≈ 3 days

synchronization – circularization – orbital migration – tidal heating ➜



NASA



http://www2.astro.psu.edu/mrichards/research/binarypict.gif

Hot Jupiters

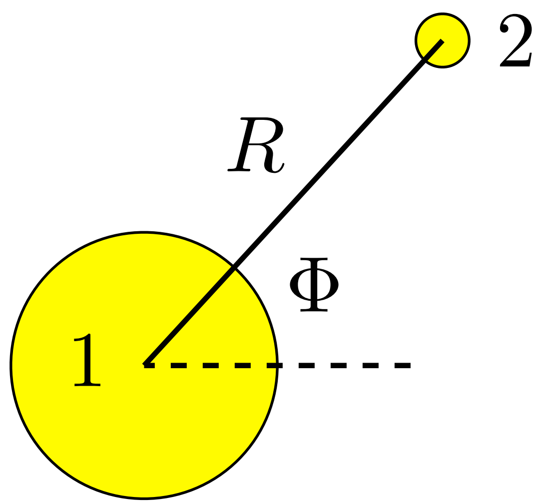
- Gravitational, thermal and magnetic interactions

http://bellerophonchimera.files.wordpress.com/2008/07/solar-maximum-september-1-20011.gif

Tidal forcing

- Tidal potential experienced by body 1

$$\Psi = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} \Psi_{l,m}(t) \left(\frac{r}{R_1}\right)^l Y_{l,m}(\theta,\phi)$$

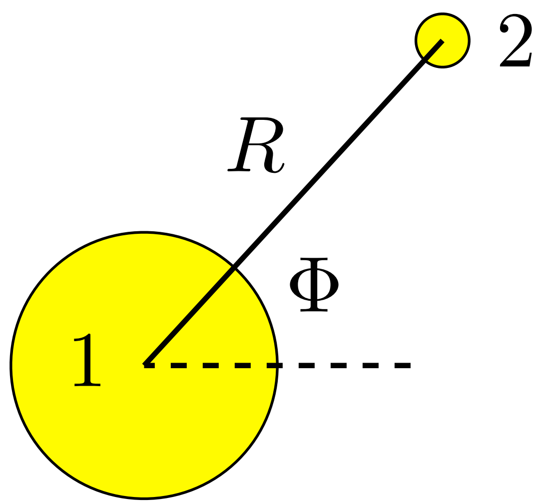


Tidal forcing

- Tidal potential experienced by body 1

$$\Psi = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} \Psi_{l,m}(t) \left(\frac{r}{R_1}\right)^l Y_{l,m}(\theta,\phi)$$

$$\Psi_{l,m} = -C_{l,m} \frac{GM_2}{R_1} \left(\frac{R}{R_1}\right)^{-(l+1)} \mathrm{e}^{-\mathrm{i}m\Phi}$$

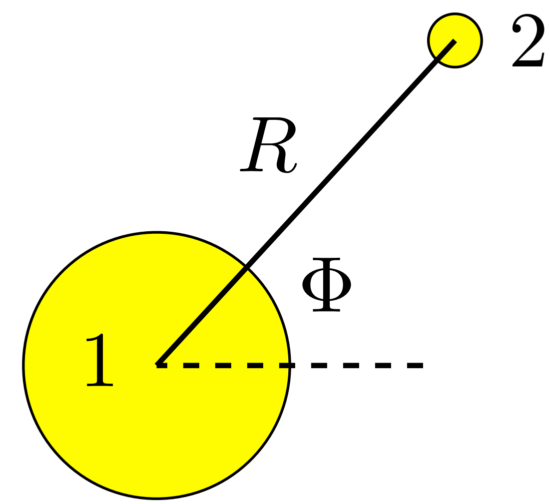


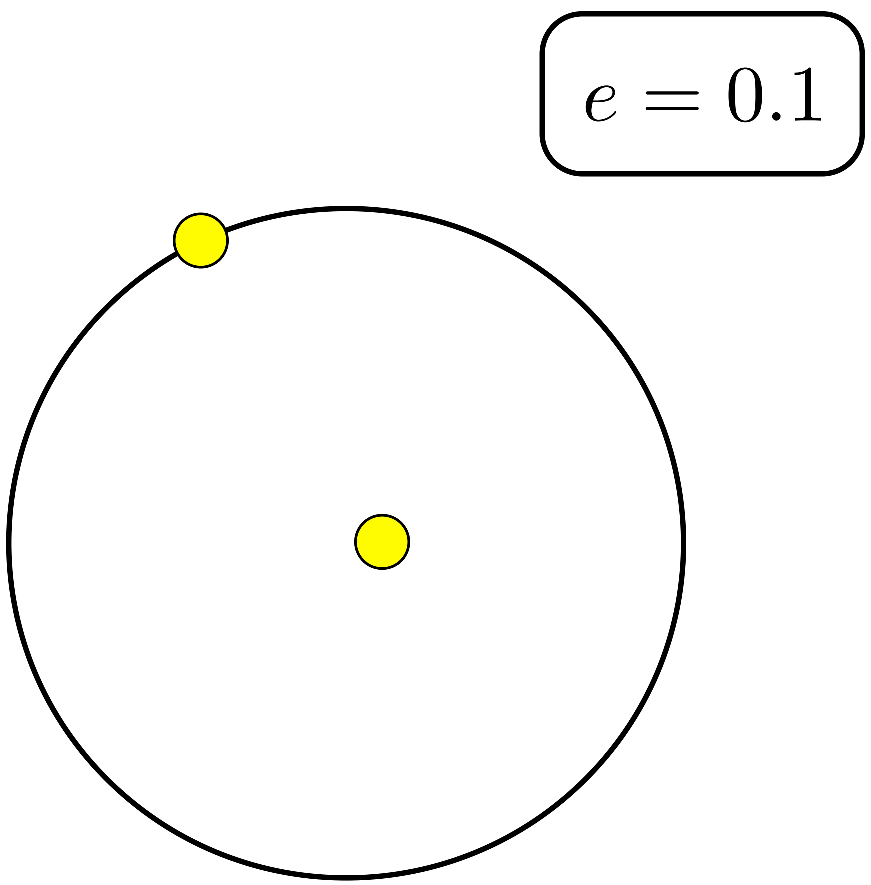
Tidal forcing

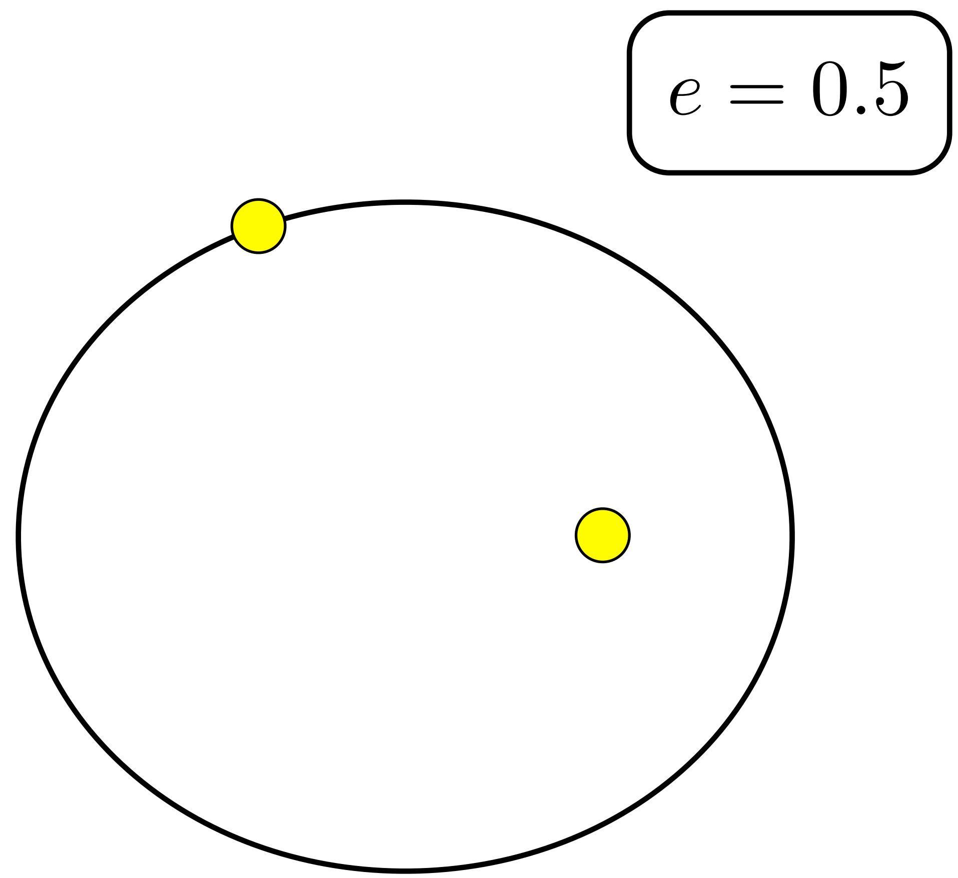
- Tidal potential experienced by body 1

$$\Psi = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} \Psi_{l,m}(t) \left(\frac{r}{R_1}\right)^l Y_{l,m}(\theta,\phi)$$

$$\Psi_{l,m} = -C_{l,m} \frac{GM_2}{R_1} \left(\frac{R}{R_1}\right)^{-(l+1)} \mathrm{e}^{-\mathrm{i}m\Phi}$$





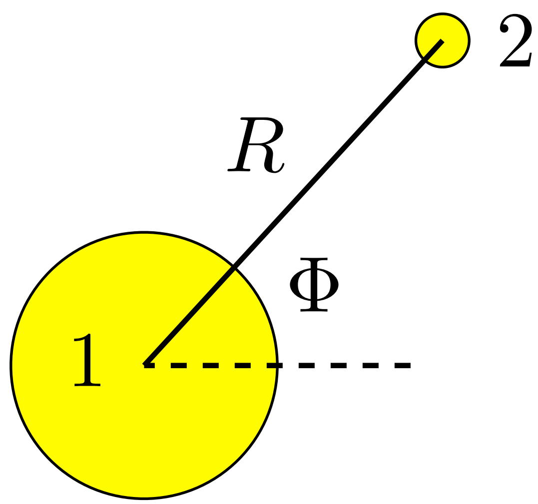


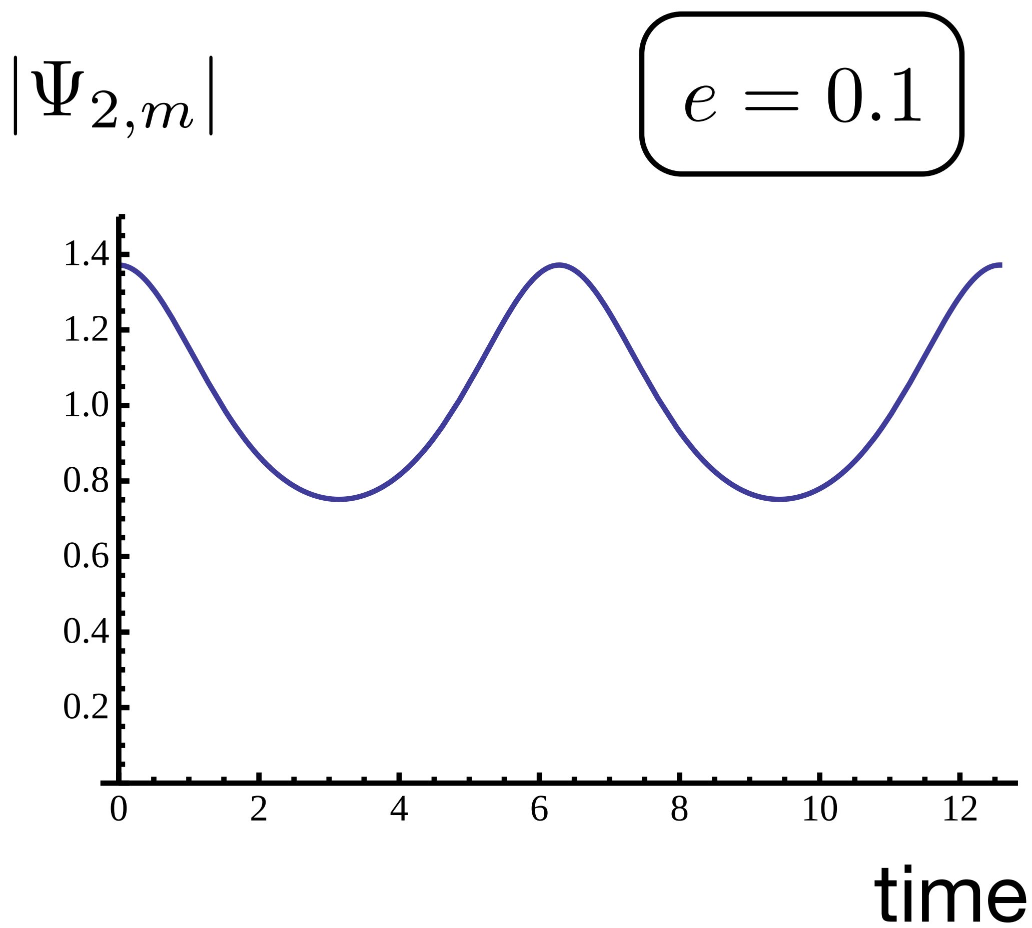
Tidal forcing

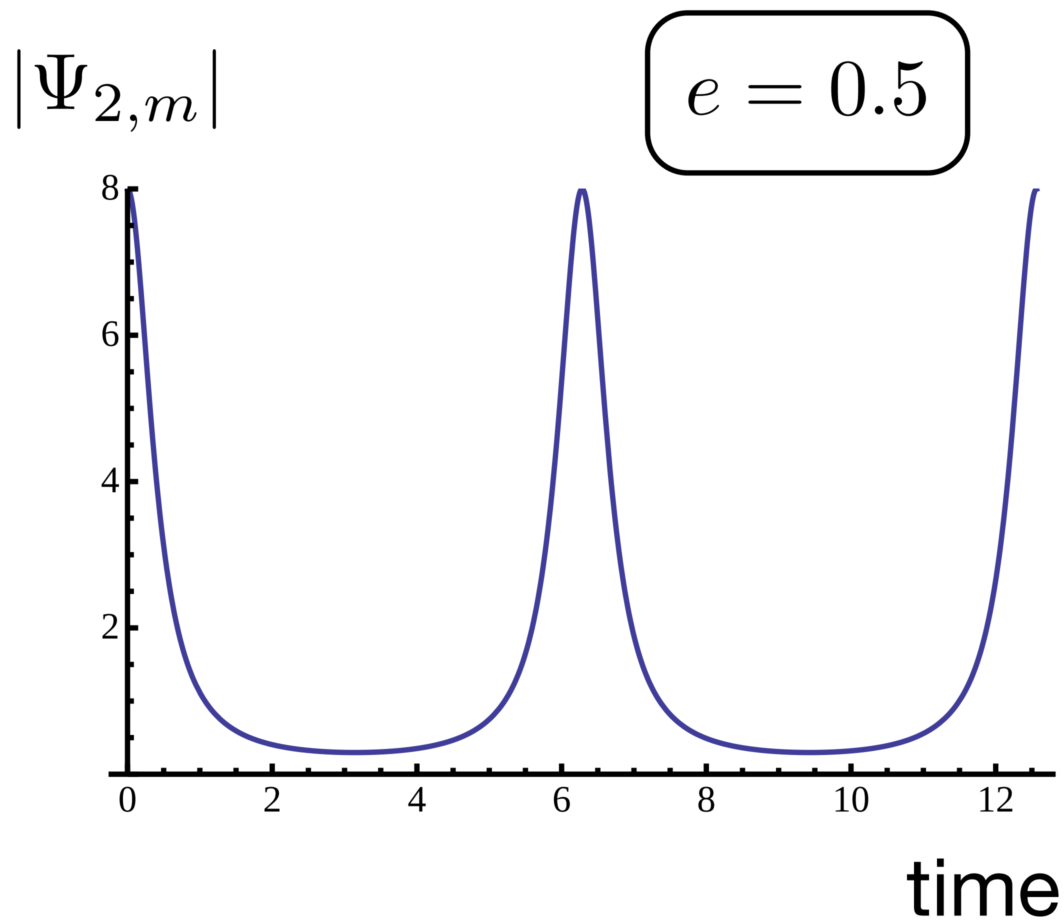
- Tidal potential experienced by body 1

$$\Psi = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} \Psi_{l,m}(t) \left(\frac{r}{R_1}\right)^l Y_{l,m}(\theta,\phi)$$

$$\Psi_{l,m} = -C_{l,m} \frac{GM_2}{R_1} \left(\frac{R}{R_1}\right)^{-(l+1)} \mathrm{e}^{-im\Phi}$$



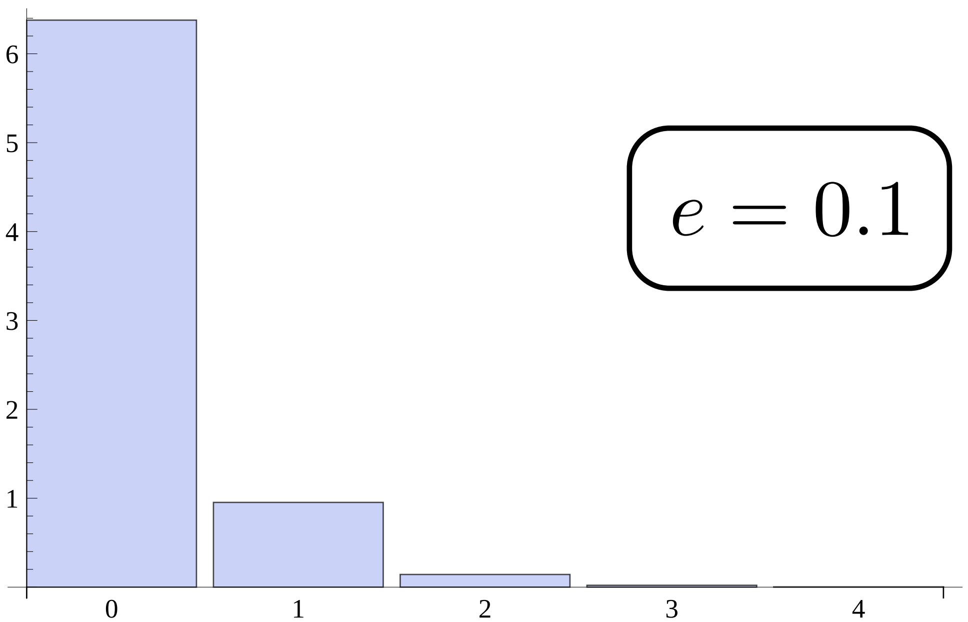


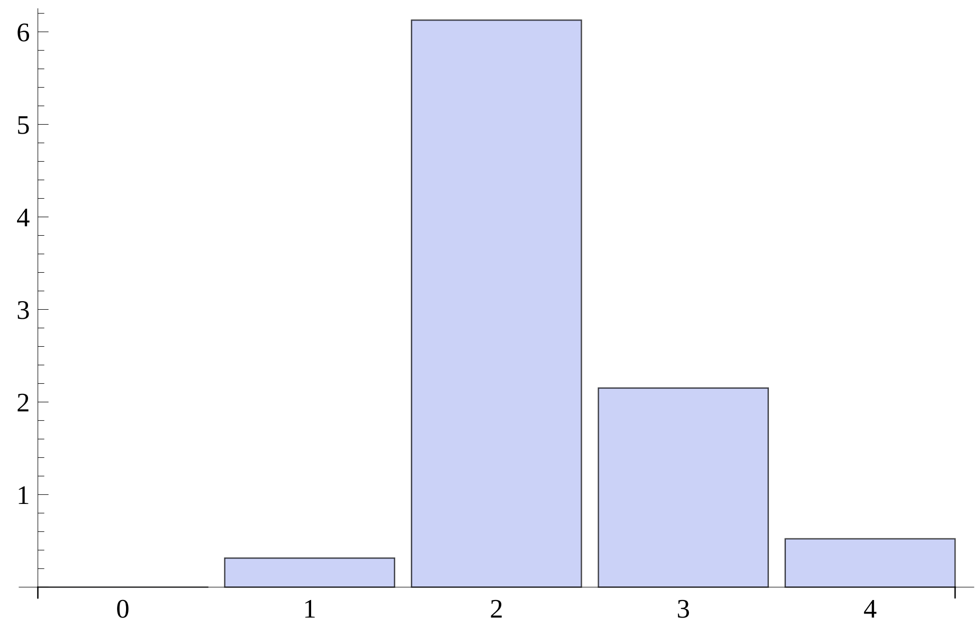


Fourier analysis

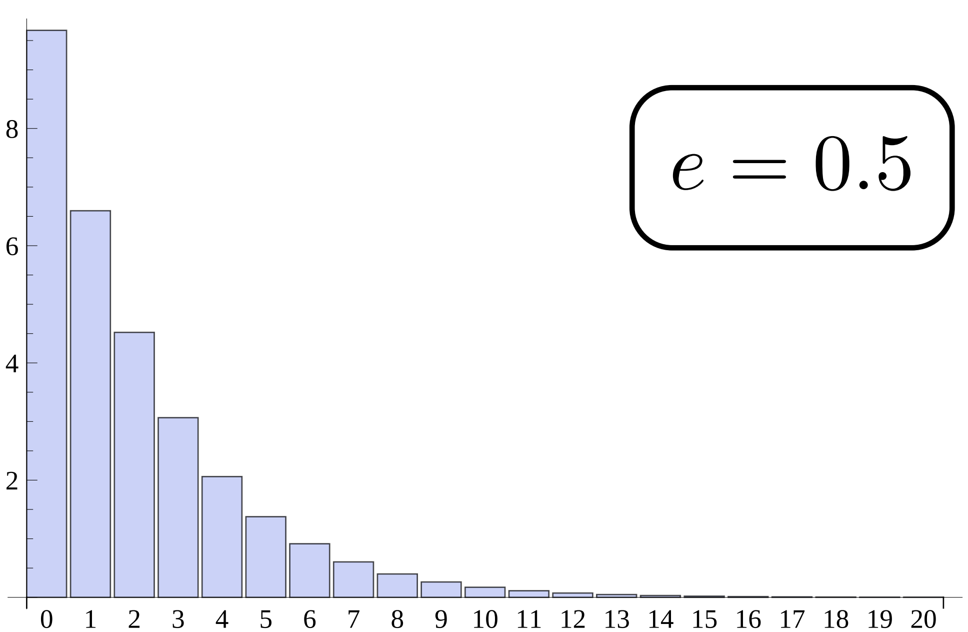
$|\tilde{\Psi}_{2,0}|$

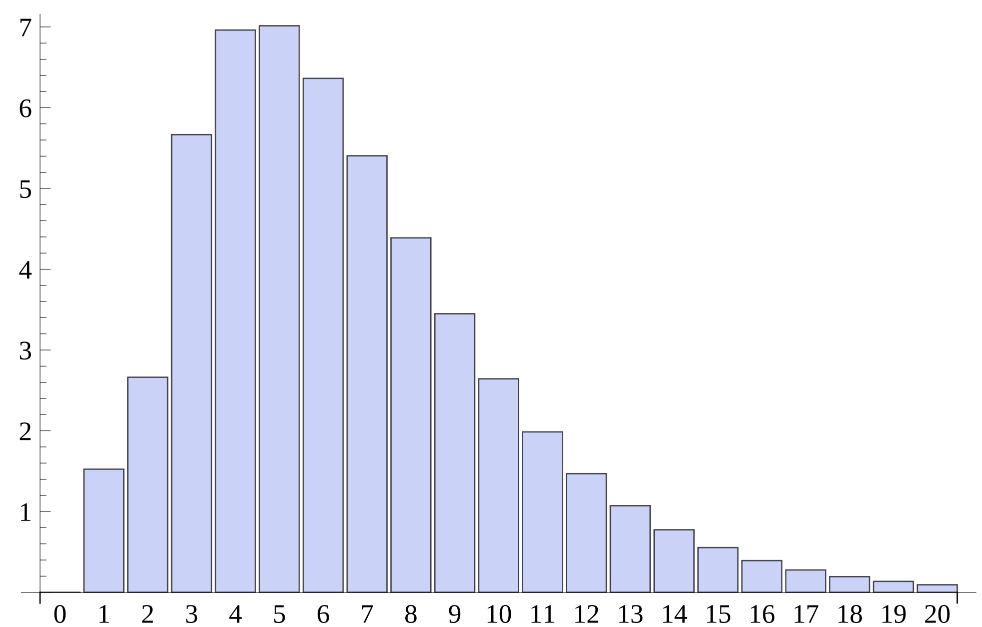
$|\tilde{\Psi}_{2,2}|$





$e = 0.1$





$e = 0.5$

frequency

frequency

Love number and "tidal Q"

- Consider each potential component experienced by body 1

$$\Psi = \tilde{\Psi}_{l,m} \left(\frac{r}{R_1} \right)^l Y_{l,m}(\theta,\phi)\, \mathrm{e}^{-\mathrm{i}\omega t}$$

Love number and "tidal Q"

- Consider each potential component experienced by body 1

$$\Psi = \tilde{\Psi}_{l,m} \left(\frac{r}{R_1} \right)^{l} Y_{l,m}(\theta,\phi)\, \mathrm{e}^{-\mathrm{i}\omega t}$$

- Body 1 is deformed and generates an external potential

$$\Phi' = \underline{k_{l,m}(\omega)}\, \tilde{\Psi}_{l,m} \left(\frac{r}{R_1} \right)^{-(l+1)} Y_{l,m}(\theta,\phi)\, \mathrm{e}^{-\mathrm{i}\omega t}$$

(+ orthogonal terms)

- Potential Love number (linear response function)

Love number and "tidal Q"

- Consider each potential component experienced by body 1

$$\Psi = \tilde{\Psi}_{l,m} \left(\frac{r}{R_1} \right)^{l} Y_{l,m}(\theta,\phi)\, \mathrm{e}^{-\mathrm{i}\omega t}$$

- Body 1 is deformed and generates an external potential

$$\Phi' = \underline{k_{l,m}(\omega)}\, \tilde{\Psi}_{l,m} \left(\frac{r}{R_1} \right)^{-(l+1)} Y_{l,m}(\theta,\phi)\, \mathrm{e}^{-\mathrm{i}\omega t}$$

(+ orthogonal terms)

- Potential Love number (linear response function)
- Energy transfer to orbit $\propto \omega\, \mathrm{Im}(k_{l,m})\, |\tilde{\Psi}_{l,m}|^2$
- Angular momentum transfer $\propto m\, \mathrm{Im}(k_{l,m})\, |\tilde{\Psi}_{l,m}|^2$

Love number and "tidal Q"

- Consider each potential component experienced by body 1

$$\Psi = \tilde{\Psi}_{l,m} \left(\frac{r}{R_1} \right)^l Y_{l,m}(\theta,\phi)\, \mathrm{e}^{-\mathrm{i}\omega t}$$

- Body 1 is deformed and generates an external potential

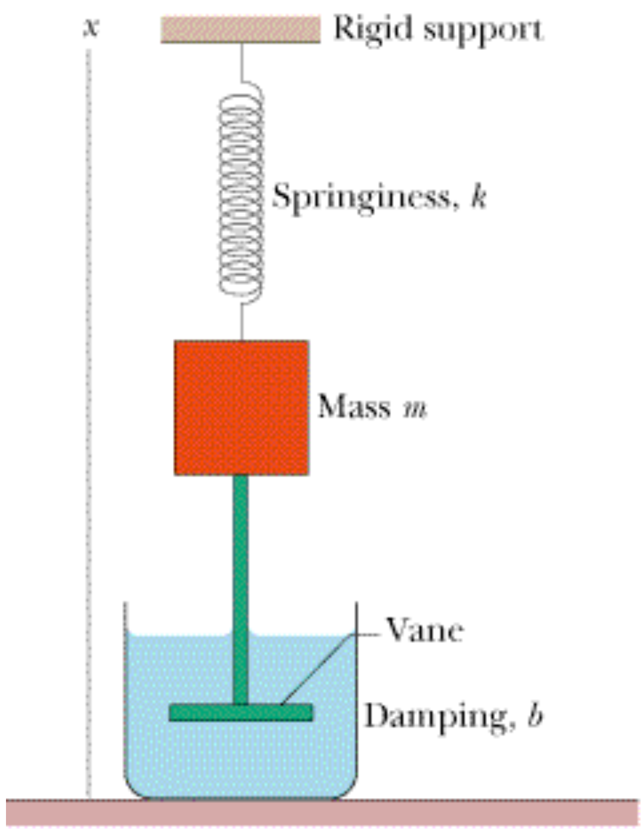
$$\Phi' = \underline{k_{l,m}(\omega)}\, \tilde{\Psi}_{l,m} \left(\frac{r}{R_1} \right)^{-(l+1)} Y_{l,m}(\theta,\phi)\, \mathrm{e}^{-\mathrm{i}\omega t}$$

(+ orthogonal terms)

- Potential Love number (linear response function)
- Energy transfer to orbit $\propto \omega\, \mathrm{Im}(k_{l,m})\, |\tilde{\Psi}_{l,m}|^2$
- Angular momentum transfer $\propto m\, \mathrm{Im}(k_{l,m})\, |\tilde{\Psi}_{l,m}|^2$
- $\underline{\mathrm{Im}(k) \approx \frac{k}{Q} \approx \frac{1}{Q'} \ll 1}$ depends on ω, l, m (usually $l = m = 2$)

Analogy: forced harmonic oscillator

$$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = f\,\mathrm{e}^{-\mathrm{i}\omega t}$$



http://www.webassign.net/hrw/hrw7_15-15.gif

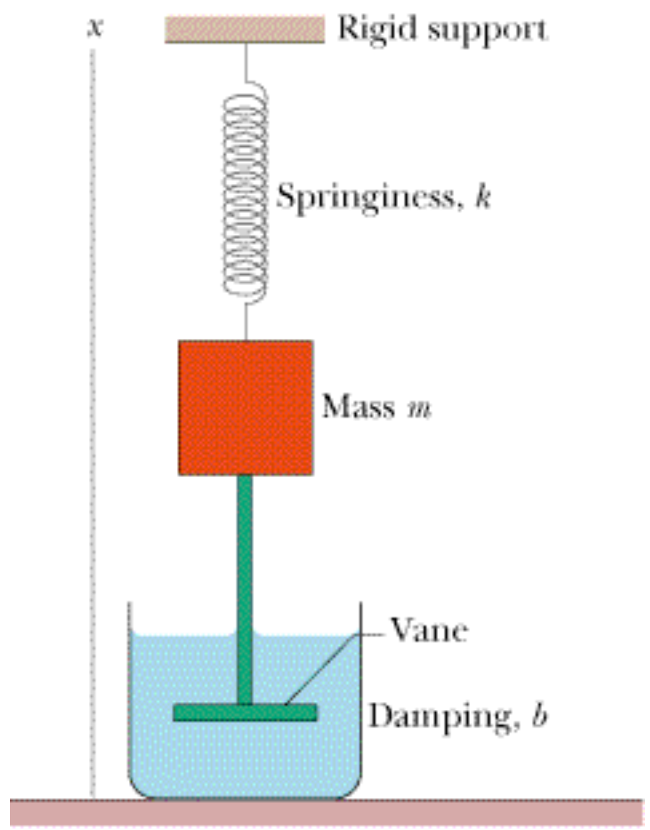
Analogy: forced harmonic oscillator

$$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = f\,\mathrm{e}^{-\mathrm{i}\omega t}$$

$$x = k\,\frac{f}{\omega_0^2}\,\mathrm{e}^{-\mathrm{i}\omega t}$$

$$\begin{aligned} k &= \left(1 - \frac{2\mathrm{i}\omega\gamma}{\omega_0^2} - \frac{\omega^2}{\omega_0^2}\right)^{-1} \\ &\approx \left(1 + \mathrm{i}\,Q^{-1}\right) \end{aligned}$$

$$[\,\omega,\gamma \ll \omega_0\,]$$



$$Q = \frac{\omega_0^2}{2\omega\gamma} = \frac{2\pi \times \text{maximum potential energy}}{\text{energy dissipated per cycle}} \gg 1$$

http://www.webassign.net/hrw/hrw7_15-15.gif

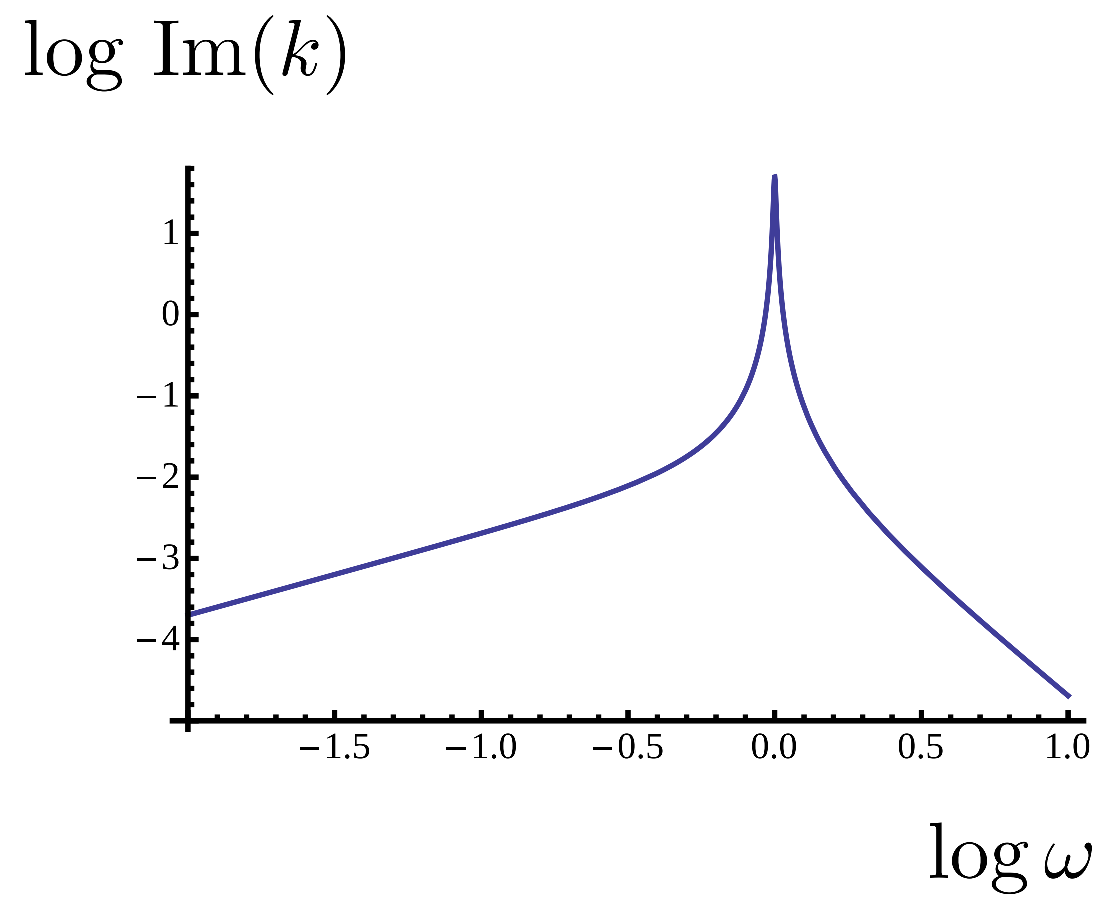
Analogy: forced harmonic oscillator

$$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = f\,\mathrm{e}^{-\mathrm{i}\omega t}$$

$$x = k\,\frac{f}{\omega_0^2}\,\mathrm{e}^{-\mathrm{i}\omega t}$$

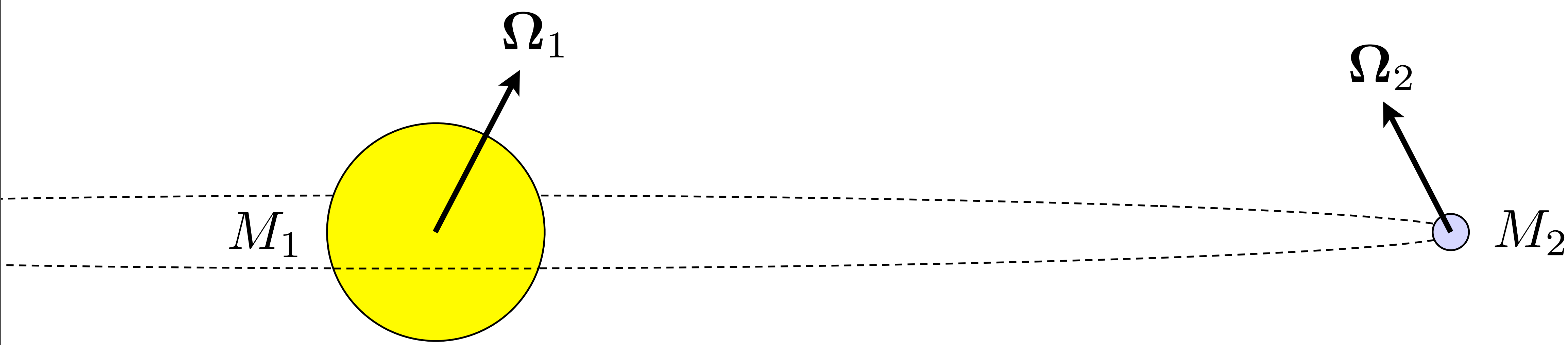
$$\begin{aligned} k &= \left(1 - \frac{2\mathrm{i}\omega\gamma}{\omega_0^2} - \frac{\omega^2}{\omega_0^2}\right)^{-1} \\ &\approx \left(1 + \mathrm{i}\,Q^{-1}\right) \end{aligned}$$

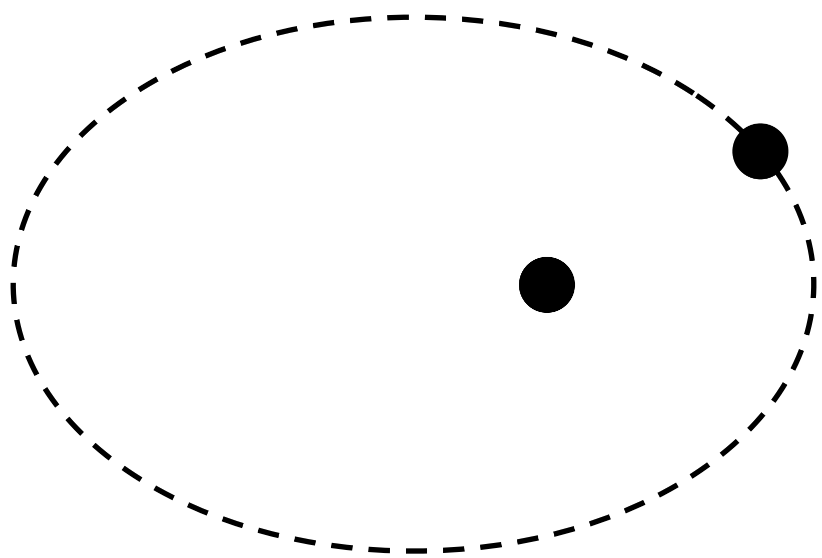
$$[\,\omega,\gamma \ll \omega_0\,]$$



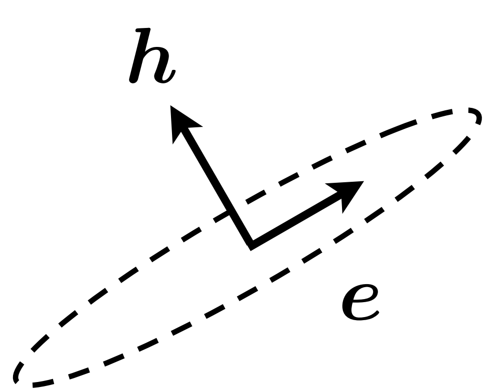
$$Q = \frac{\omega_0^2}{2\omega\gamma} = \frac{2\pi \times \text{maximum potential energy}}{\text{energy dissipated per cycle}} \gg 1$$

Spin and orbital evolution



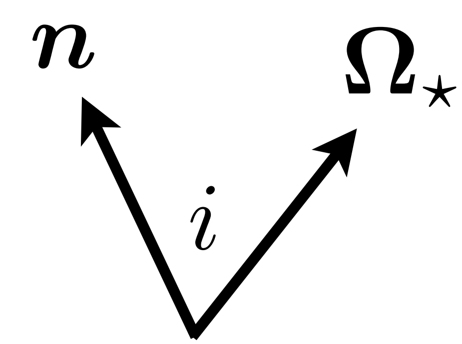


- a semimajor axis
- e eccentricity
- n mean motion
- $\boldsymbol{h}$ specific angular momentum
- $\boldsymbol{e}$ eccentricity vector



Parametrized models

- Tidal timescale $T_\star = \dfrac{2Q'_\star}{9n}\dfrac{M_\star}{M_\mathrm{p}}\left(\dfrac{a}{R_\star}\right)^5$ etc.
- Planet pseudosynchronized: $\boldsymbol{\Omega}_\mathrm{p} = \left(\dfrac{2f_4}{f_2+f_3}\right)\boldsymbol{n}$
- Magnetic braking of star



$$\frac{\mathrm{d}n}{\mathrm{d}t} = \frac{3n}{T_\star}\left[f_4 - \left(\frac{f_2+f_3}{2}\right)\frac{\Omega_\star}{n}\cos i\right]$$

$$\frac{\mathrm{d}e}{\mathrm{d}t} = -\frac{9e}{T_\star}\left[f_1 - \frac{11}{18}f_2\frac{\Omega_\star}{n}\cos i\right] - \frac{9e}{T_\mathrm{p}}\left[f_1 - \frac{11}{18}\left(\frac{2f_2f_4}{f_2+f_3}\right)\right]$$

$$\frac{\mathrm{d}\Omega_\star}{\mathrm{d}t} = \frac{\mu n a^2}{I_\star T_\star}\left[f_4\cos i - \frac{\Omega_\star}{2n}(f_2 + f_3\cos^2 i)\right] - \alpha\Omega_\star^3$$

$$\frac{\mathrm{d}i}{\mathrm{d}t} = -\frac{\Omega_\star}{2nT_\star}f_2\sin i - \frac{\mu n a^2}{I_\star\Omega_\star T_\star}\left[f_4 - \frac{\Omega_\star}{2n}f_3\cos i\right]\sin i$$

Parametrized models

- e.g. Barker & Ogilvie (2009)

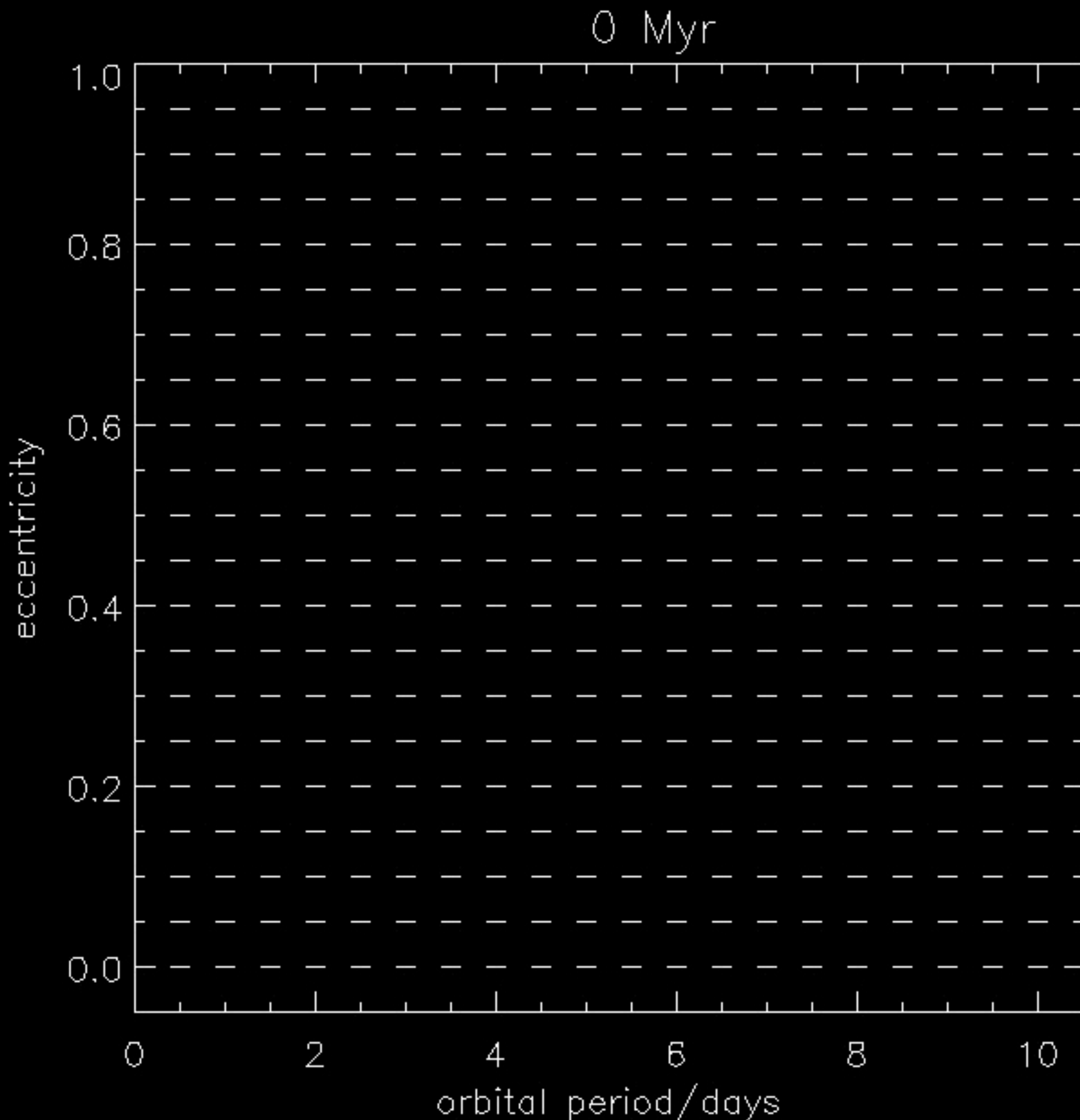
$$1\,M_\odot \qquad Q'_\star = 10^6$$

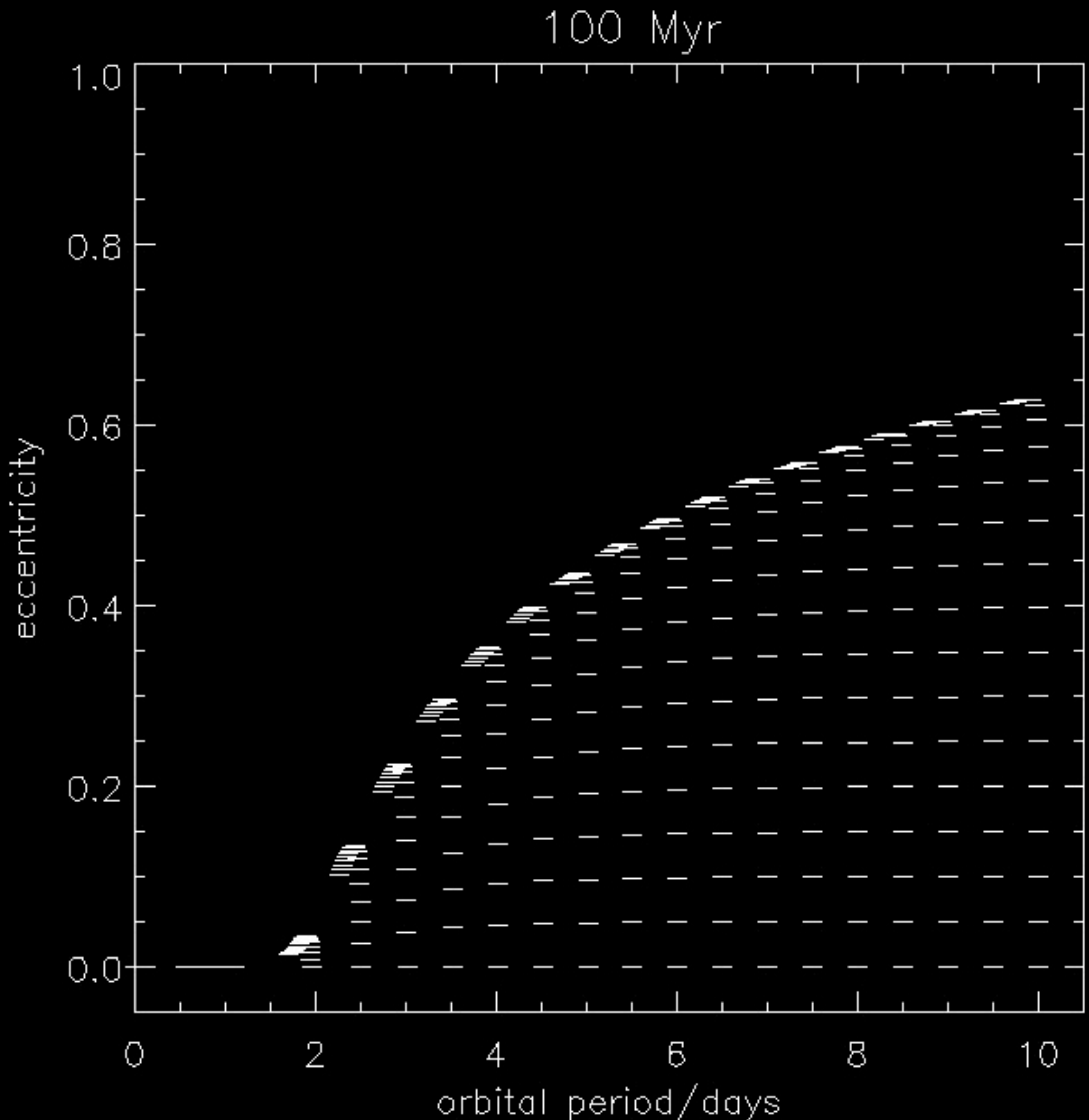
$$1\,M_\mathrm{J} \qquad Q'_\mathrm{p} = 10^6$$

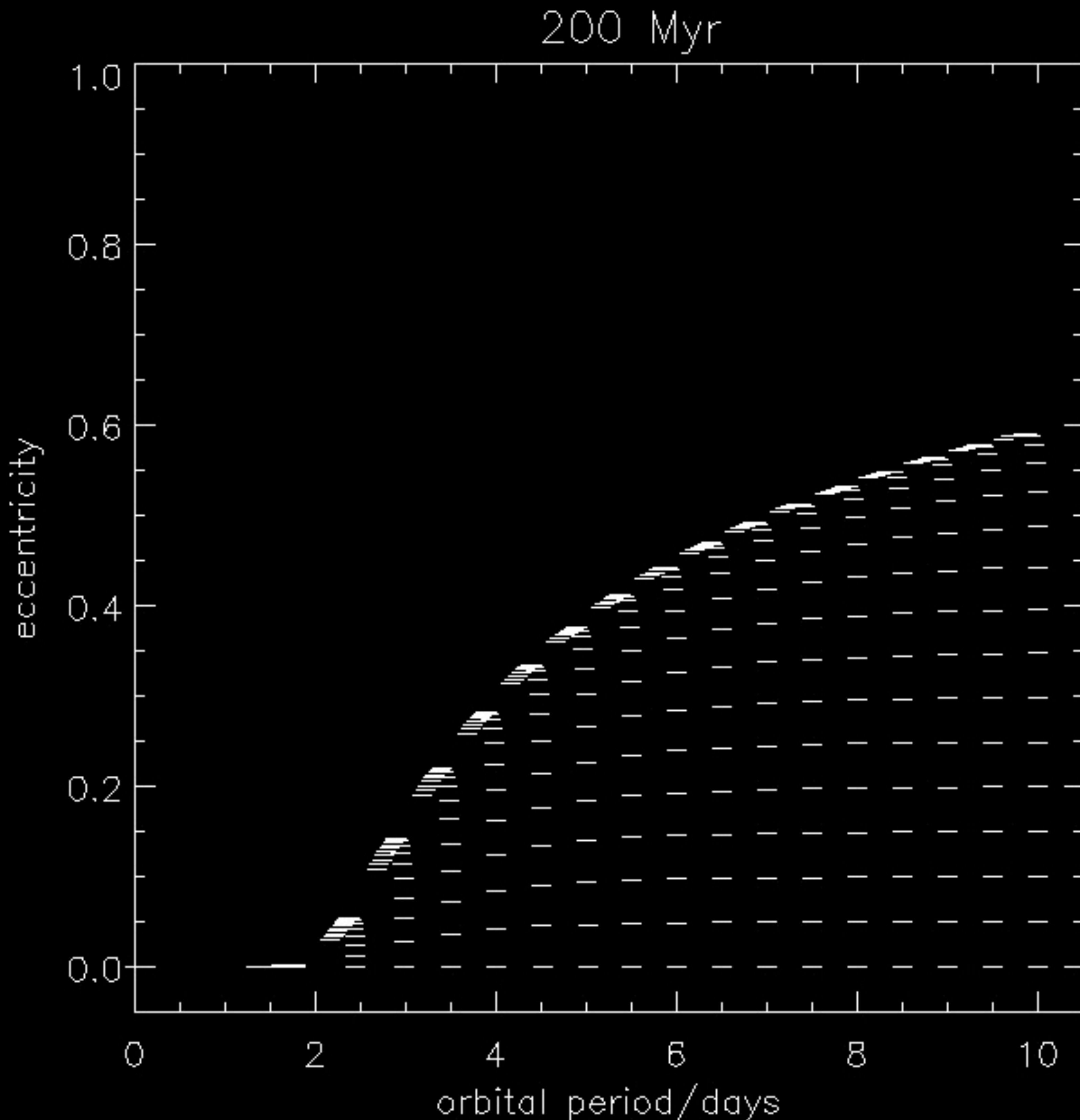
$$P_{\star 0} = 1\,\mathrm{day}$$

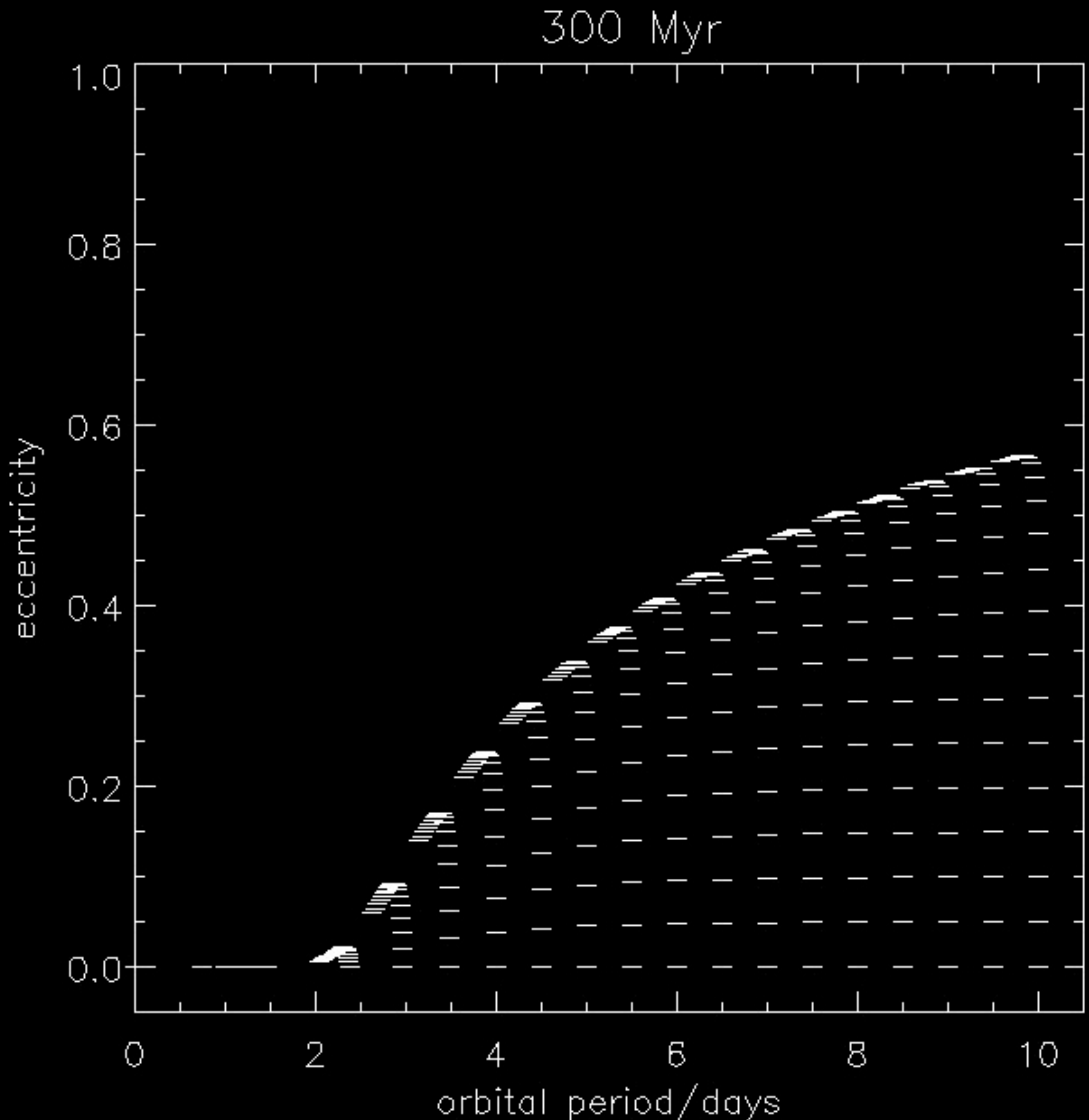
$$\alpha = 1.5 \times 10^{-14}\,\mathrm{yr}$$

Initial stellar obliquity 0°

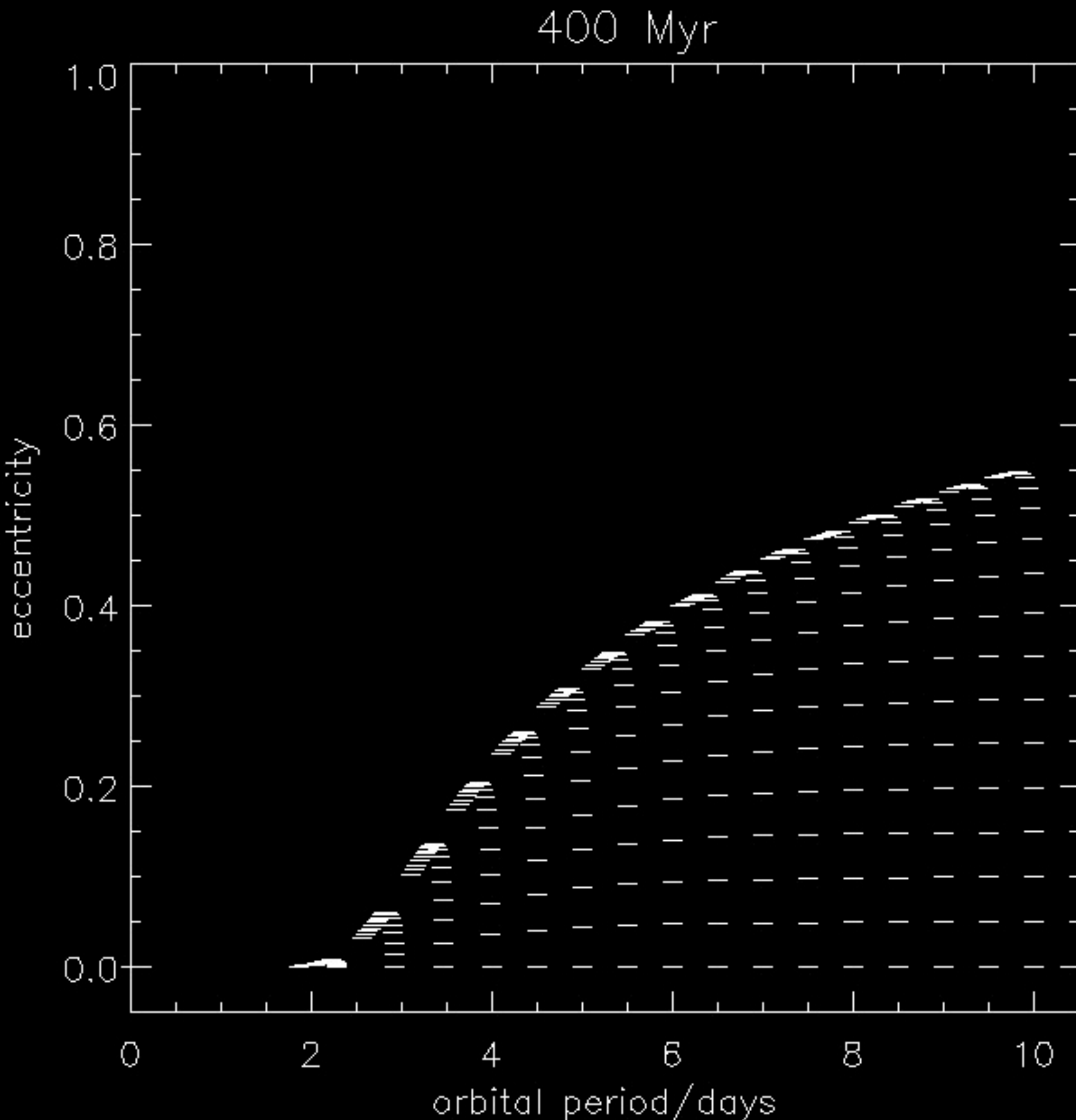

0 Myr
1.0
0.8
0.6
0.4
0.2
0.0
eccentricity
0
2
4
6
8
10
orbital period/days

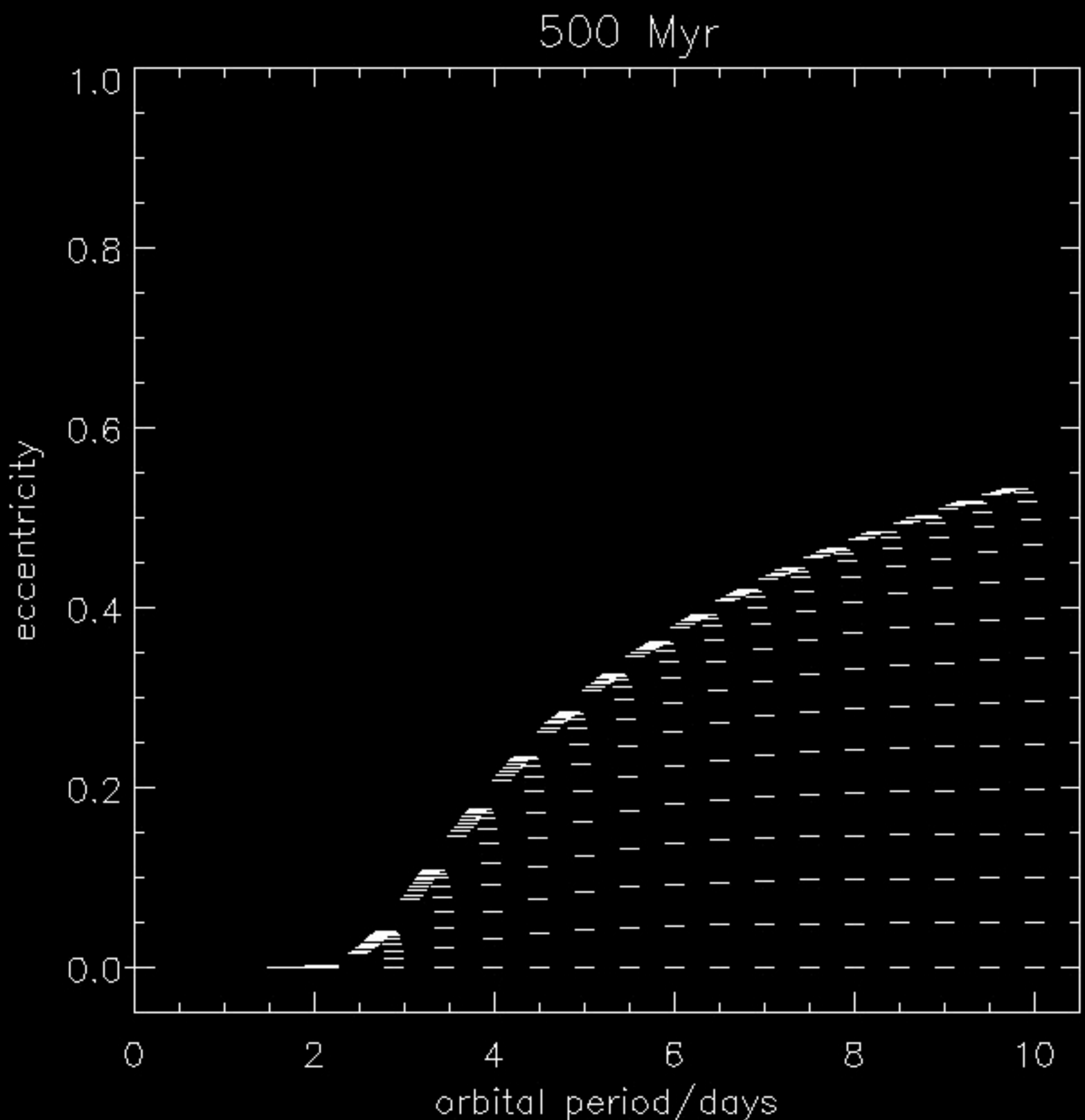

100 Myr
1.0
0.8
0.6
0.4
0.2
0.0
eccentricity
0
2
4
6
8
10
orbital period/days

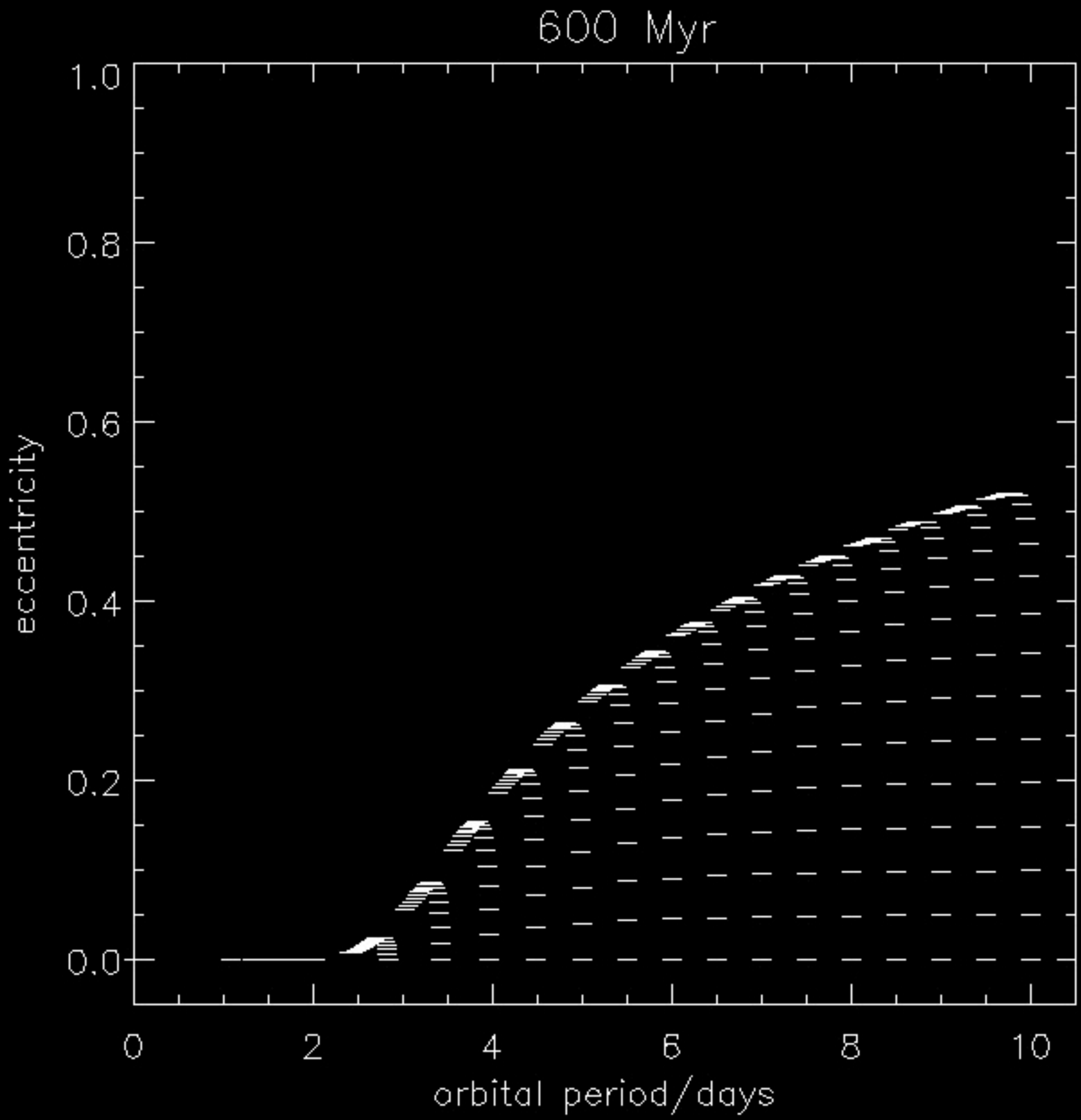

200 Myr
eccentricity
orbital period/days
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10



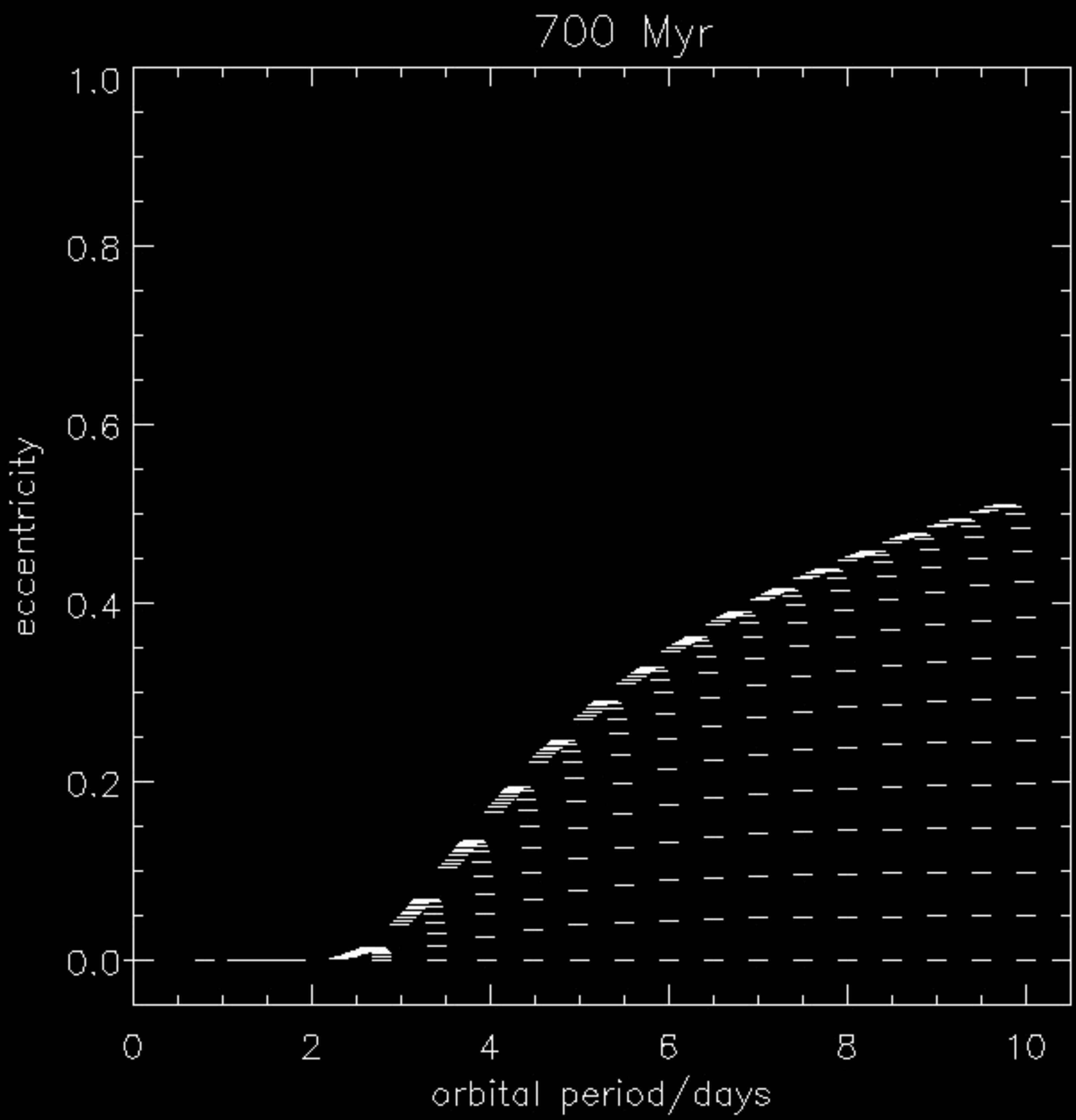
300 Myr
eccentricity
orbital period/days
0.0
0.2
0.4
0.6
0.8
1.0
0
2
4
6
8
10

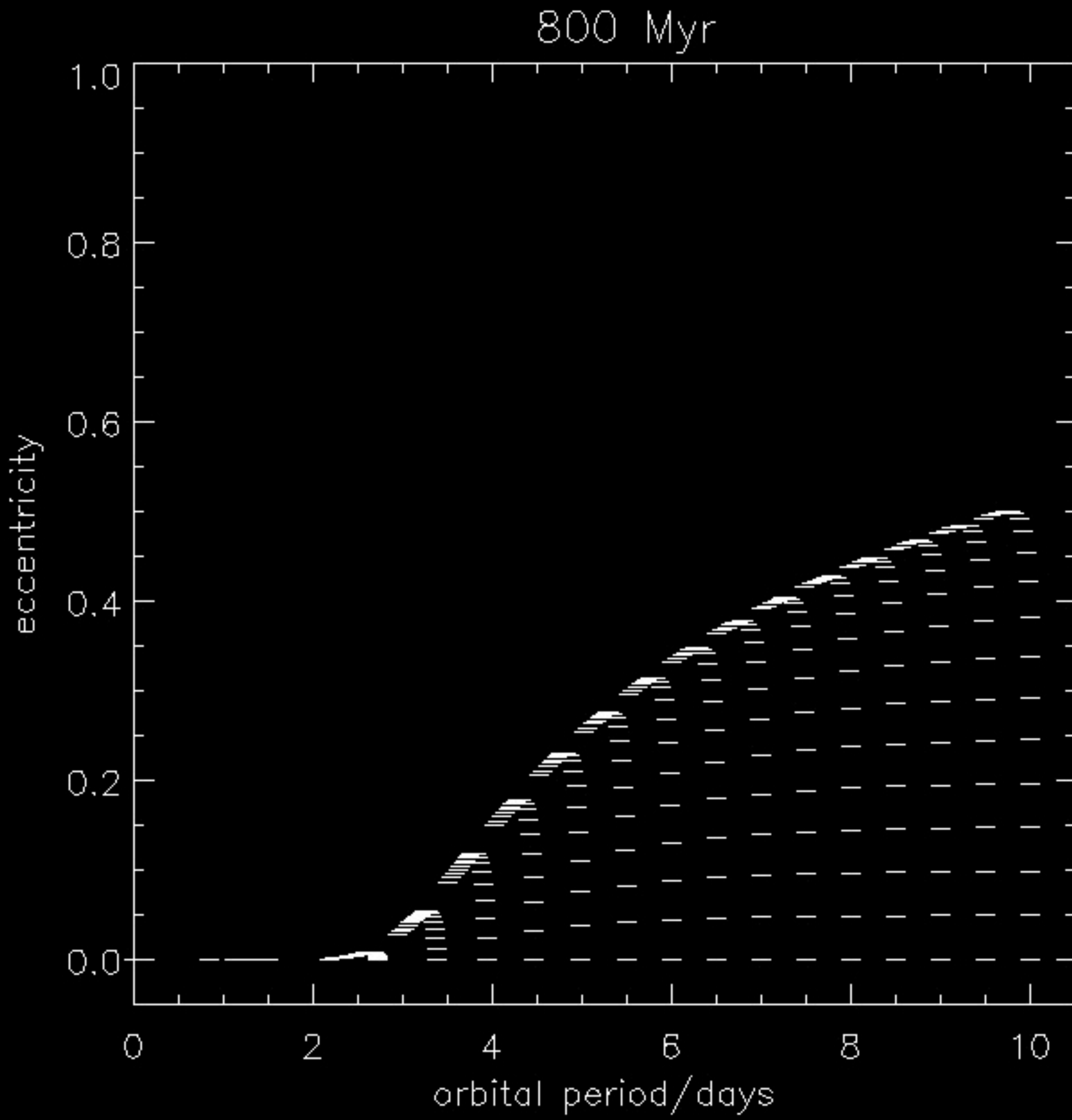

400 Myr
1.0
0.8
0.6
0.4
0.2
0.0
eccentricity
0
2
4
6
8
10
orbital period/days

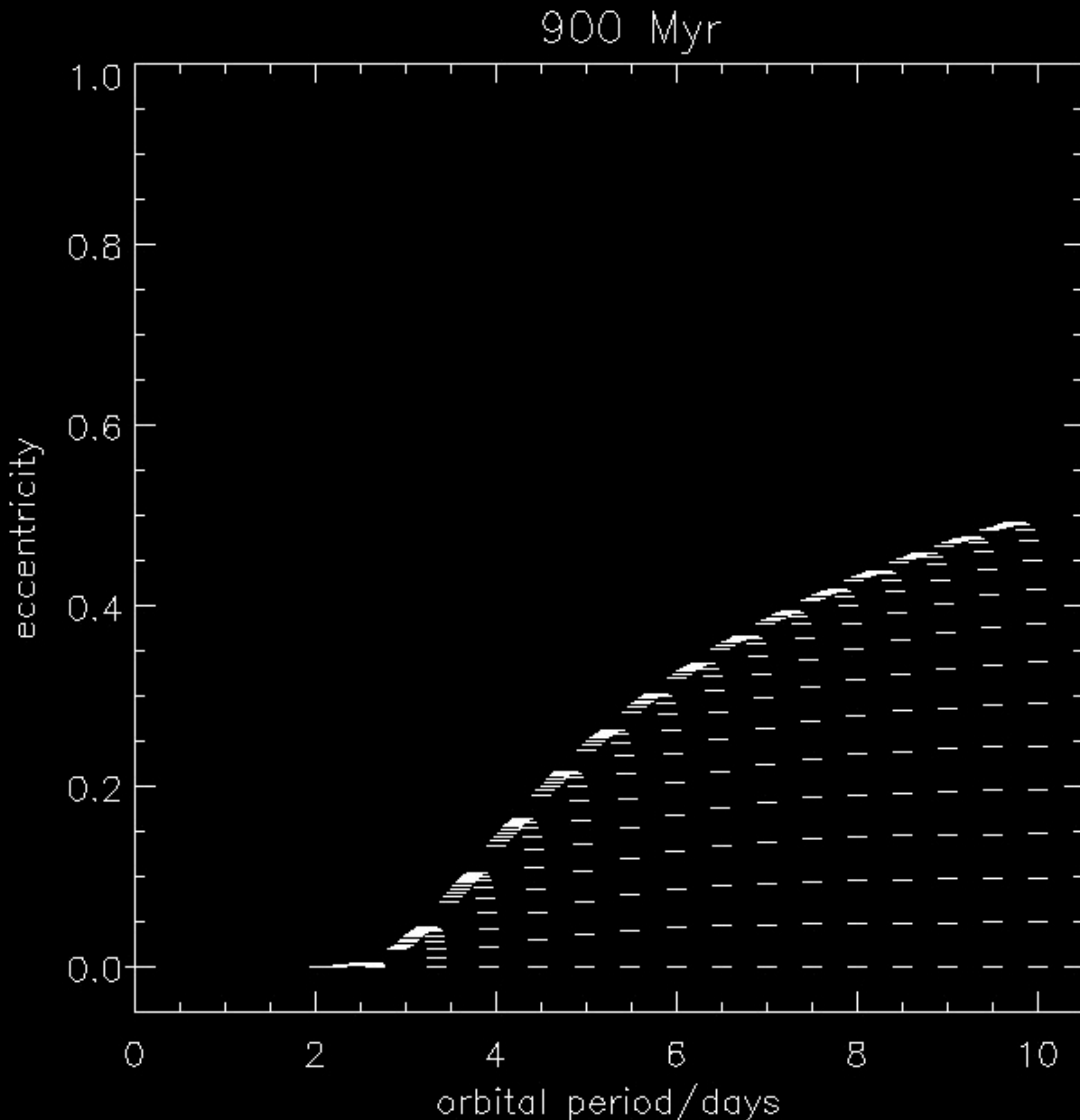

500 Myr
eccentricity
orbital period/days
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10



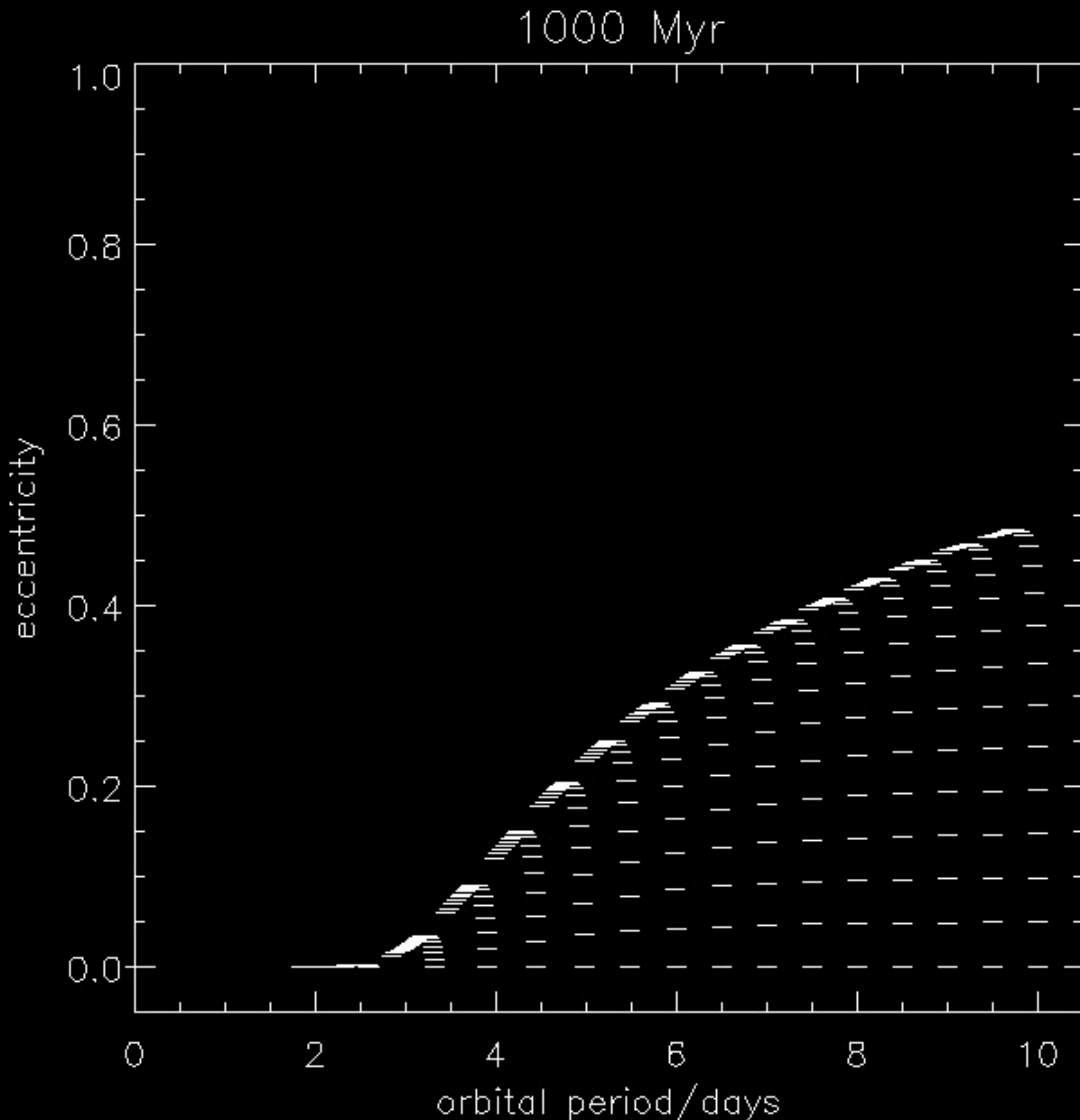
600 Myr
eccentricity
orbital period/days
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10

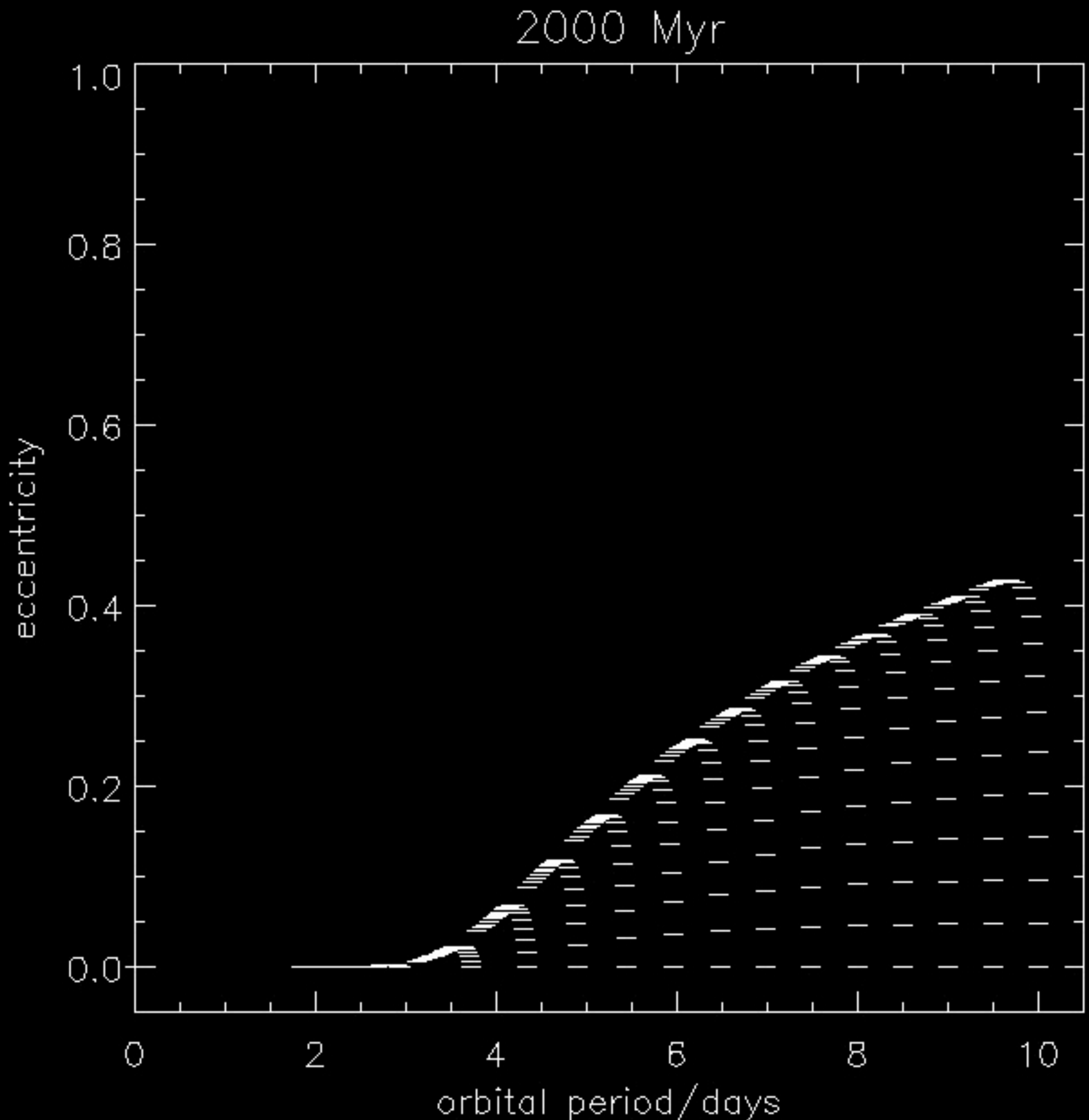

700 Myr
1.0
0.8
0.6
0.4
0.2
0.0
eccentricity
0
2
4
6
8
10
orbital period/days

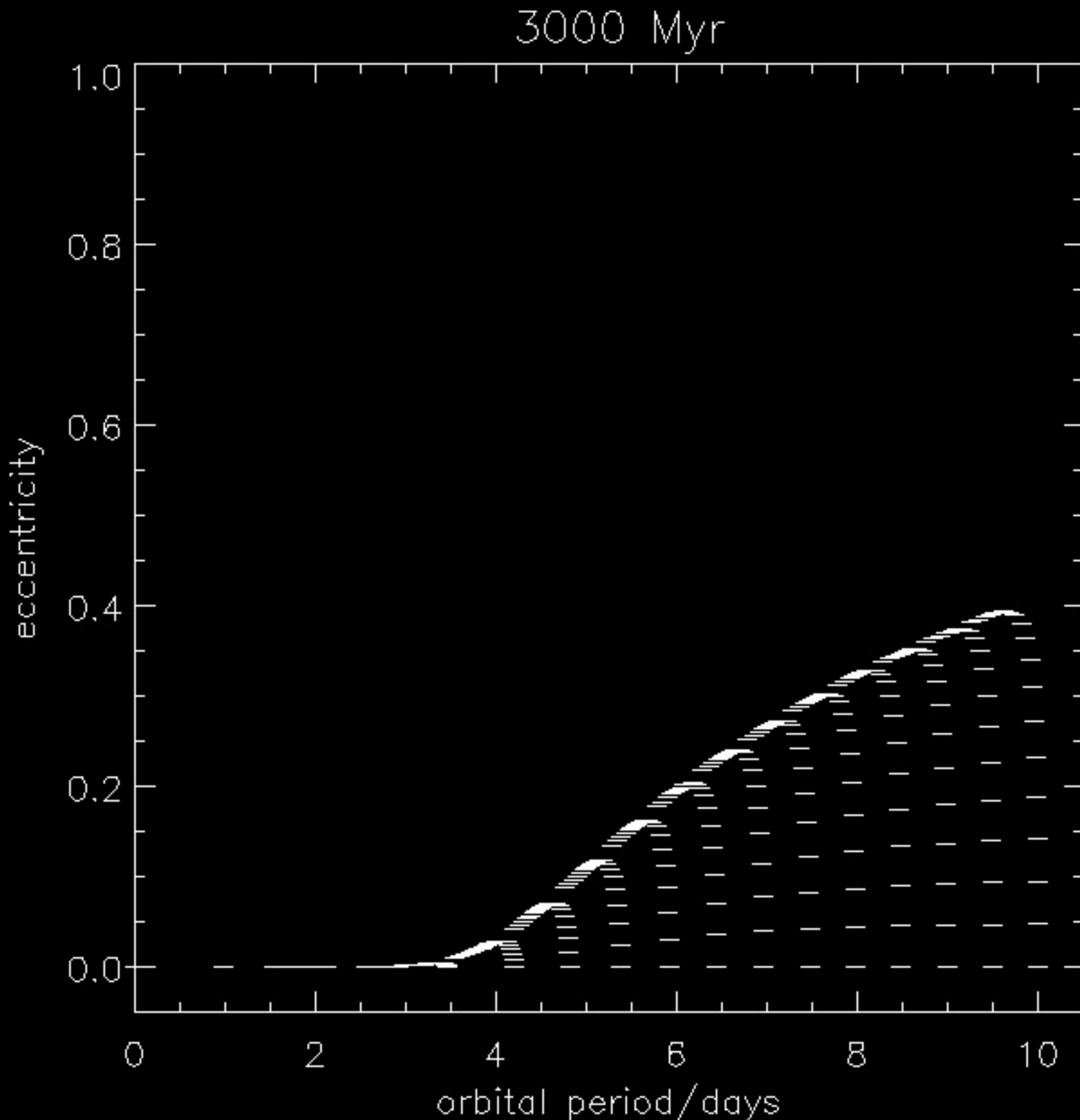

800 Myr
eccentricity
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10
orbital period/days

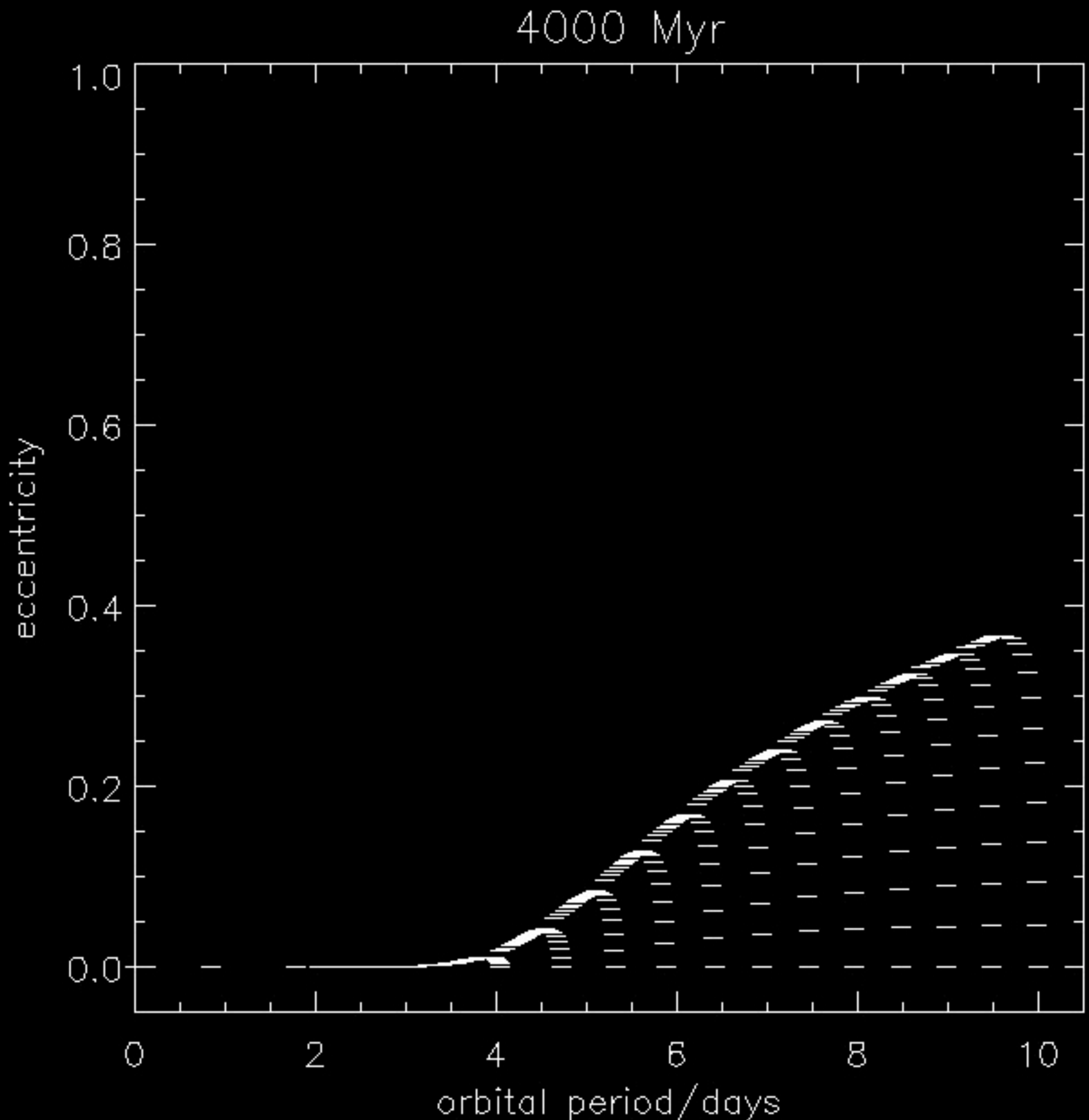


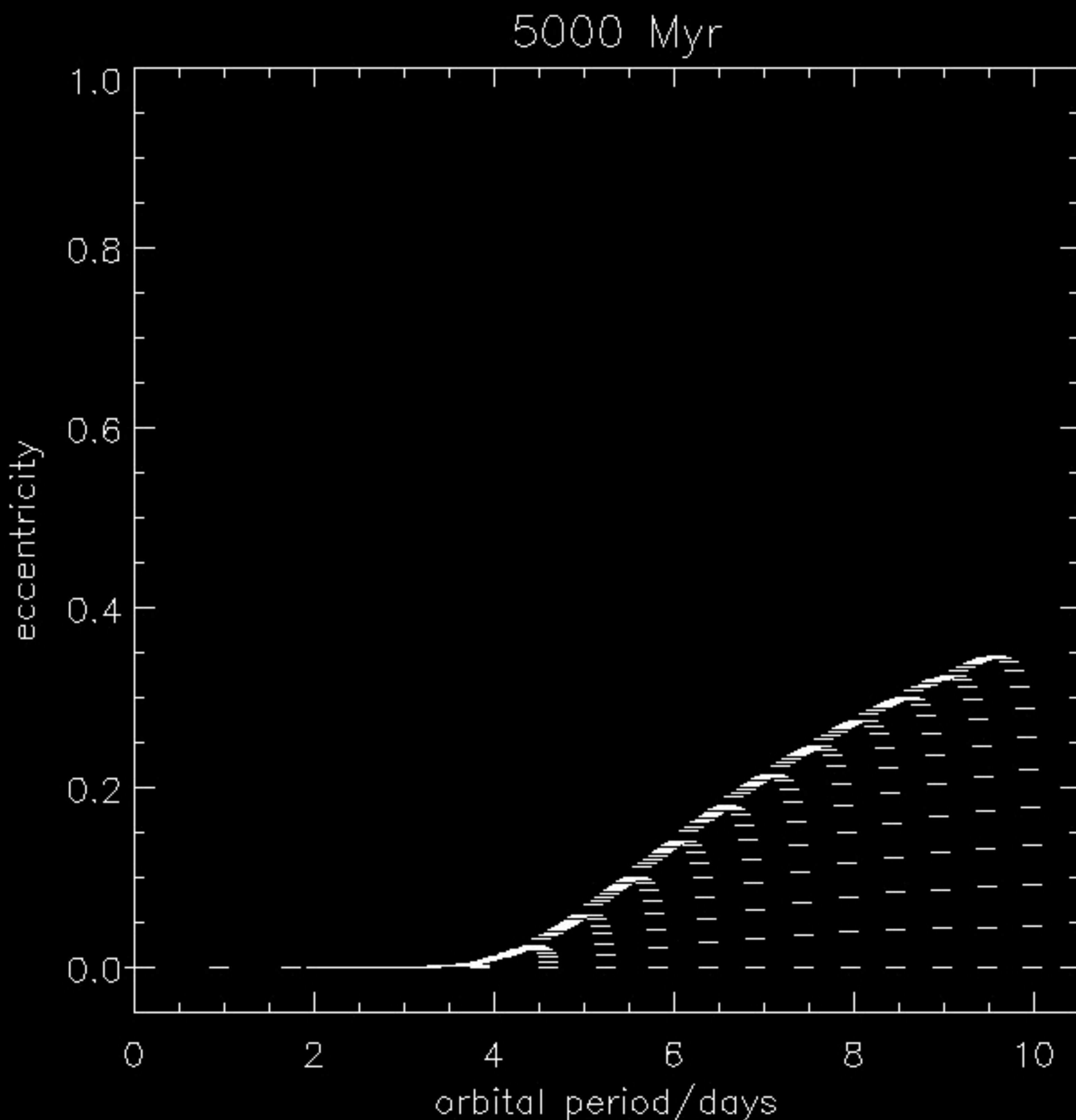
900 Myr
eccentricity
orbital period/days
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10

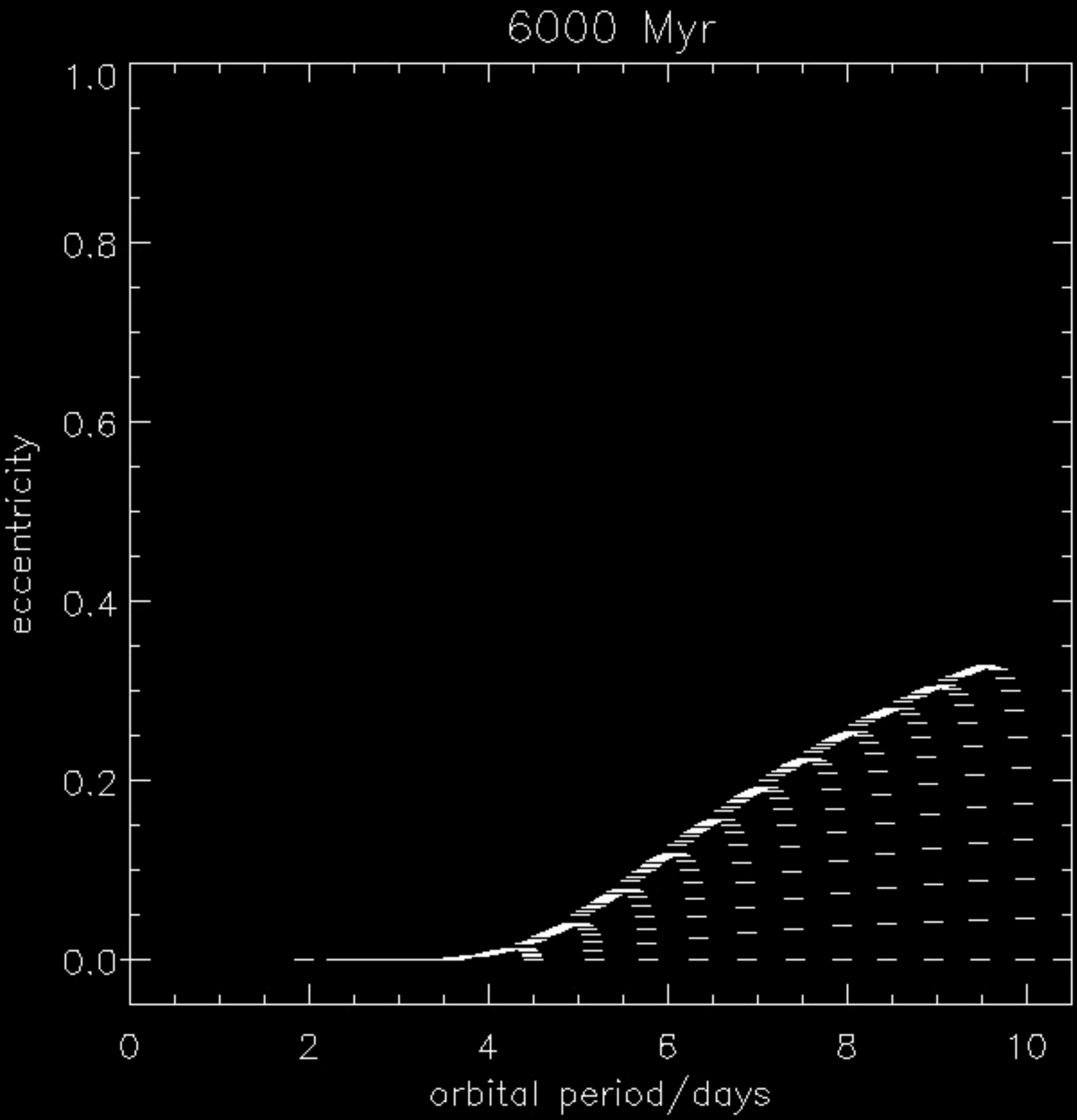

1000 Myr
eccentricity
orbital period/days
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10

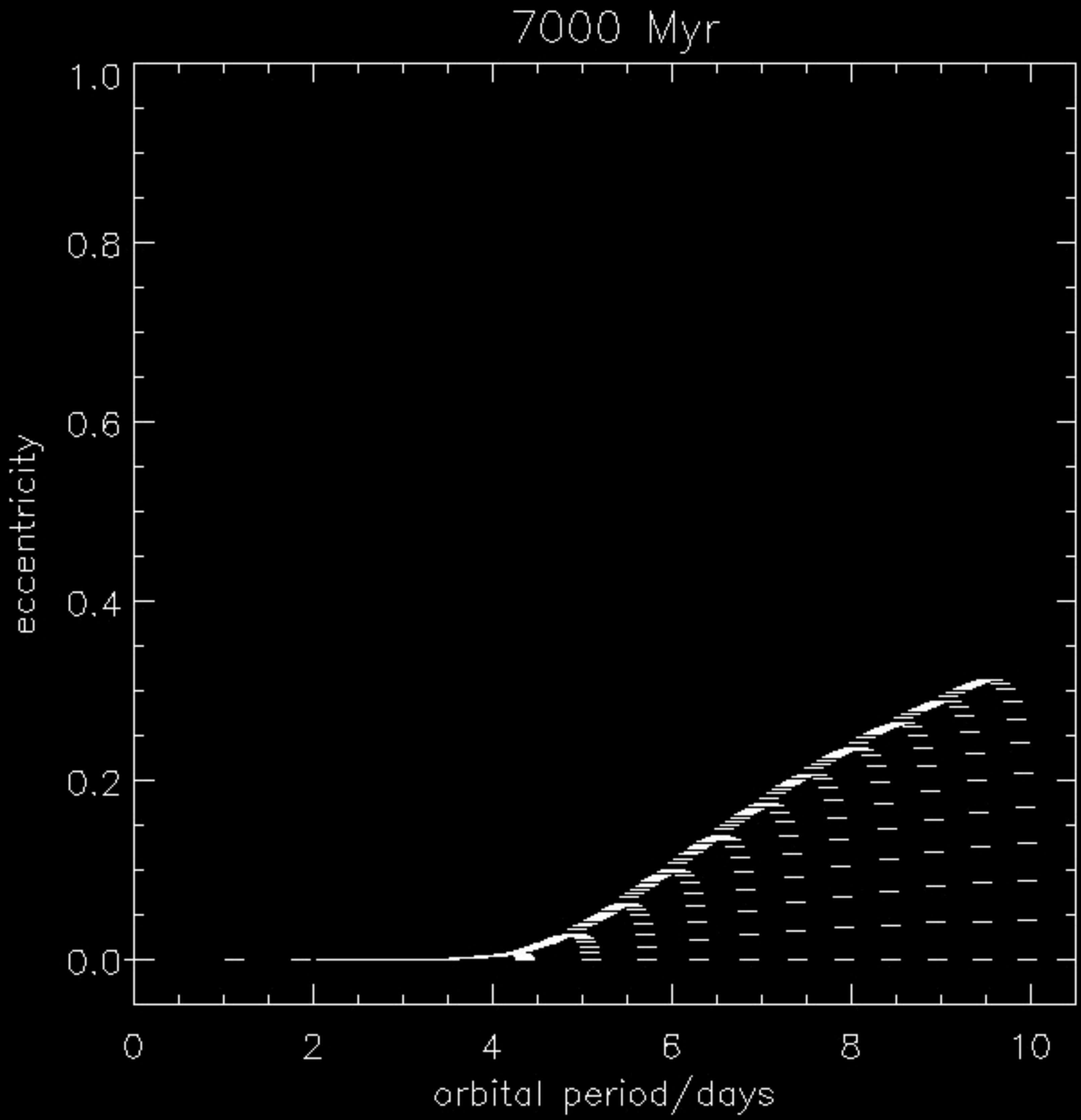

2000 Myr
eccentricity
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10
orbital period/days

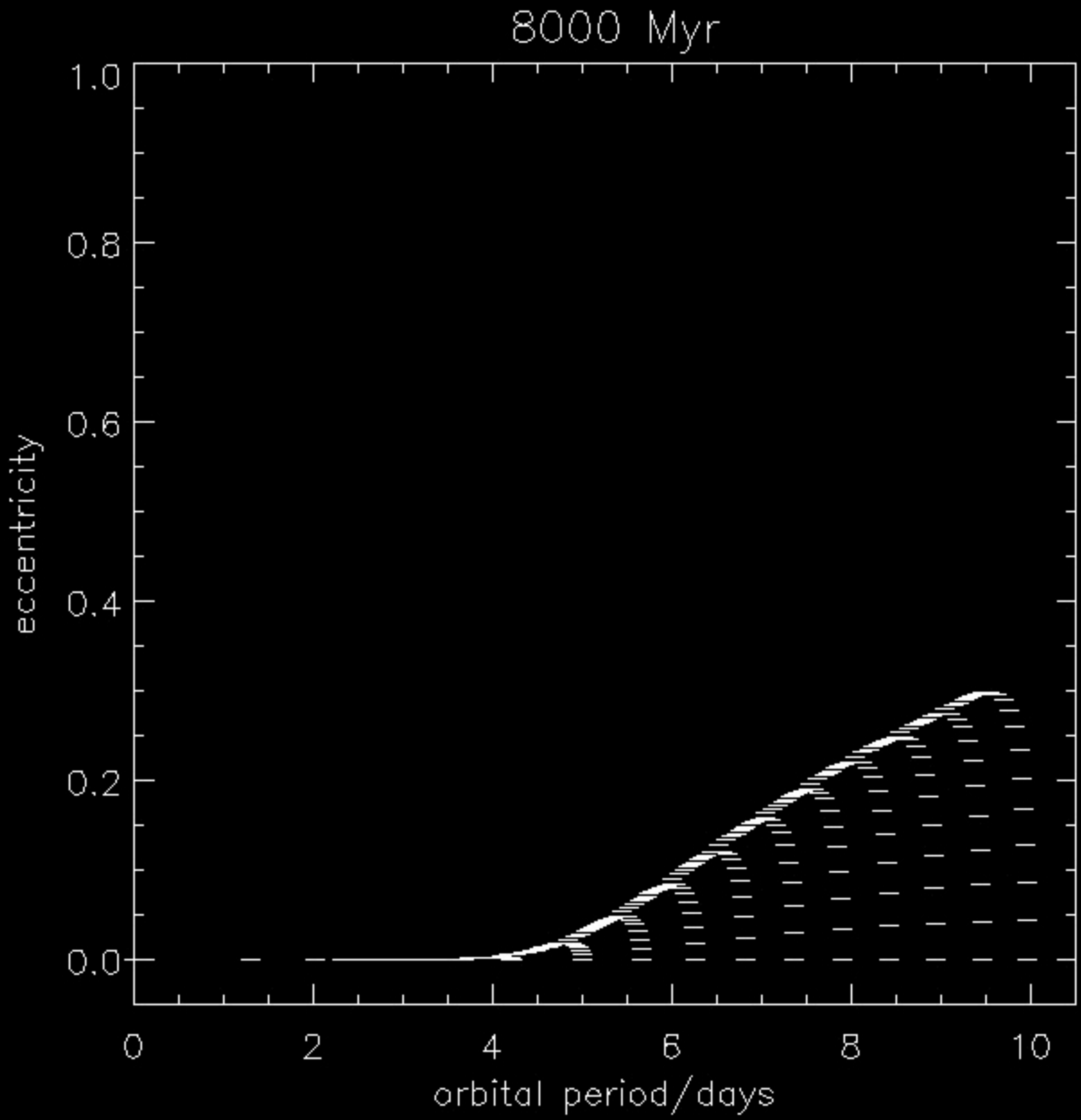

3000 Myr
1.0
0.8
0.6
0.4
0.2
0.0
eccentricity
0
2
4
6
8
10
orbital period/days

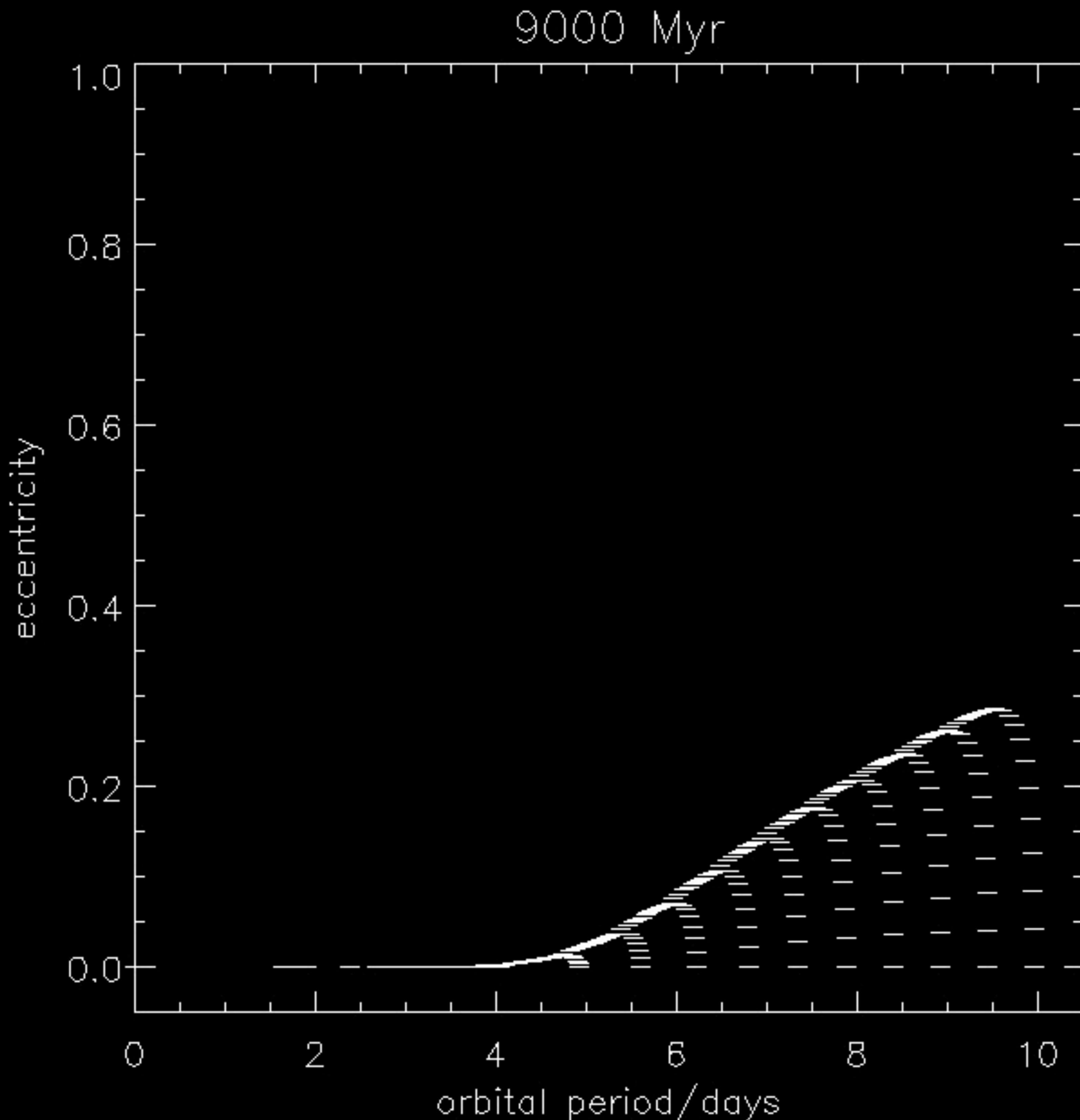

4000 Myr
1.0
0.8
0.6
0.4
0.2
0.0
eccentricity
0
2
4
6
8
10
orbital period/days

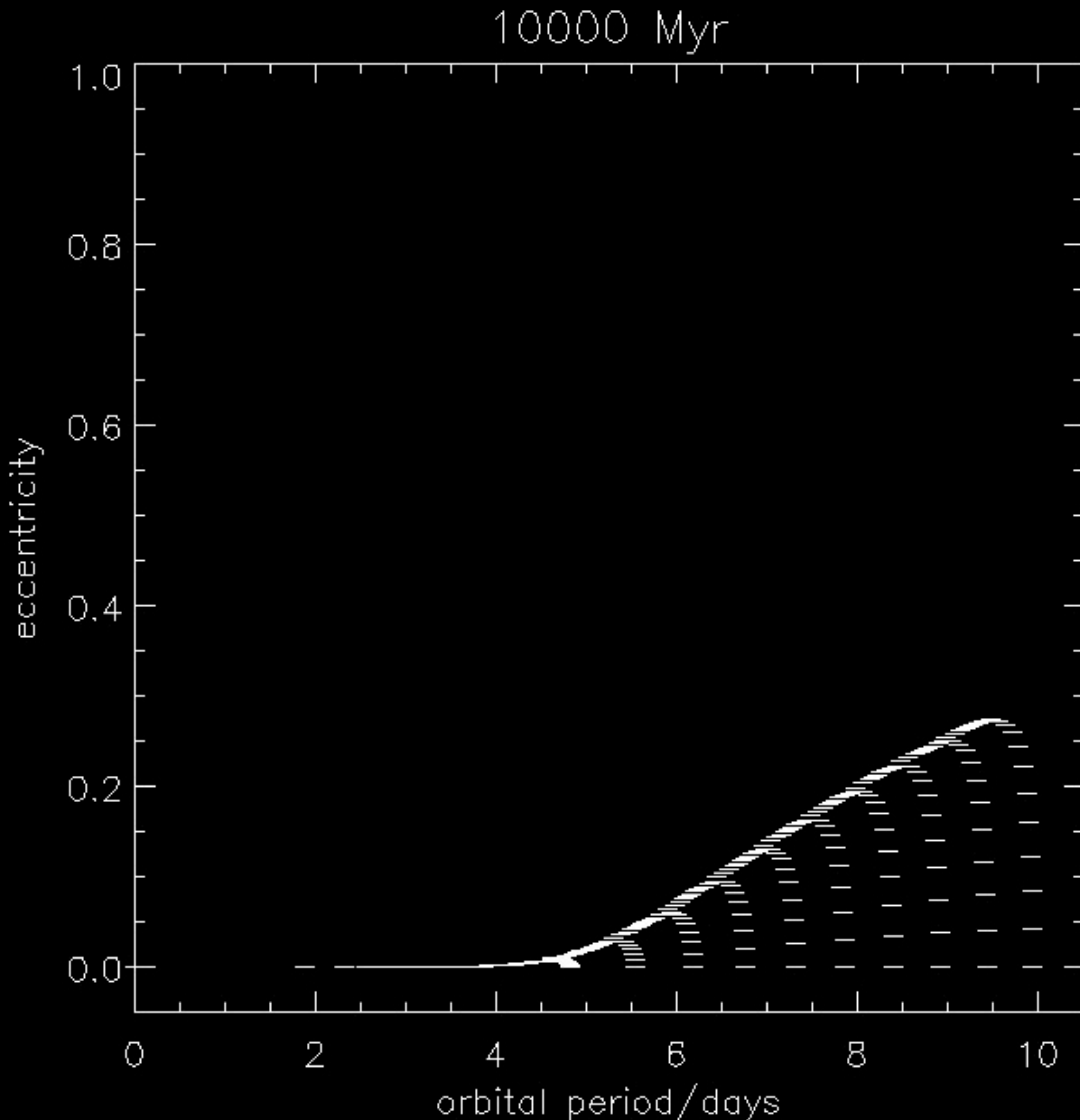

5000 Myr
1.0
0.8
0.6
0.4
0.2
0.0
eccentricity
0
2
4
6
8
10
orbital period/days


6000 Myr
eccentricity
orbital period/days
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10

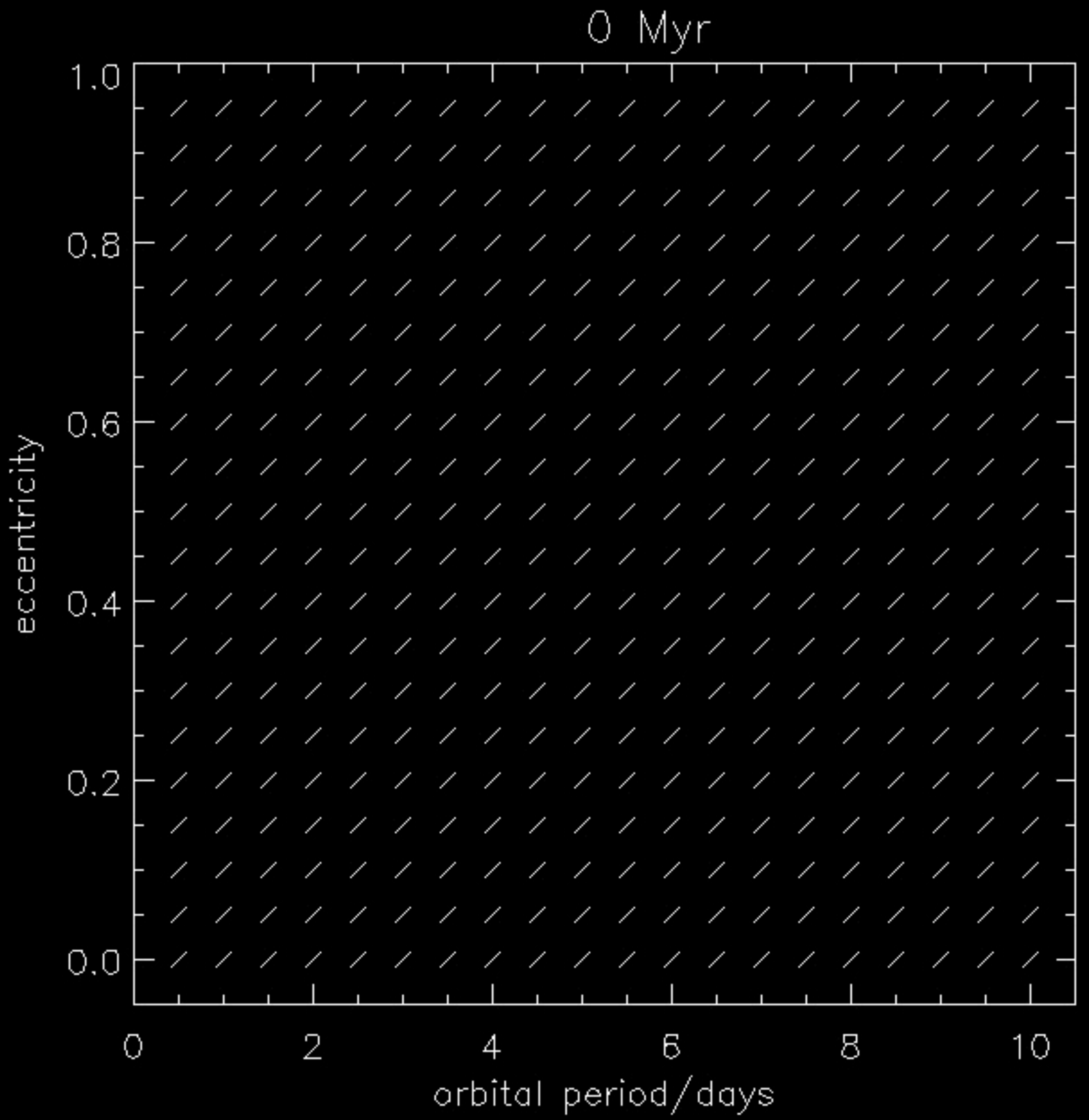

7000 Myr
eccentricity
orbital period/days
0.0
0.2
0.4
0.6
0.8
1.0
0
2
4
6
8
10

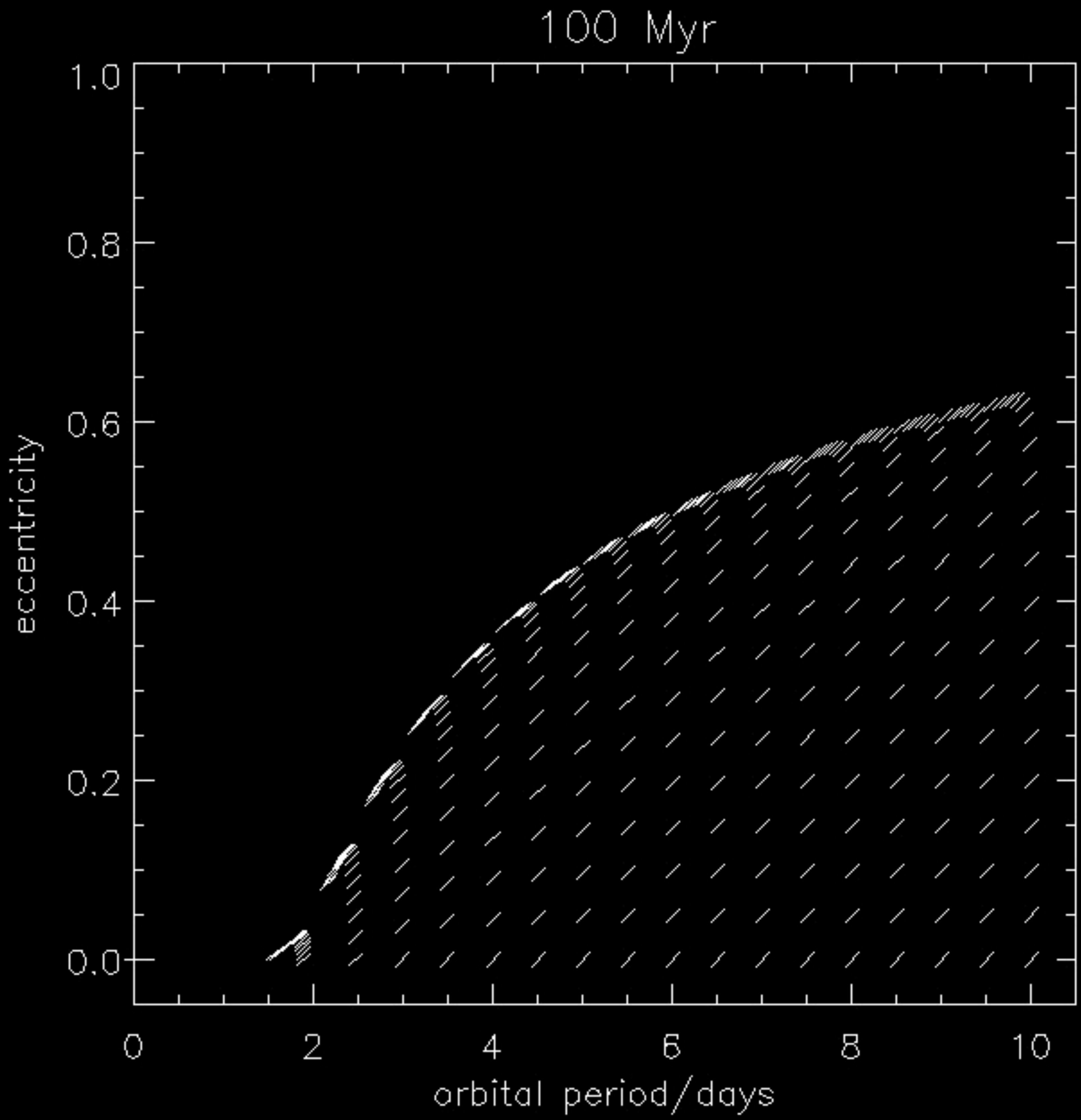

8000 Myr
eccentricity
orbital period/days
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10

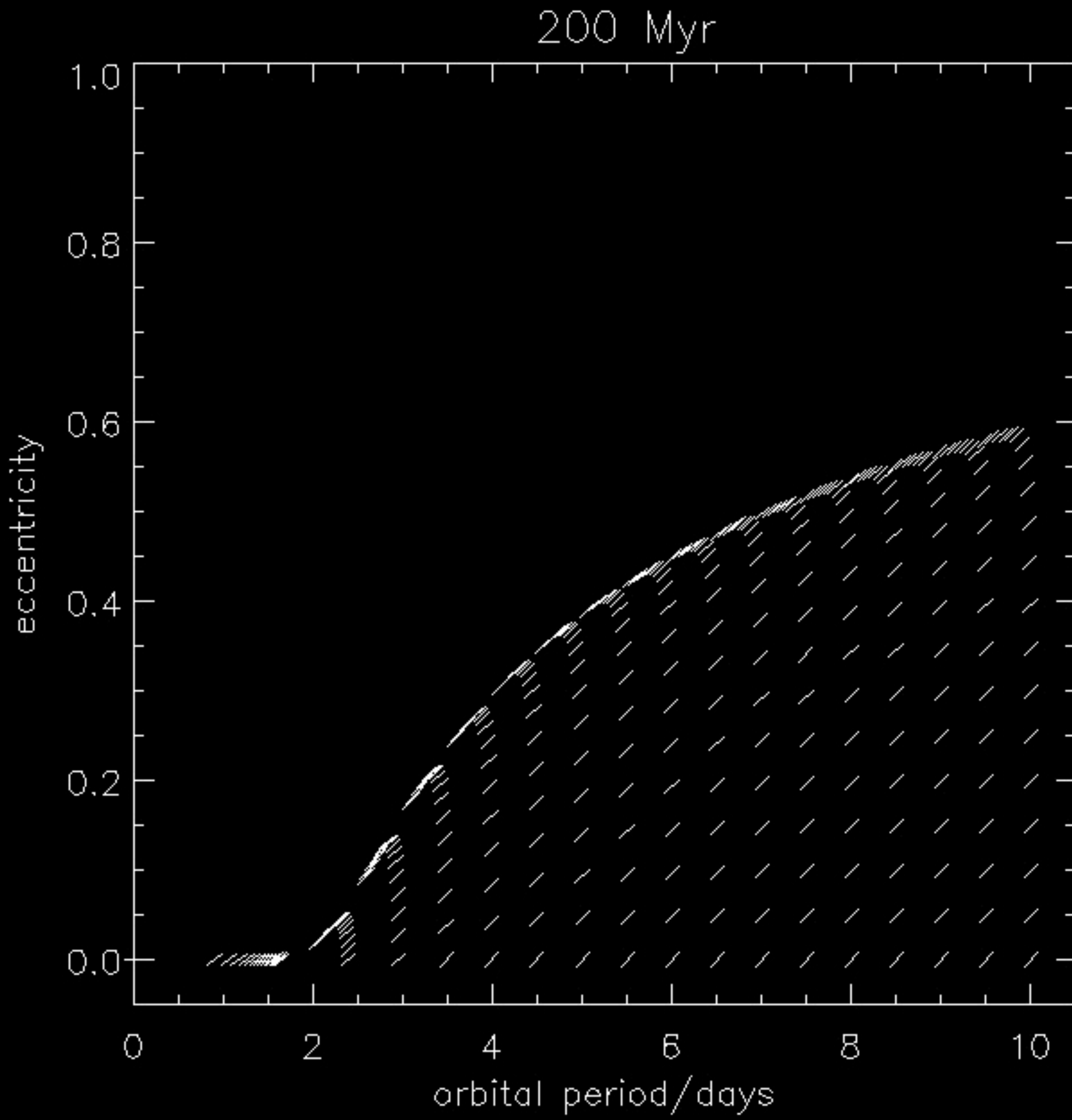

9000 Myr
eccentricity
orbital period/days
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10

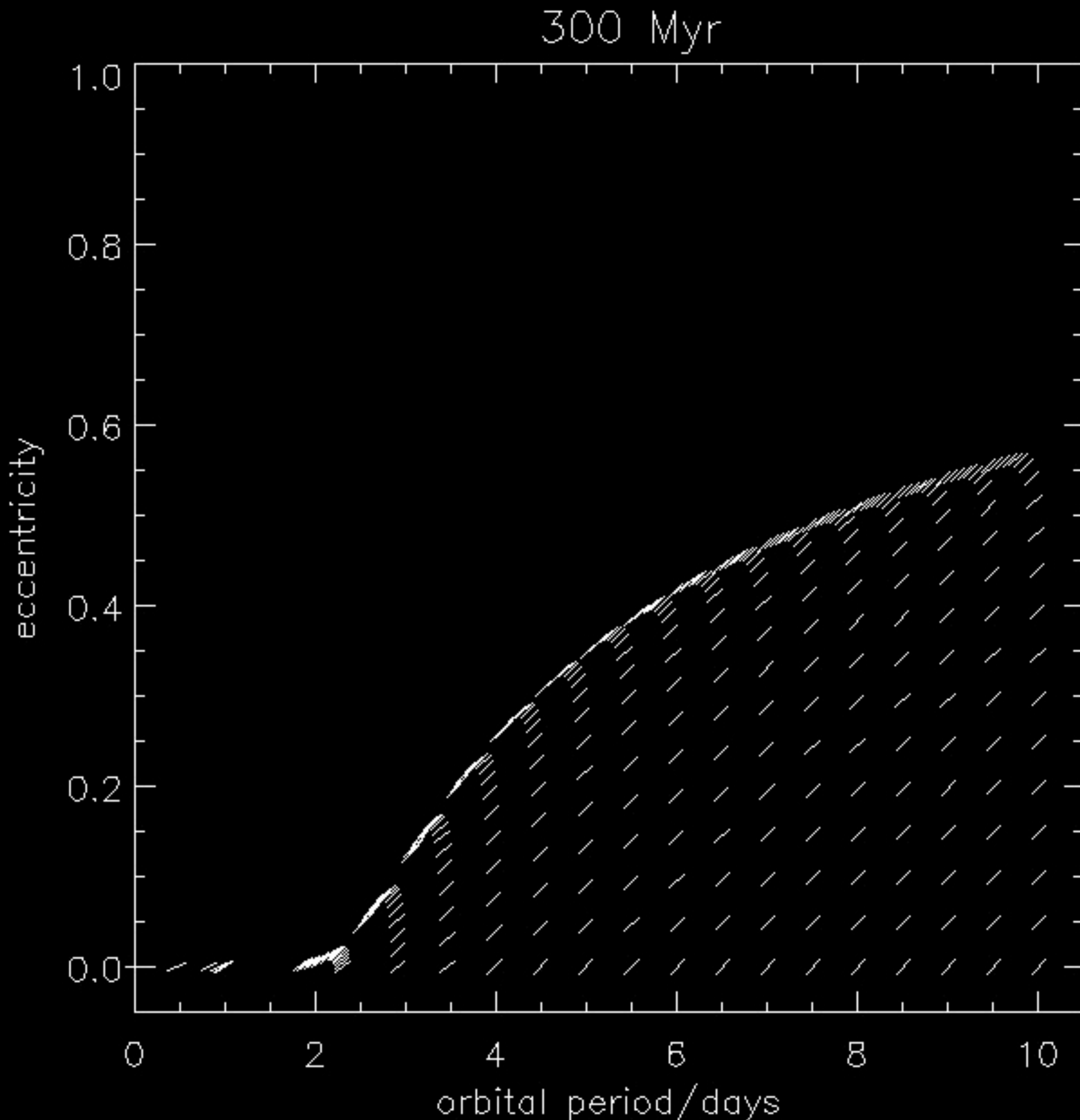

10000 Myr
eccentricity
orbital period/days
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10

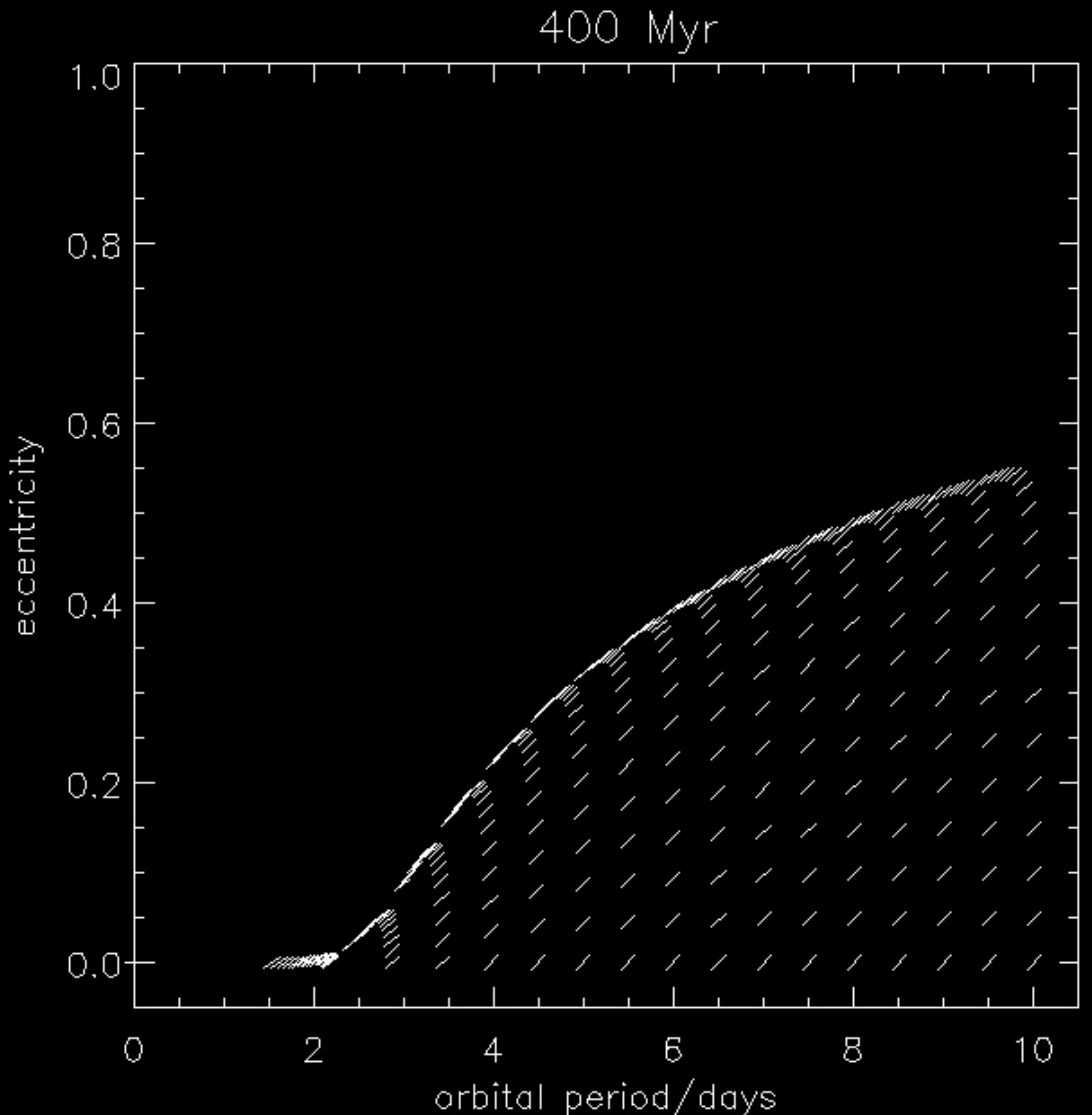
Initial stellar obliquity 45°

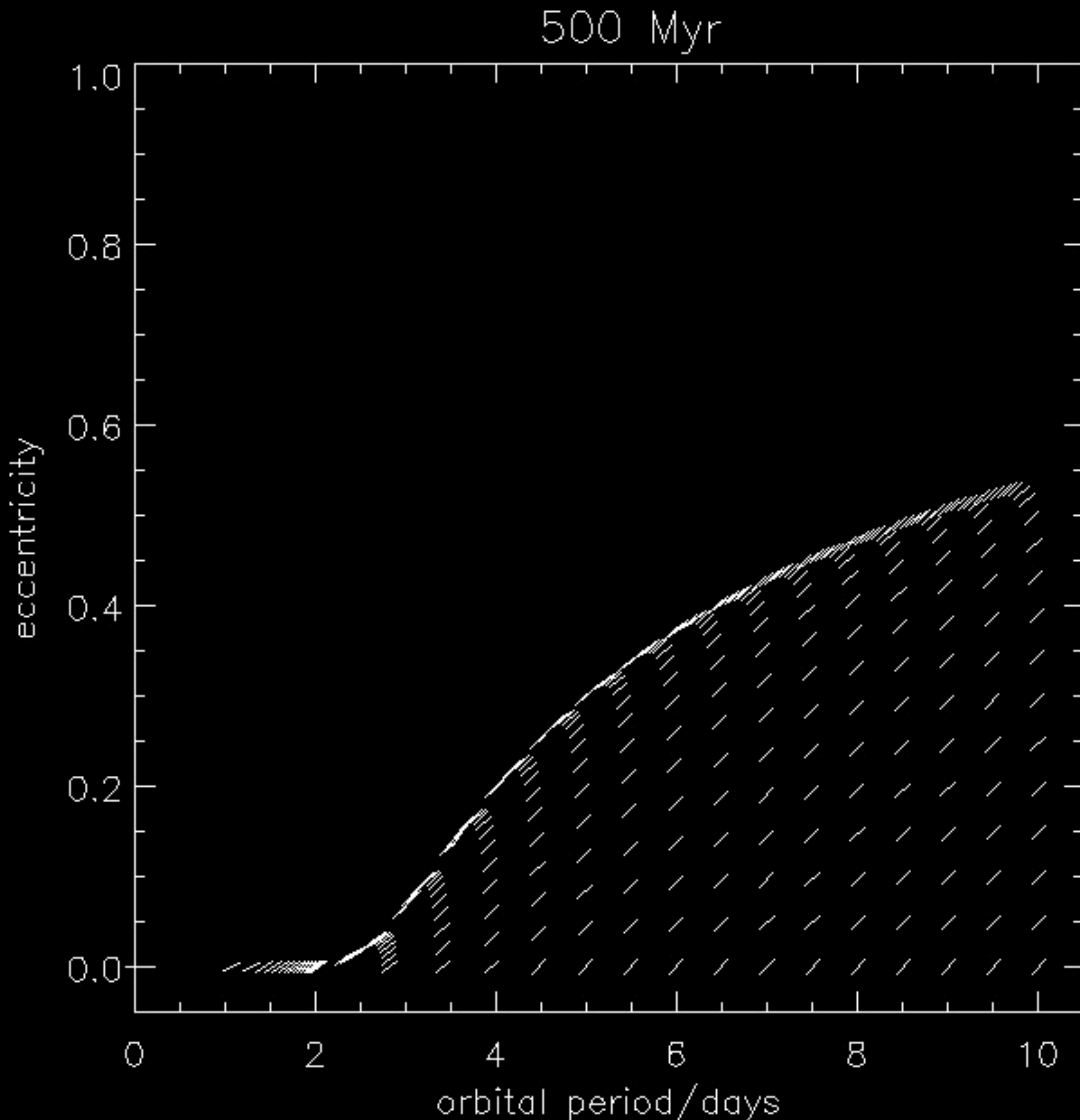

0 Myr
eccentricity
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10
orbital period/days

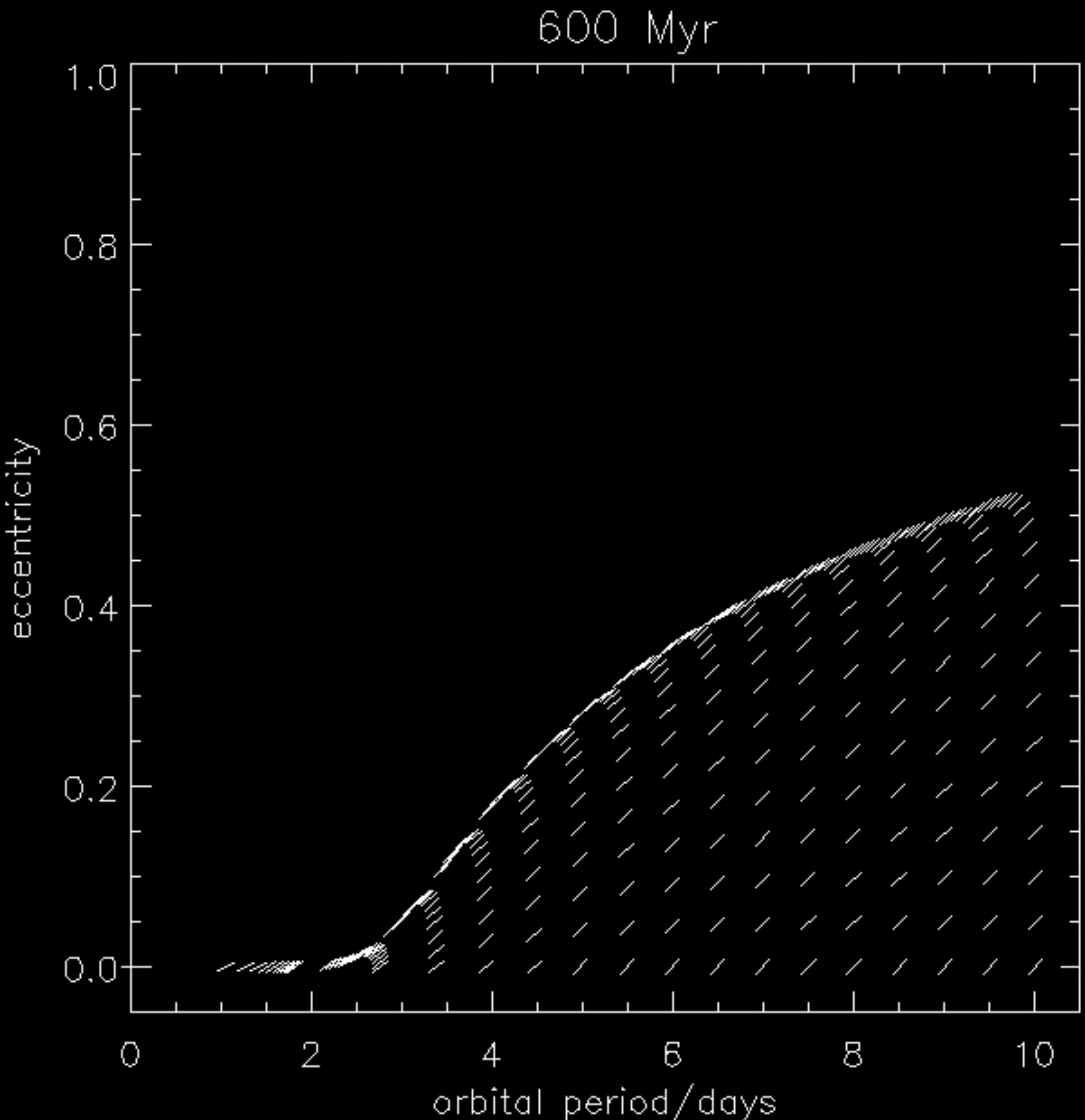

100 Myr
1.0
0.8
0.6
0.4
0.2
0.0
eccentricity
0
2
4
6
8
10
orbital period/days

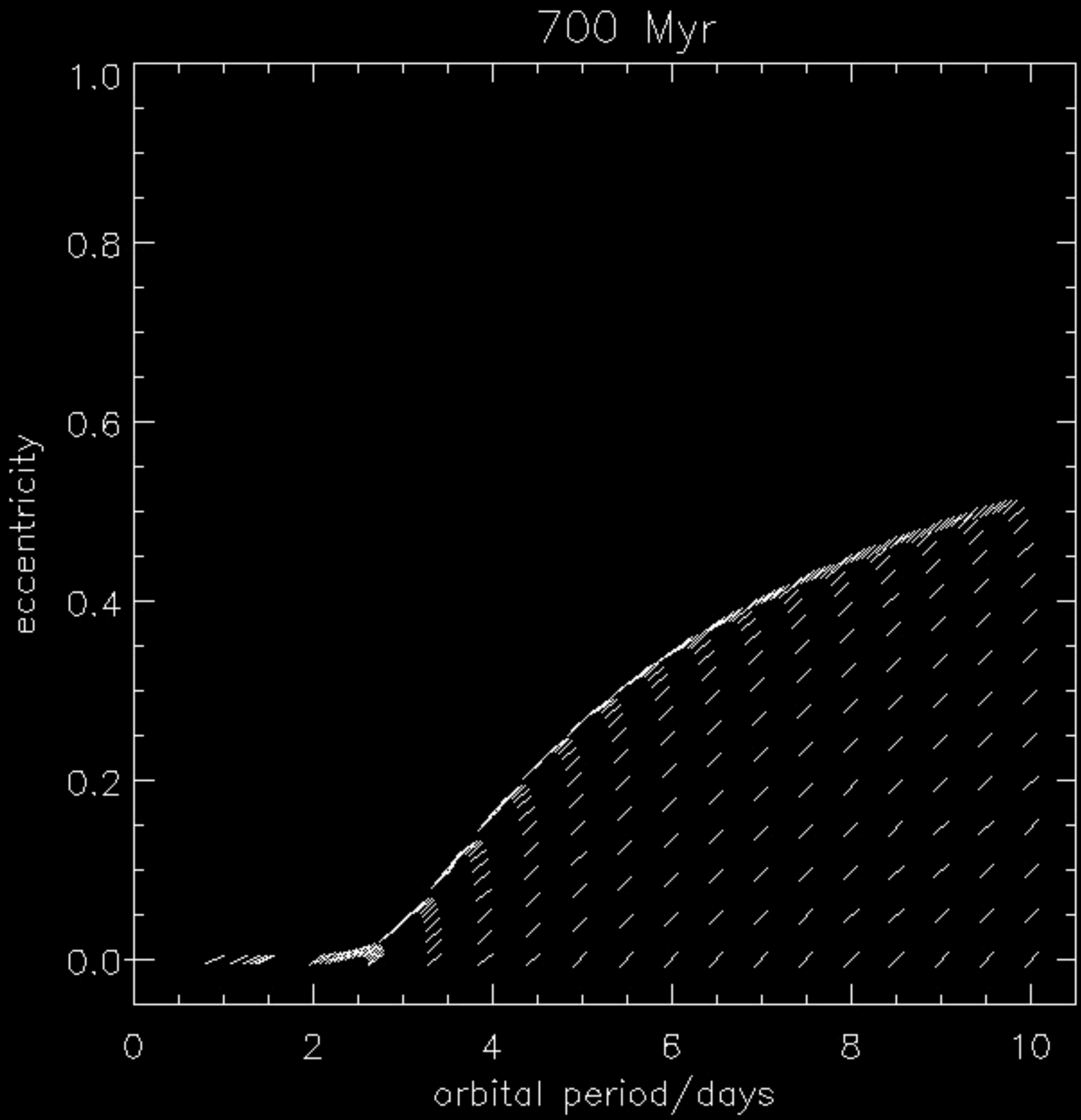

200 Myr
eccentricity
orbital period/days
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10

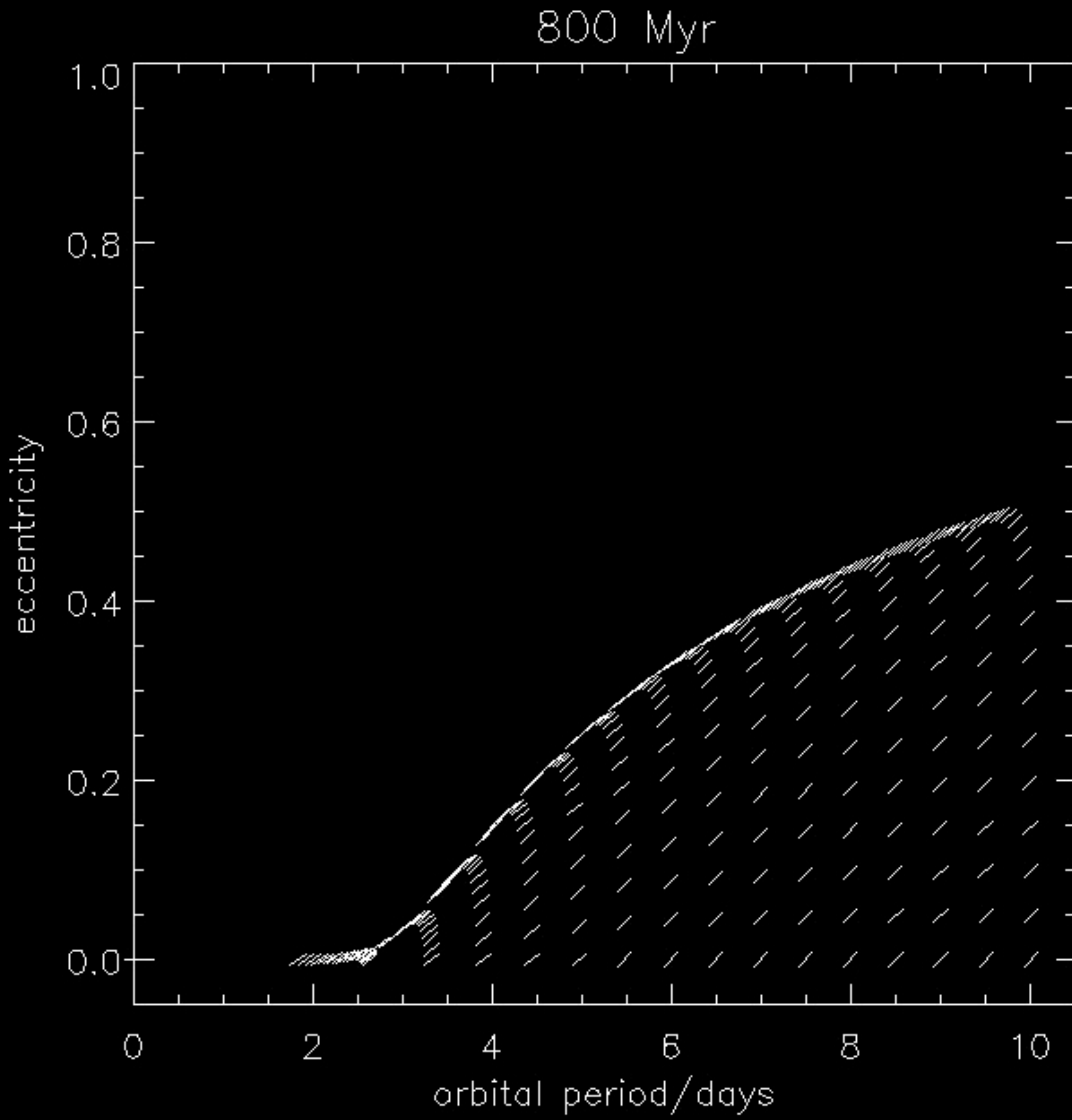

300 Myr
eccentricity
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10
orbital period/days

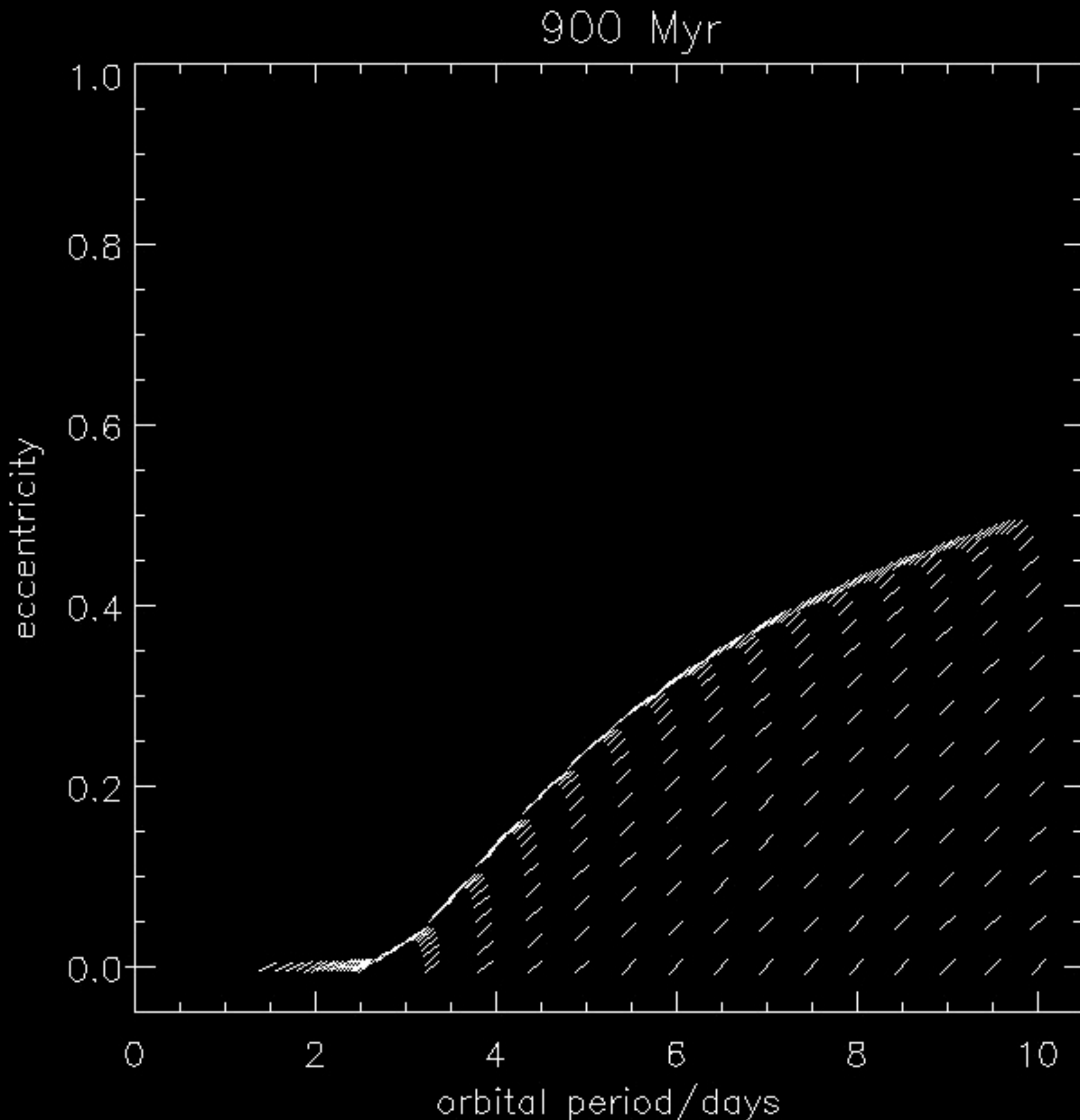

400 Myr
eccentricity
orbital period/days
0.0
0.2
0.4
0.6
0.8
1.0
0
2
4
6
8
10

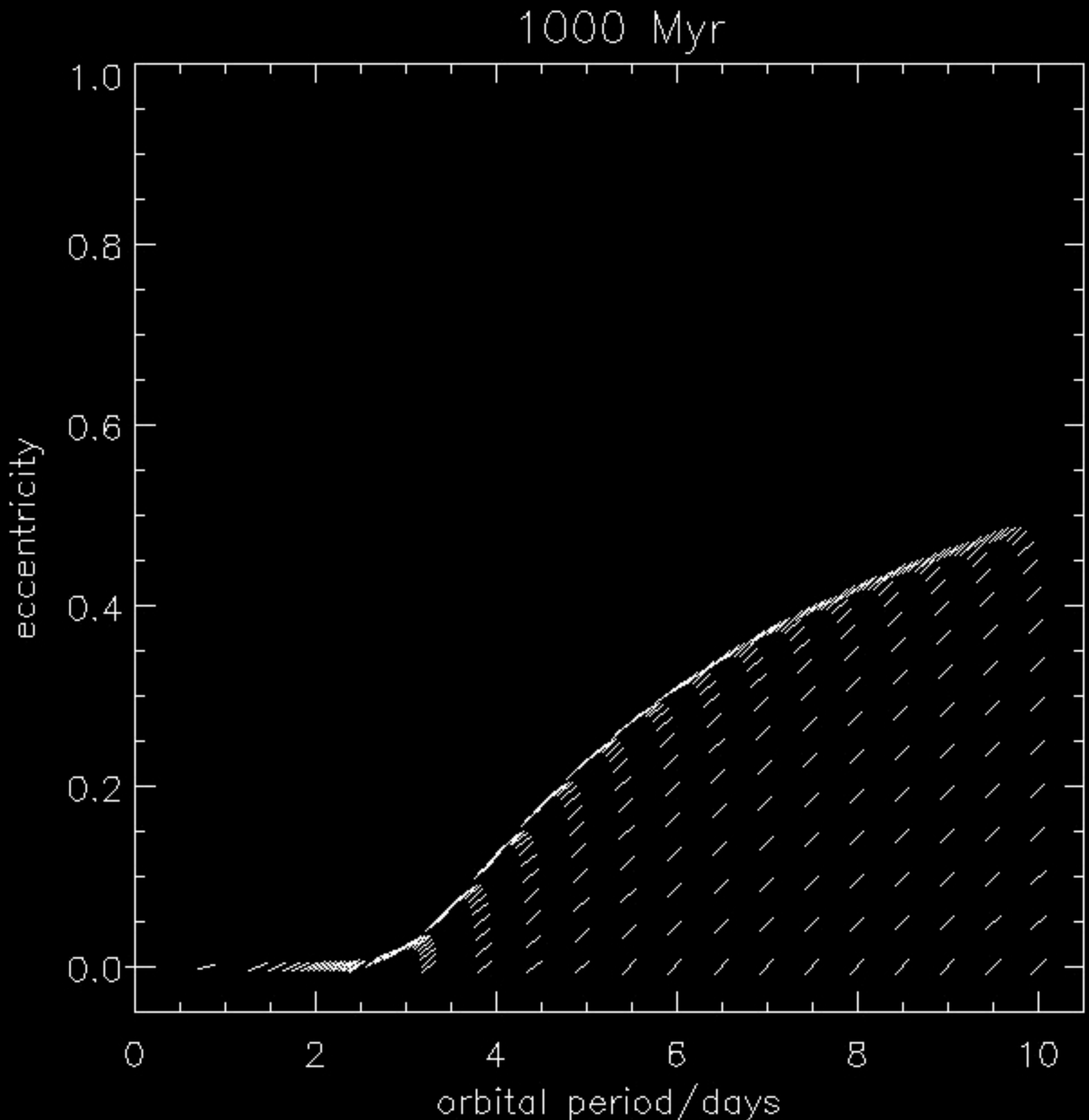

500 Myr
eccentricity
orbital period/days
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10

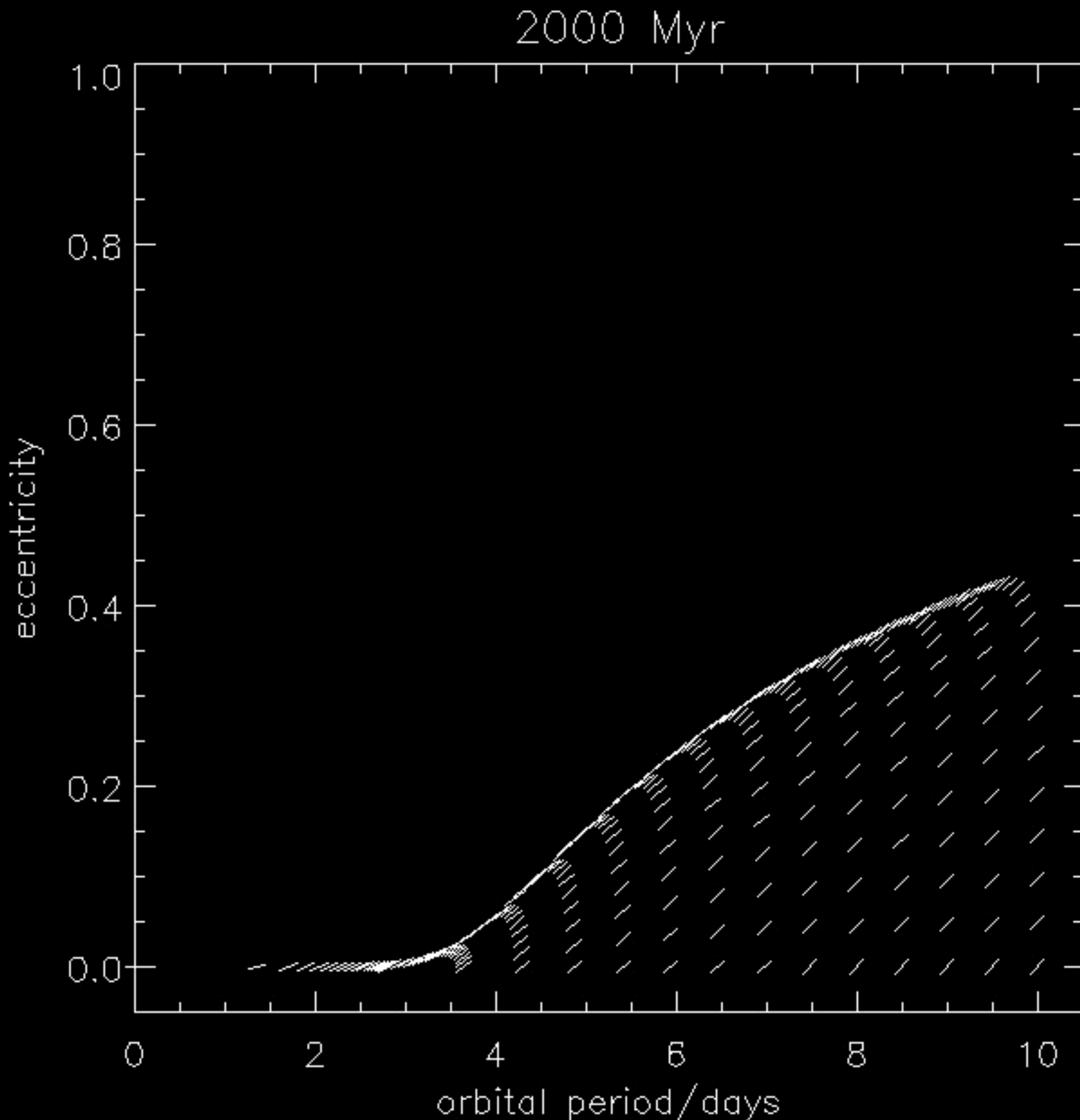

600 Myr
eccentricity
0.0
0.2
0.4
0.6
0.8
1.0
0
2
4
6
8
10
orbital period/days

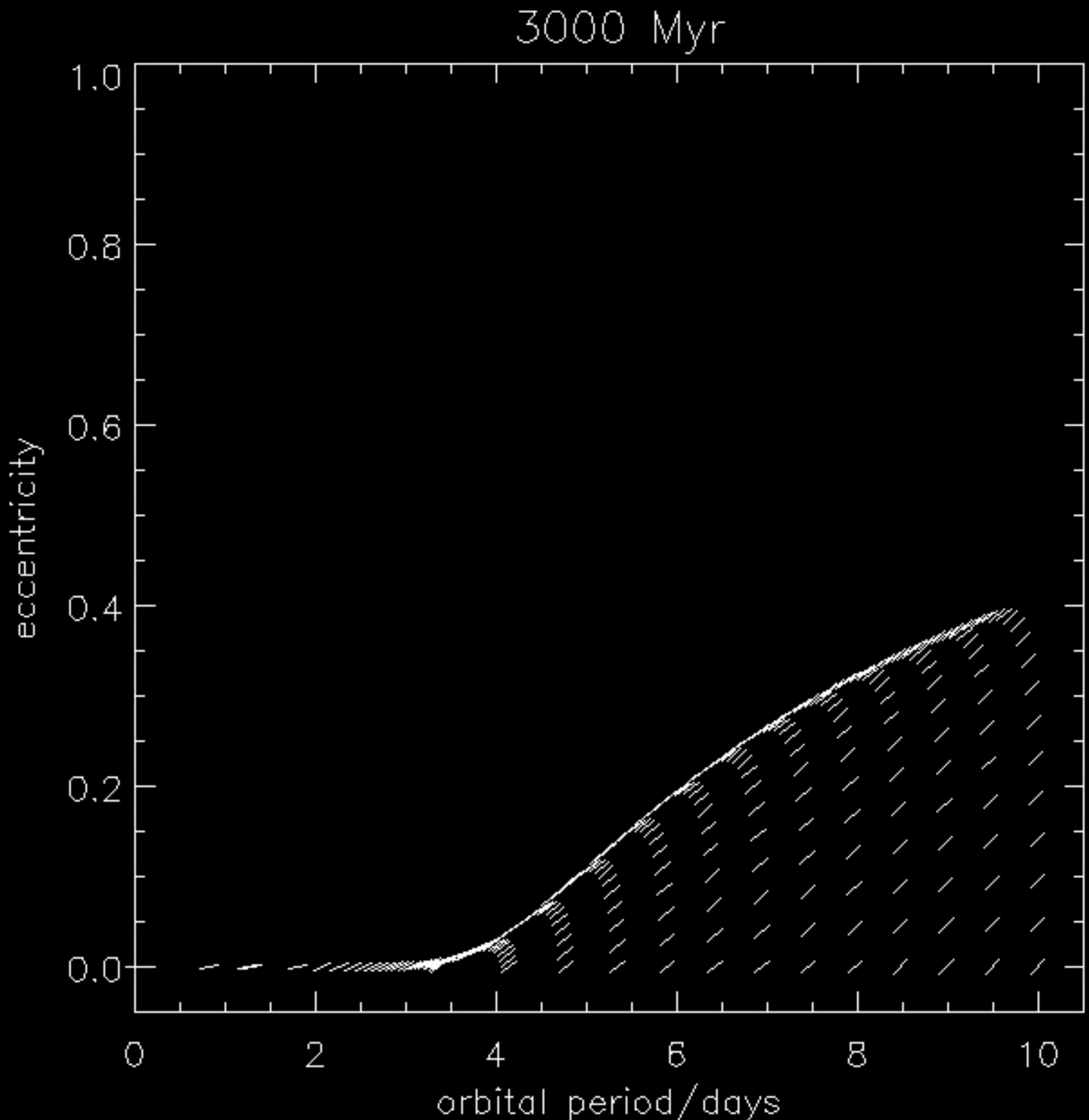

700 Myr
1.0
0.8
0.6
0.4
0.2
0.0
eccentricity
0
2
4
6
8
10
orbital period/days

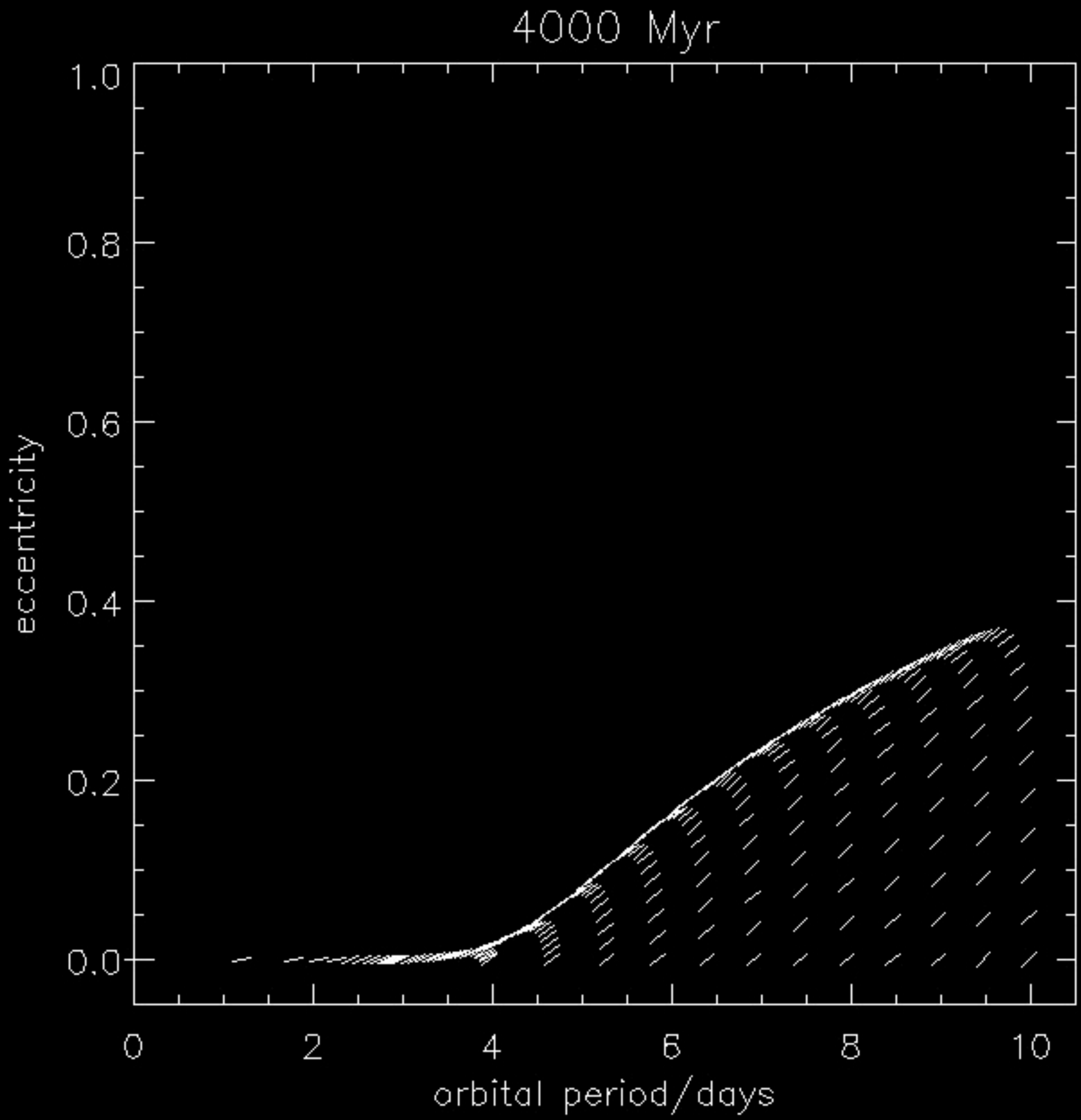

800 Myr
1.0
0.8
0.6
0.4
0.2
0.0
eccentricity
0
2
4
6
8
10
orbital period/days

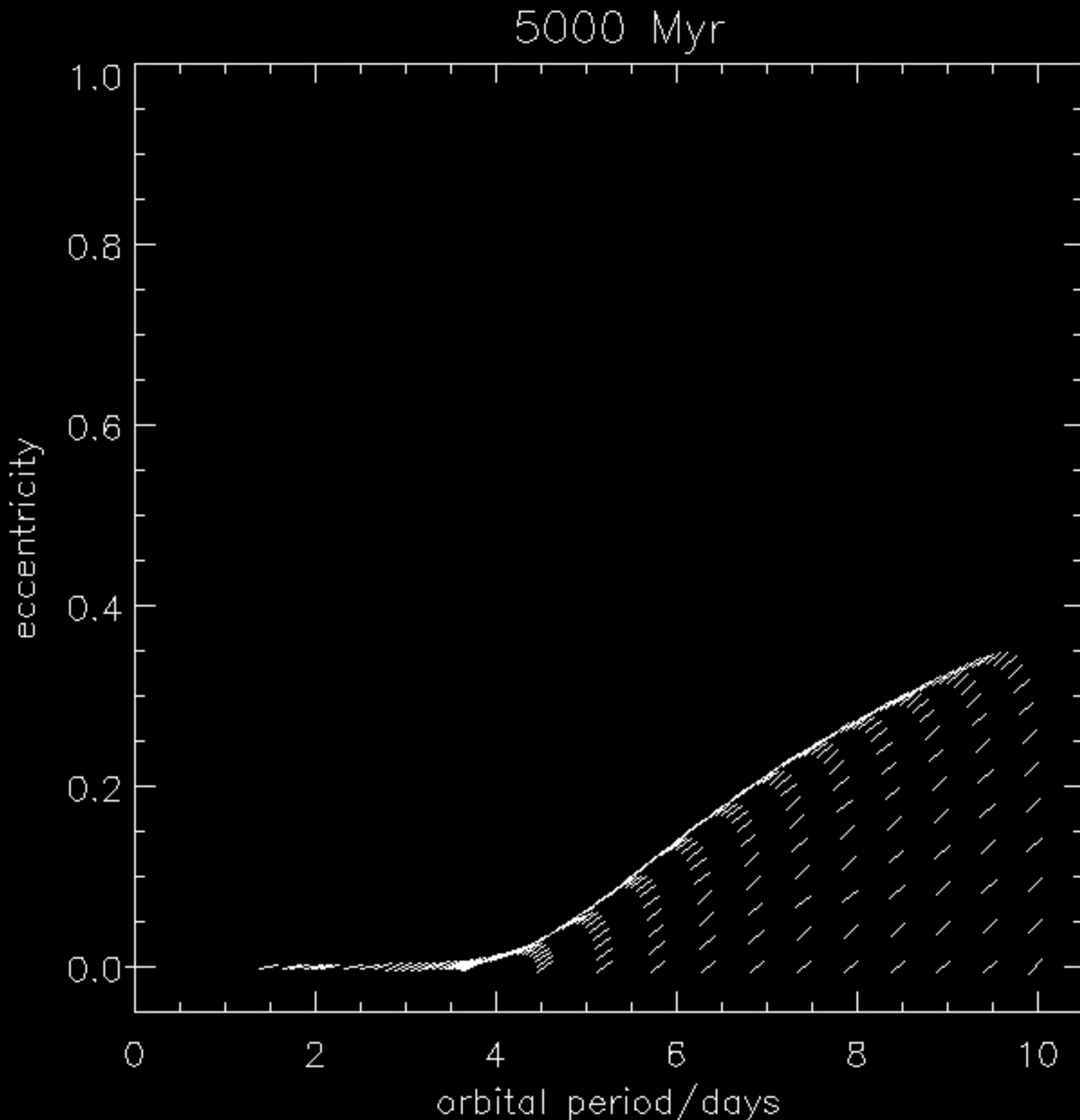

900 Myr
1.0
0.8
0.6
0.4
0.2
0.0
eccentricity
0
2
4
6
8
10
orbital period/days


1000 Myr
eccentricity
orbital period/days
0.0
0.2
0.4
0.6
0.8
1.0
0
2
4
6
8
10

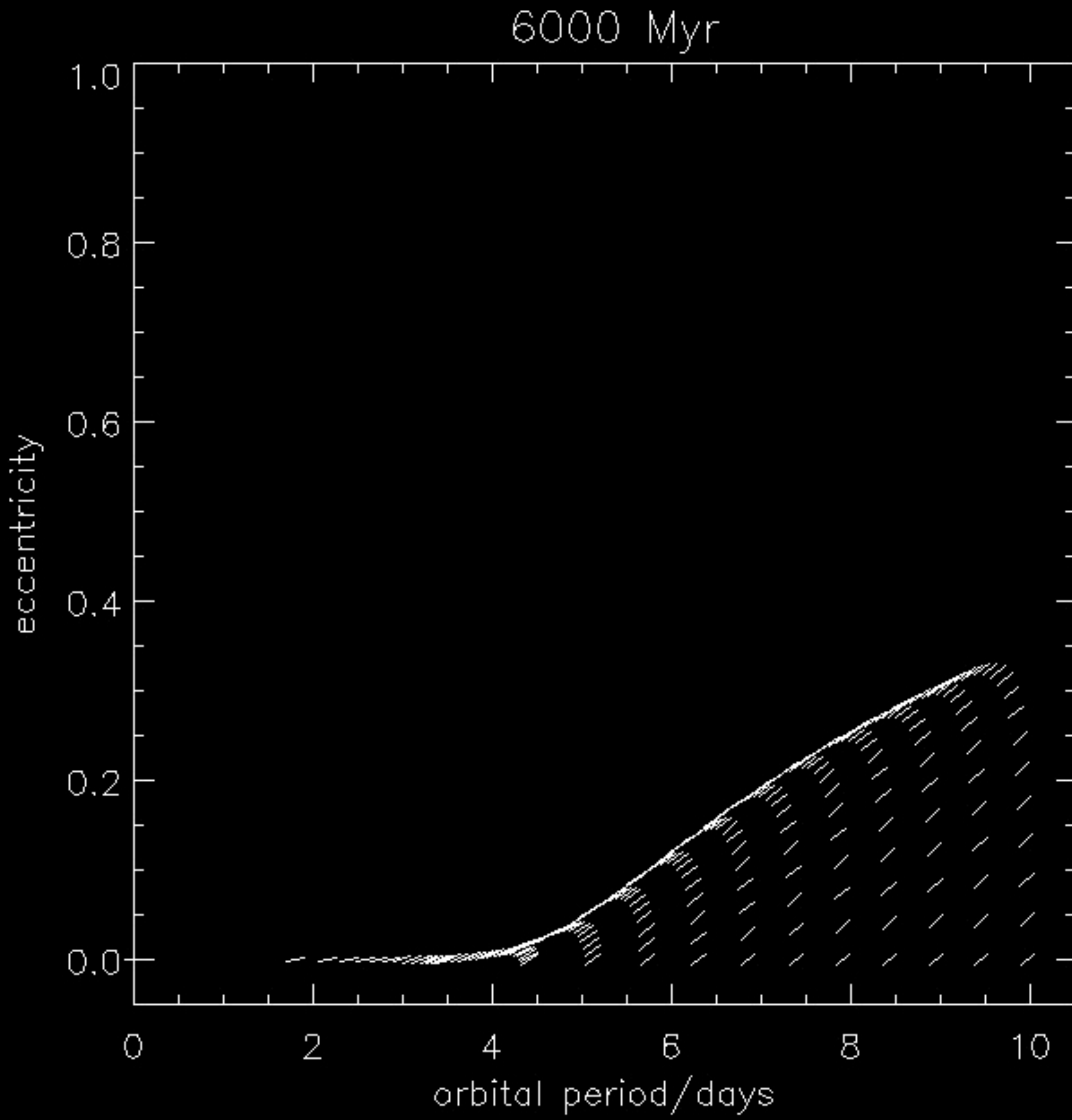

2000 Myr
eccentricity
orbital period/days
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10

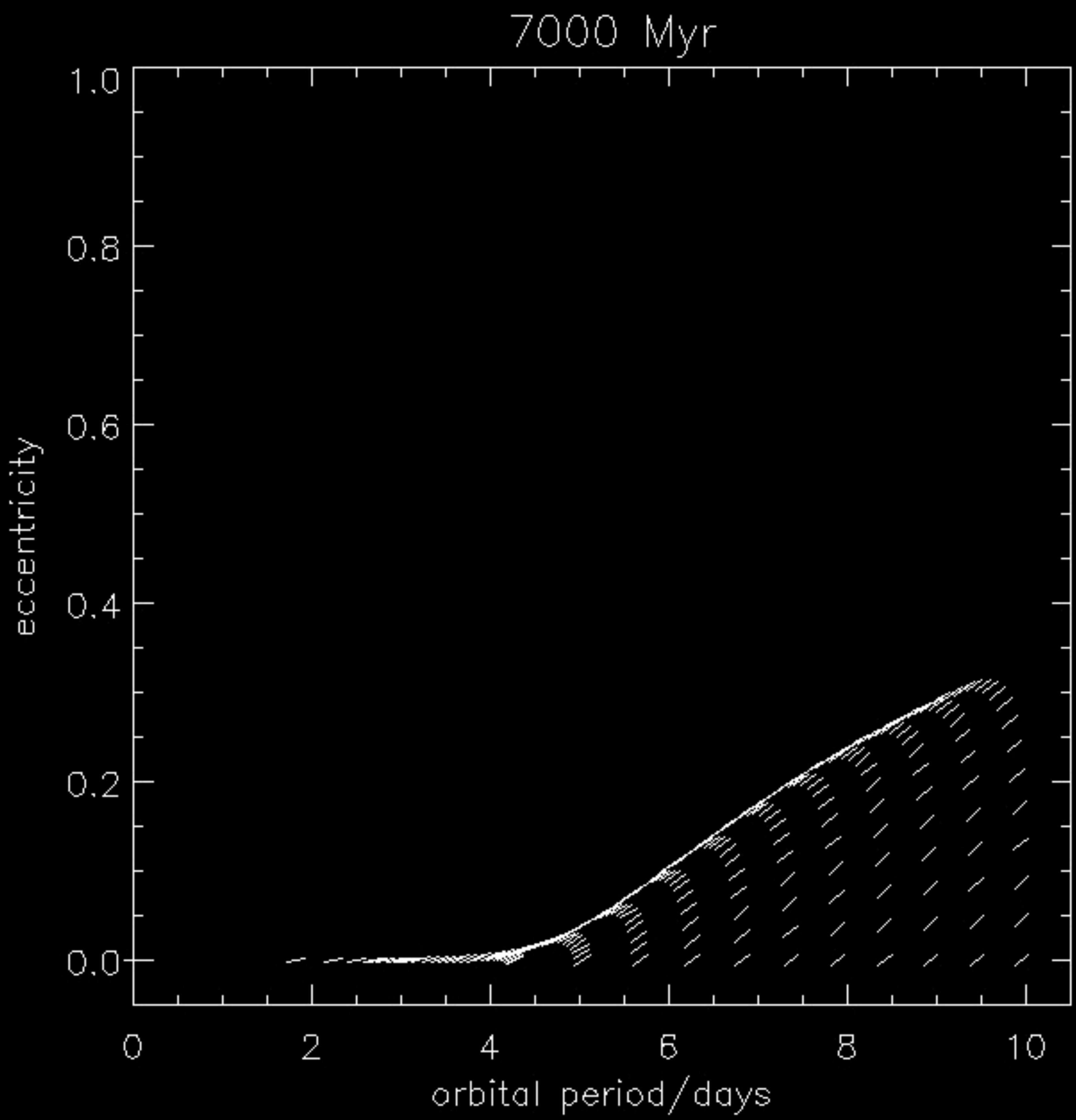

3000 Myr
eccentricity
orbital period/days
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10

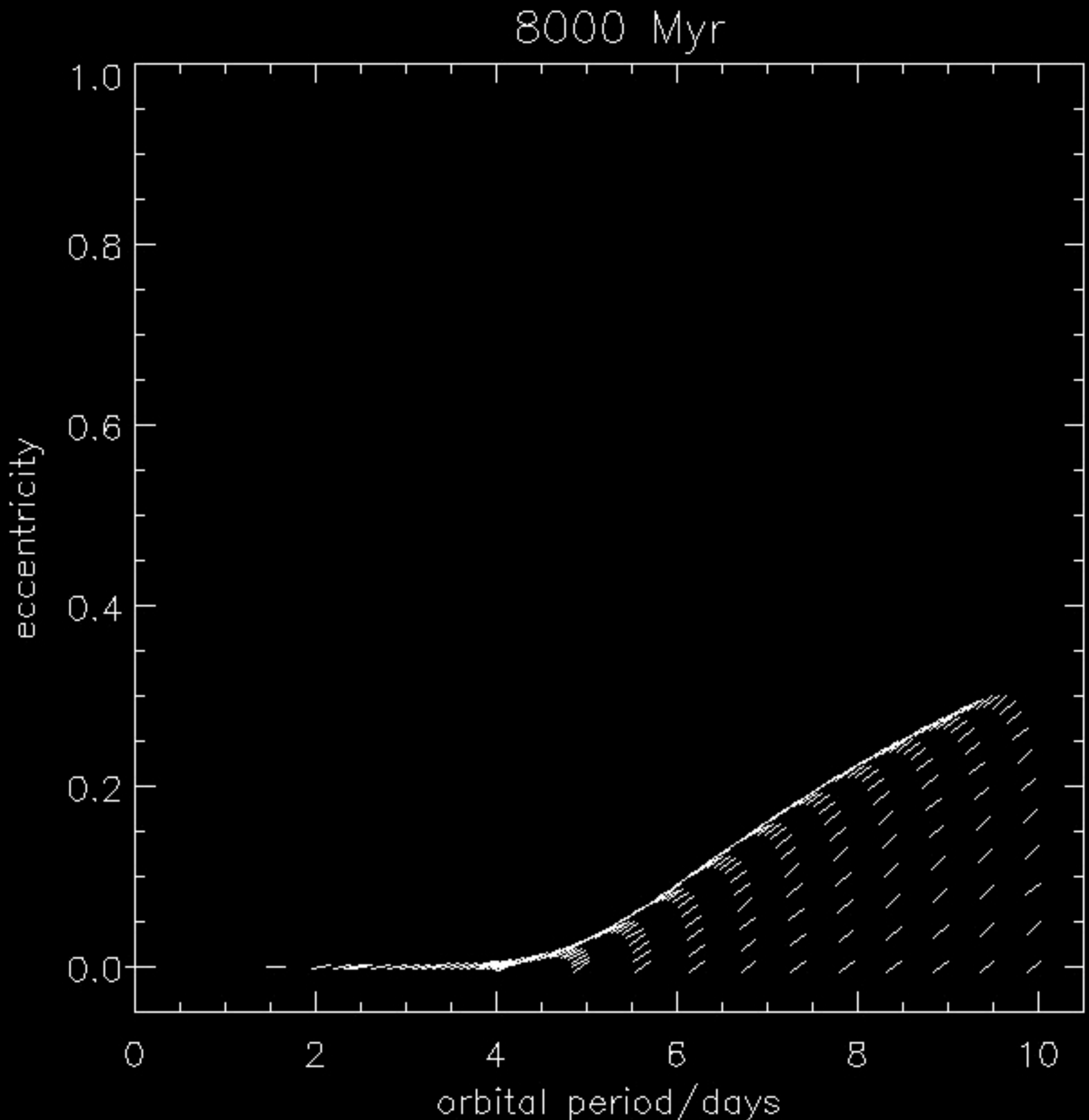

4000 Myr
eccentricity
orbital period/days
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10

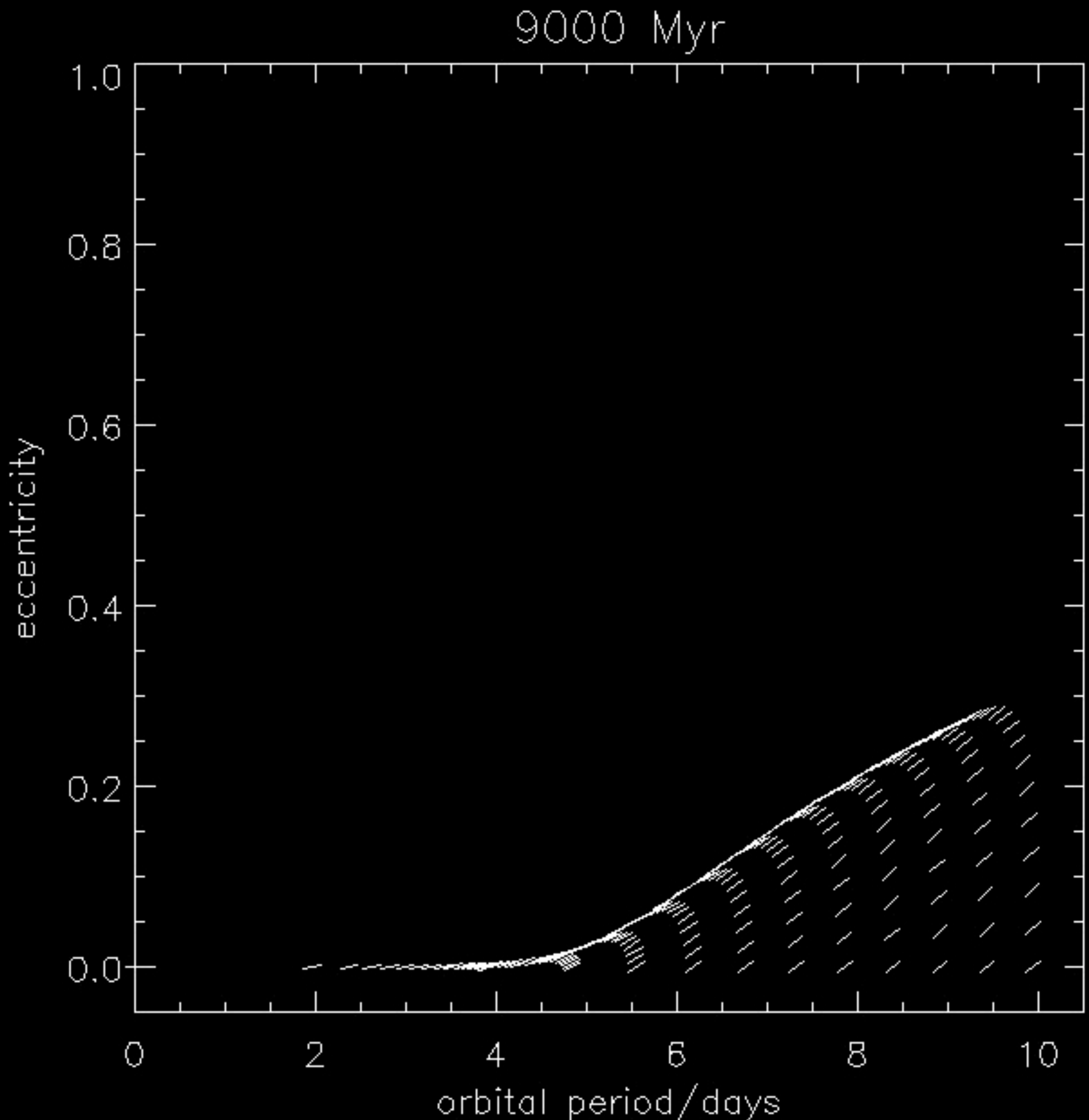


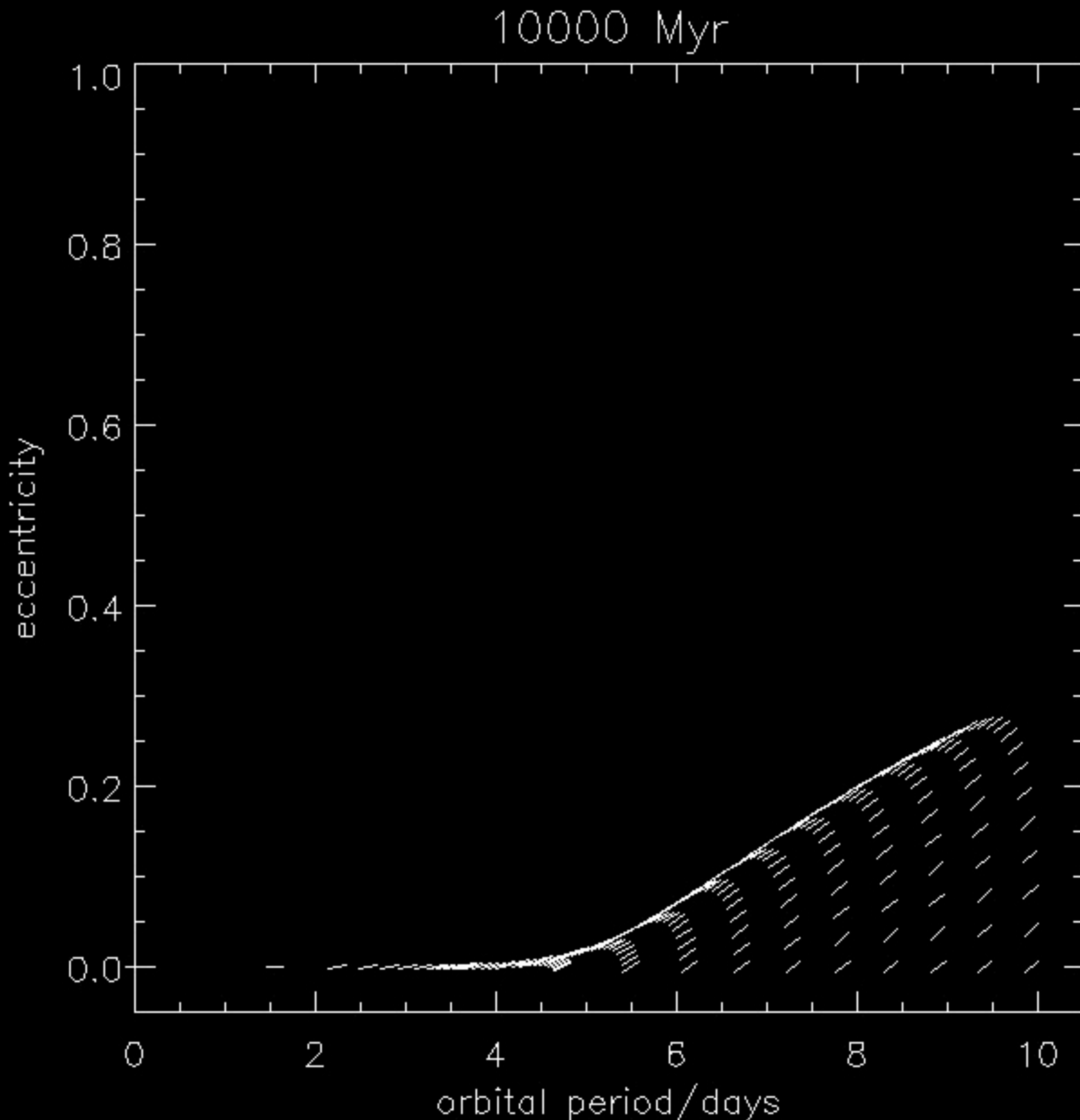
2011年12月2日金曜日


6000 Myr
eccentricity
orbital period/days
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10


7000 Myr
eccentricity
orbital period/days
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10

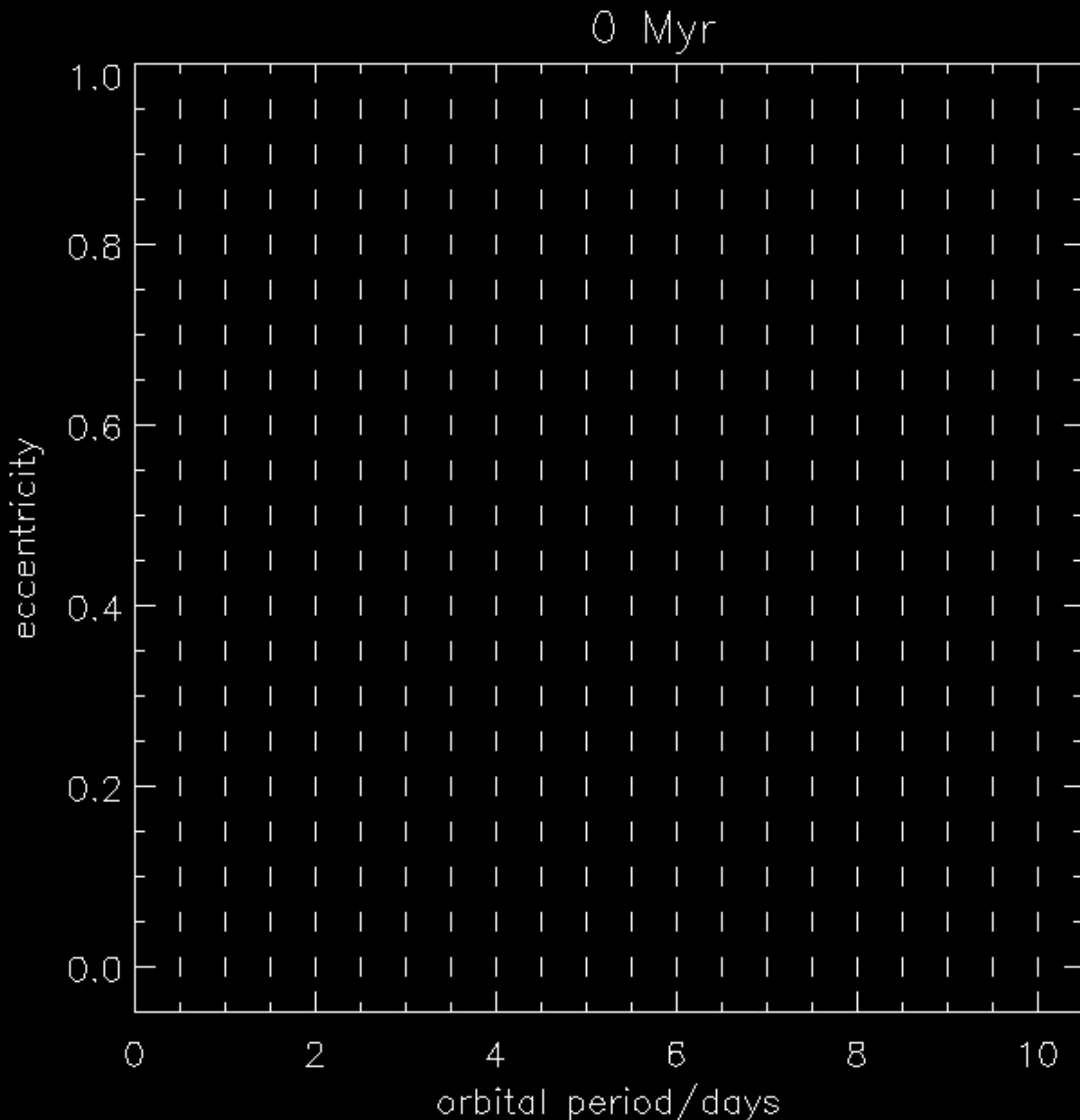

8000 Myr
eccentricity
orbital period/days
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10

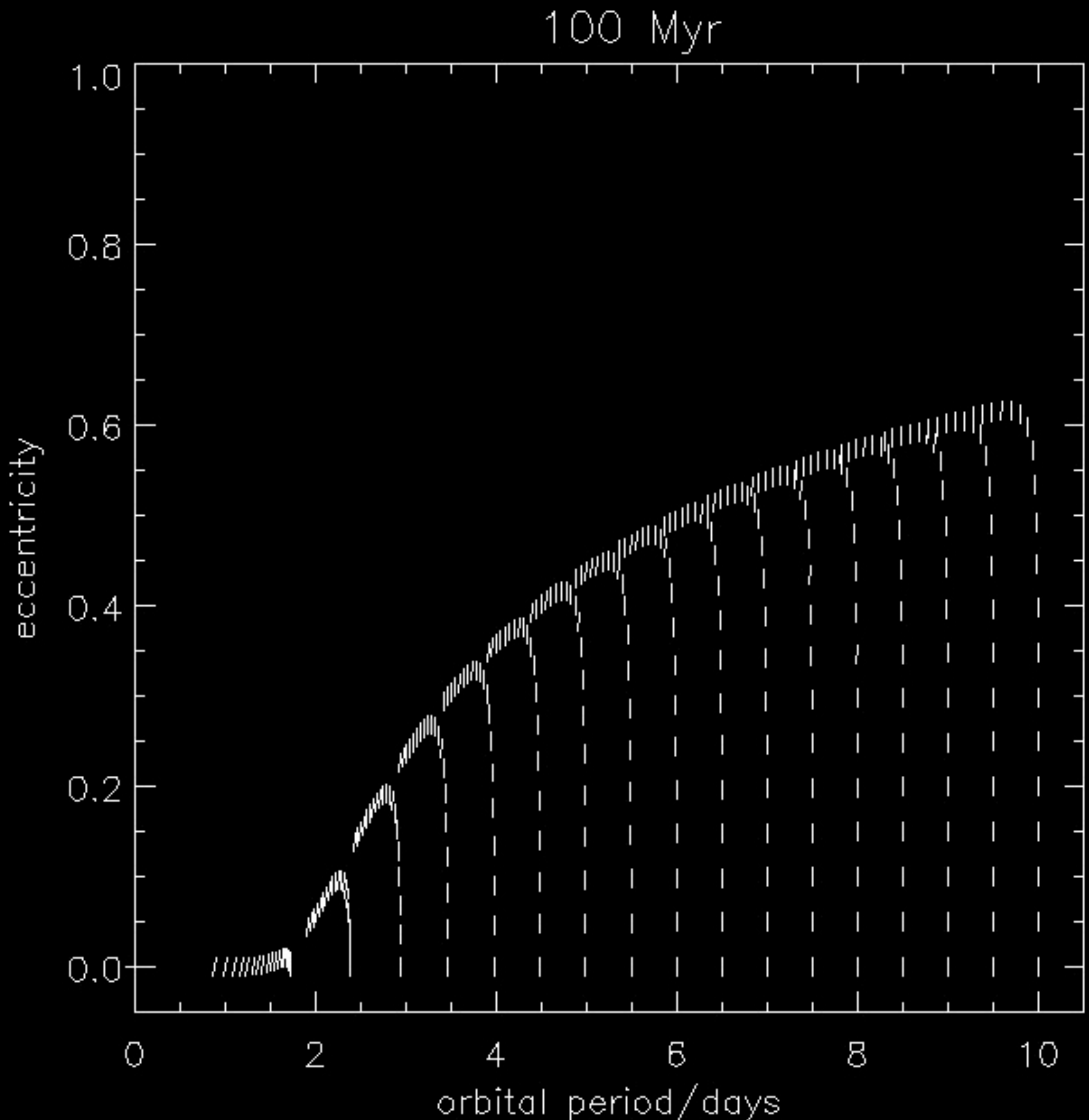

9000 Myr
1.0
0.8
0.6
0.4
0.2
0.0
eccentricity
0
2
4
6
8
10
orbital period/days



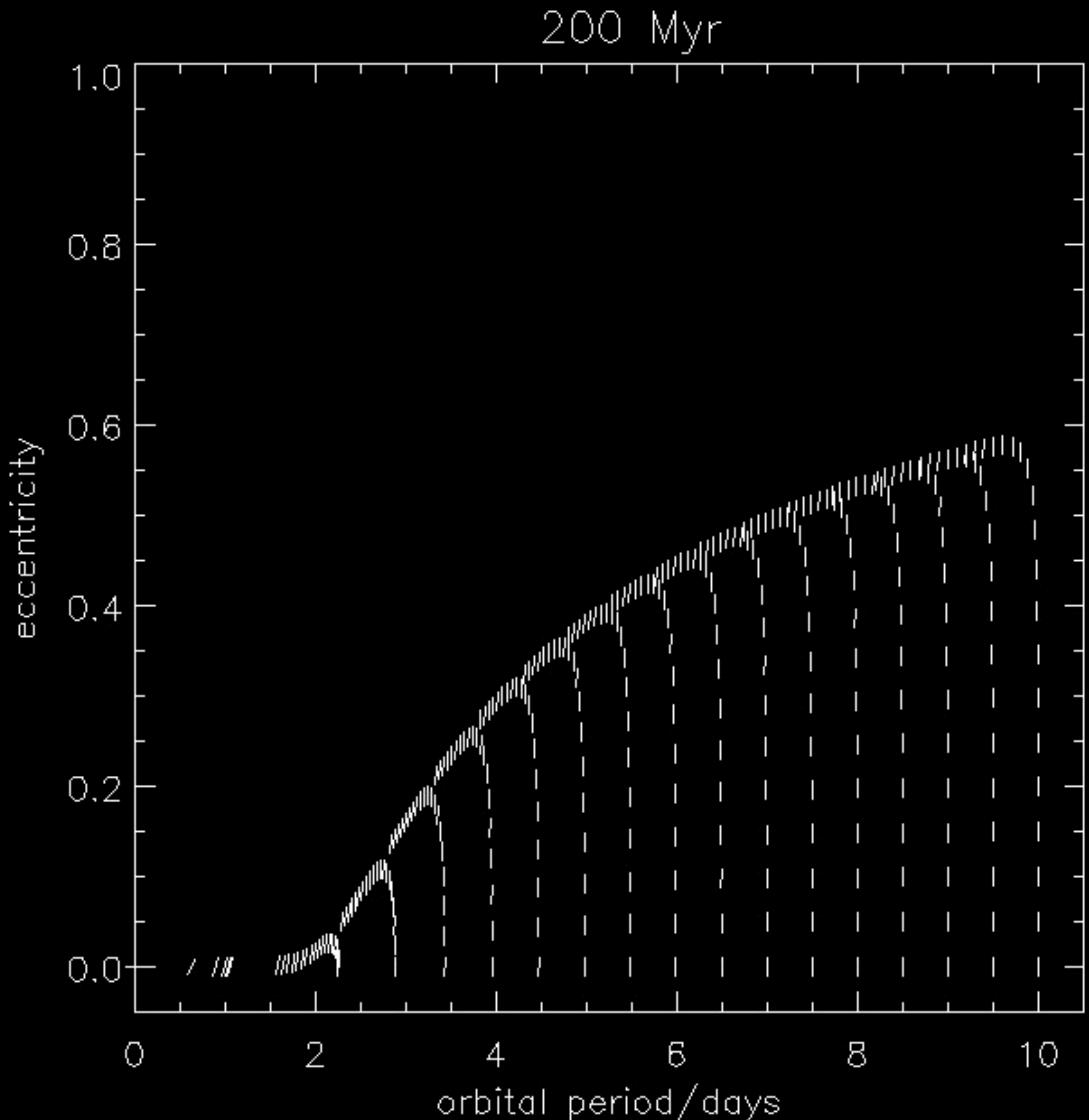
2011年12月2日金曜日

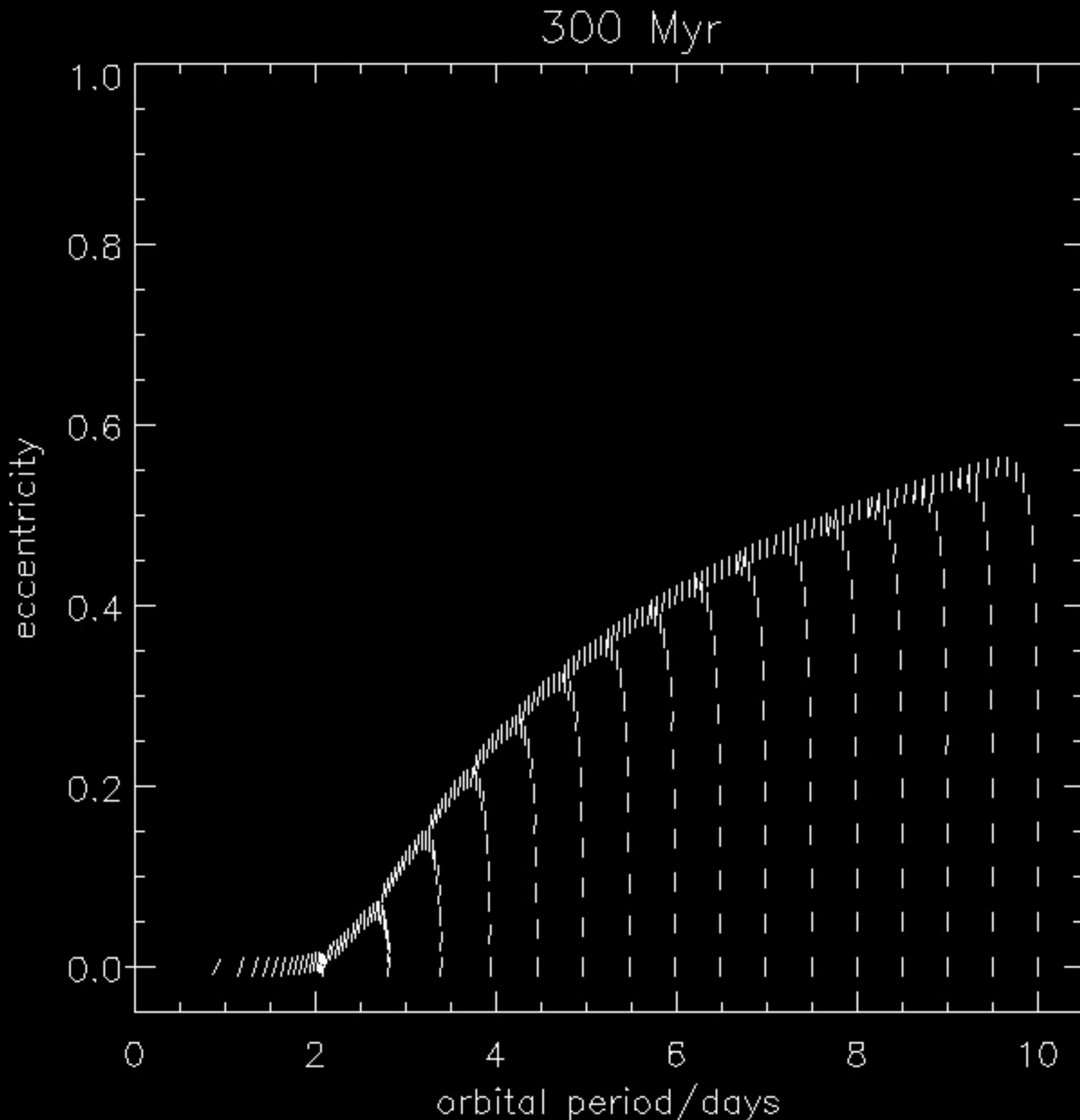
Initial stellar obliquity 90°


0 Myr
eccentricity
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10
orbital period/days

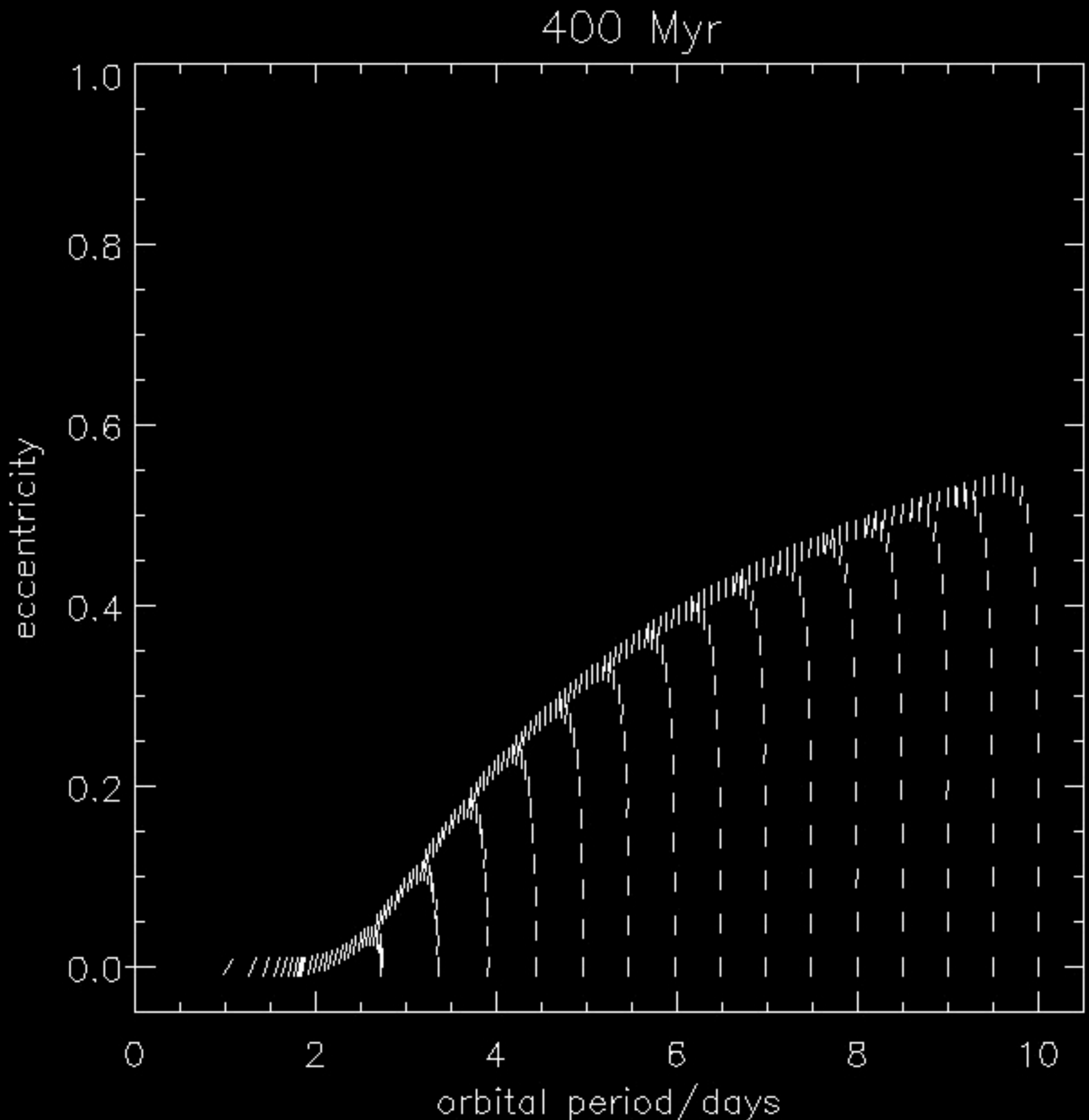


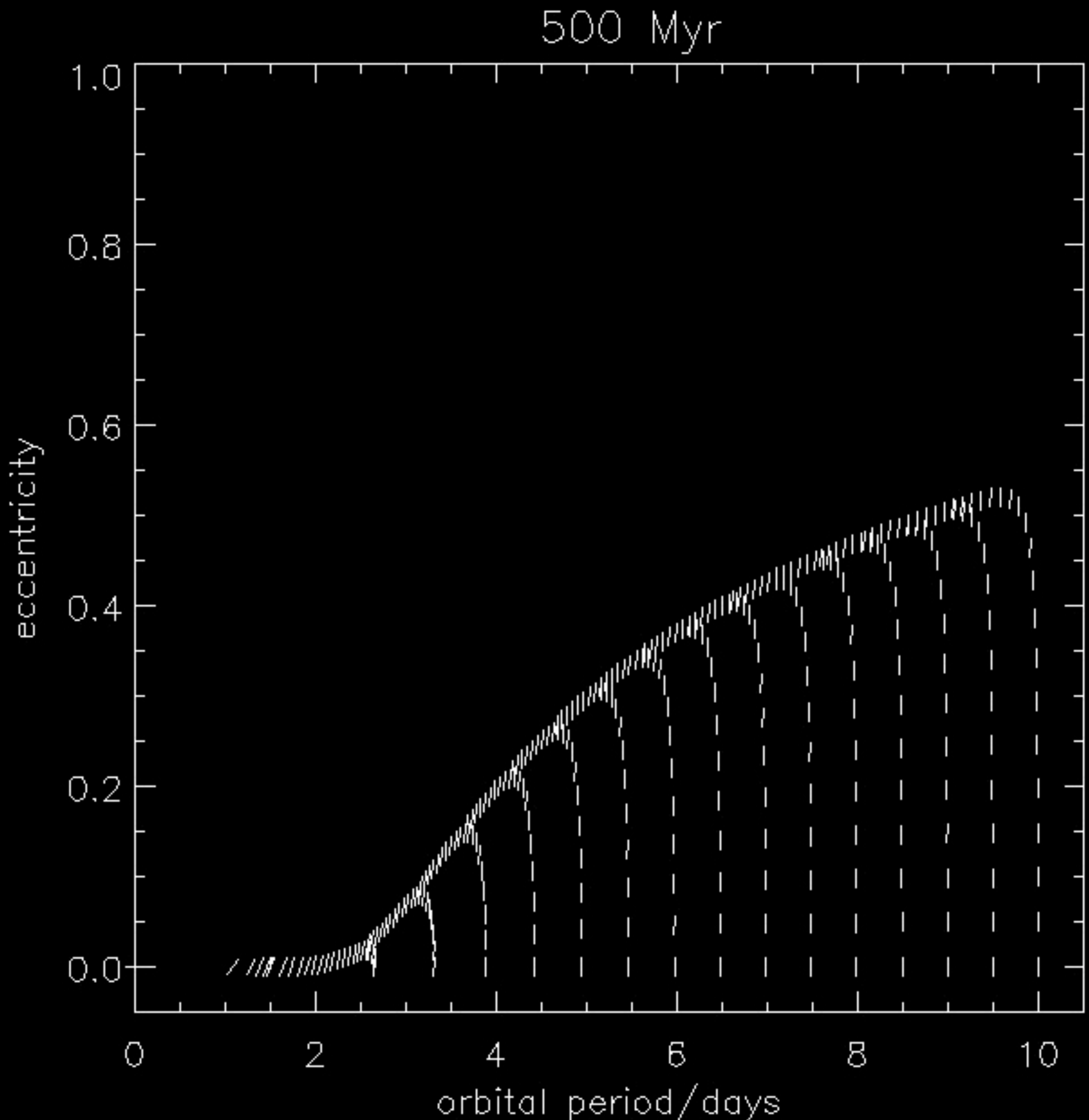
2011年12月2日金曜日

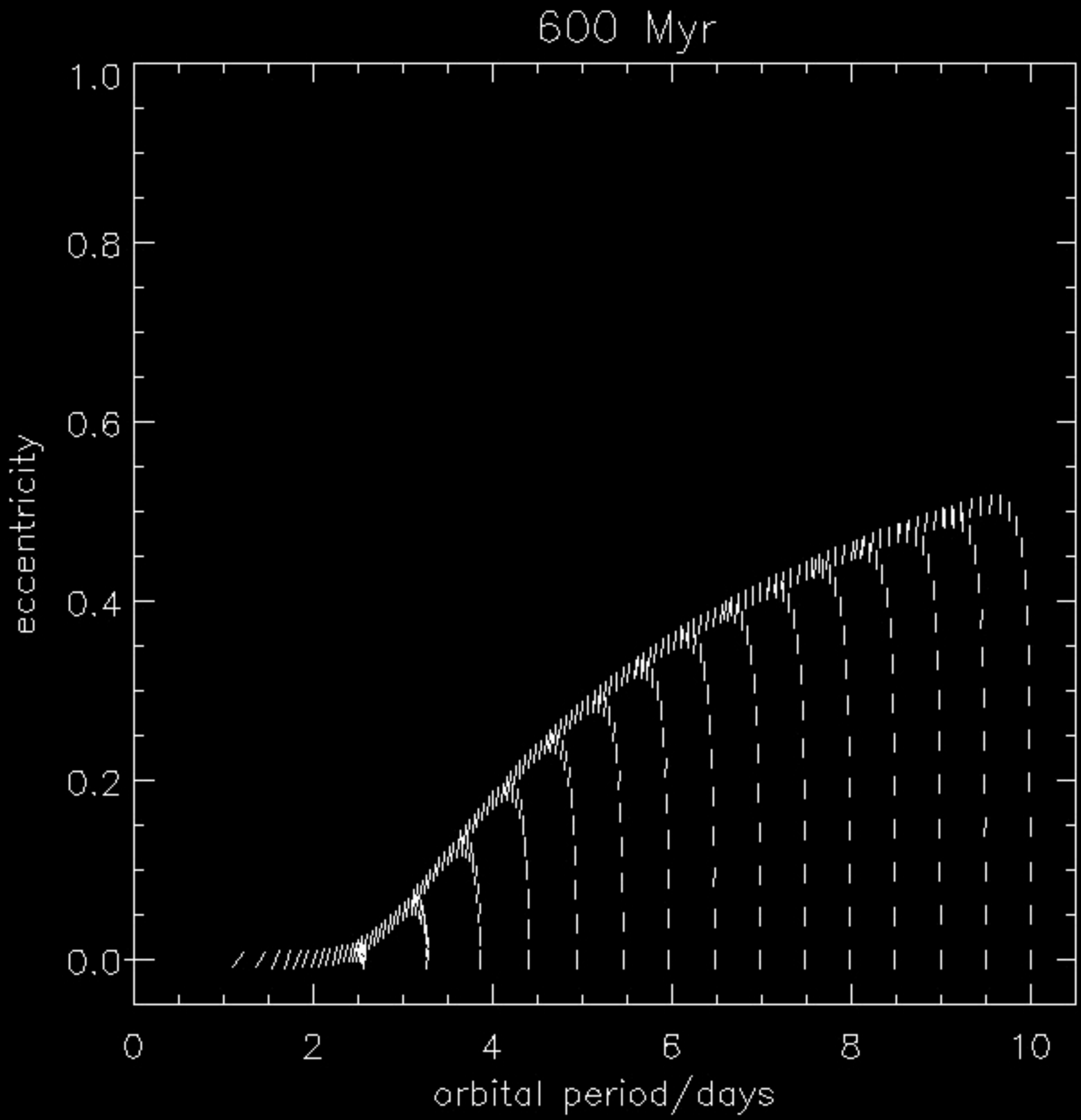

200 Myr
1.0
0.8
0.6
0.4
0.2
0.0
eccentricity
0
2
4
6
8
10
orbital period/days



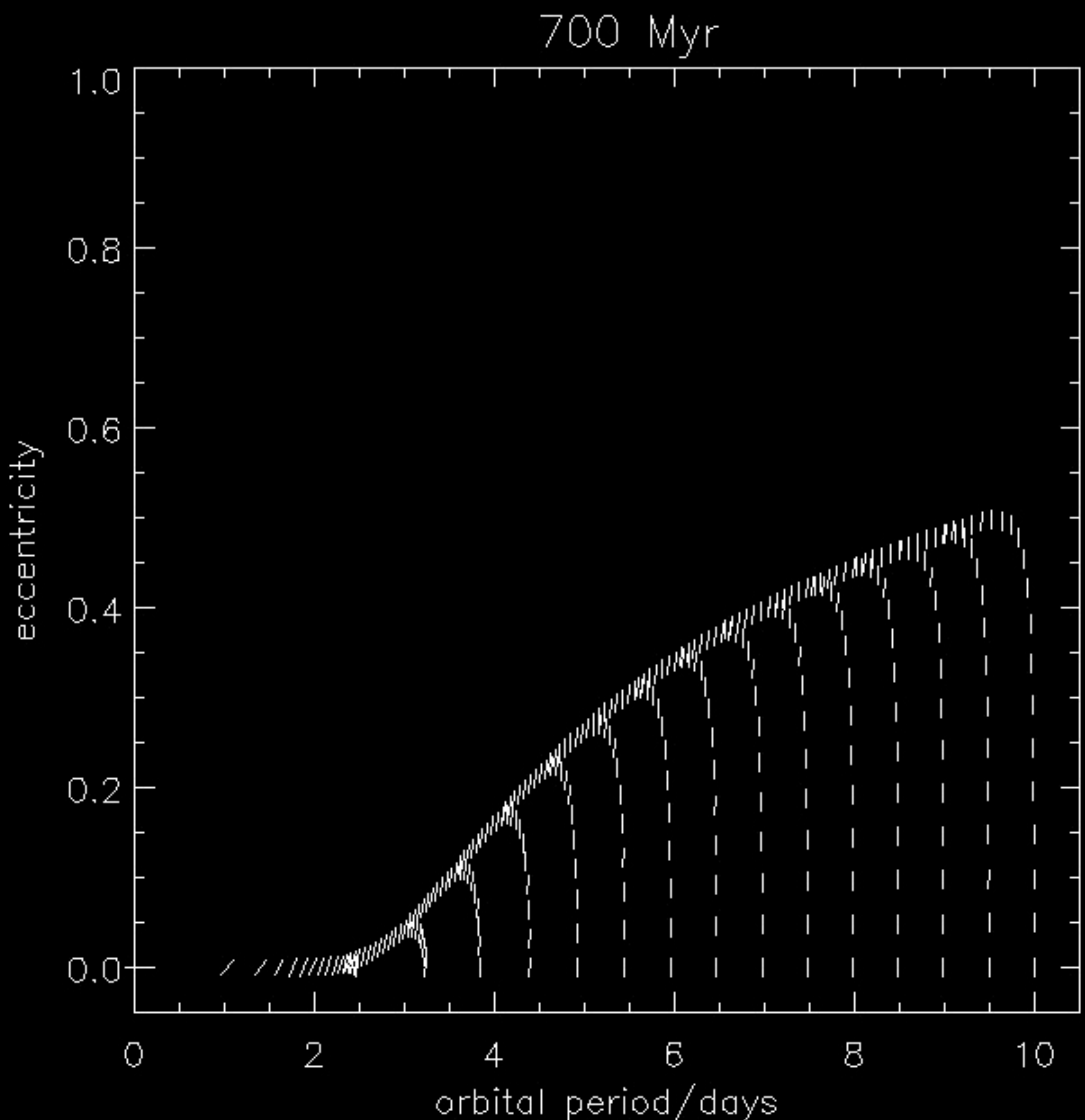
2011年12月2日金曜日

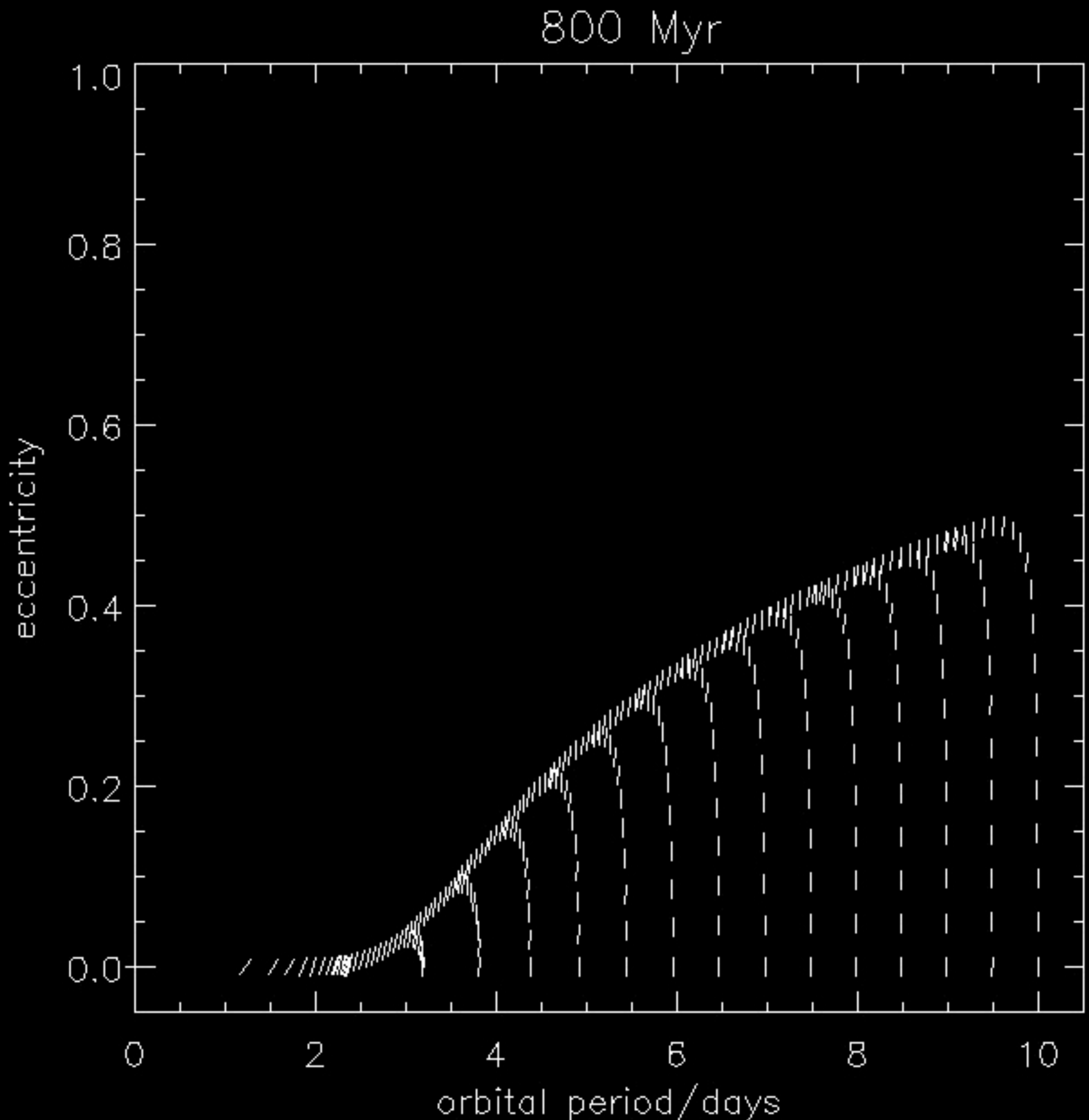

400 Myr
1.0
0.8
0.6
0.4
0.2
0.0
eccentricity
0
2
4
6
8
10
orbital period/days


500 Myr
eccentricity
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10
orbital period/days

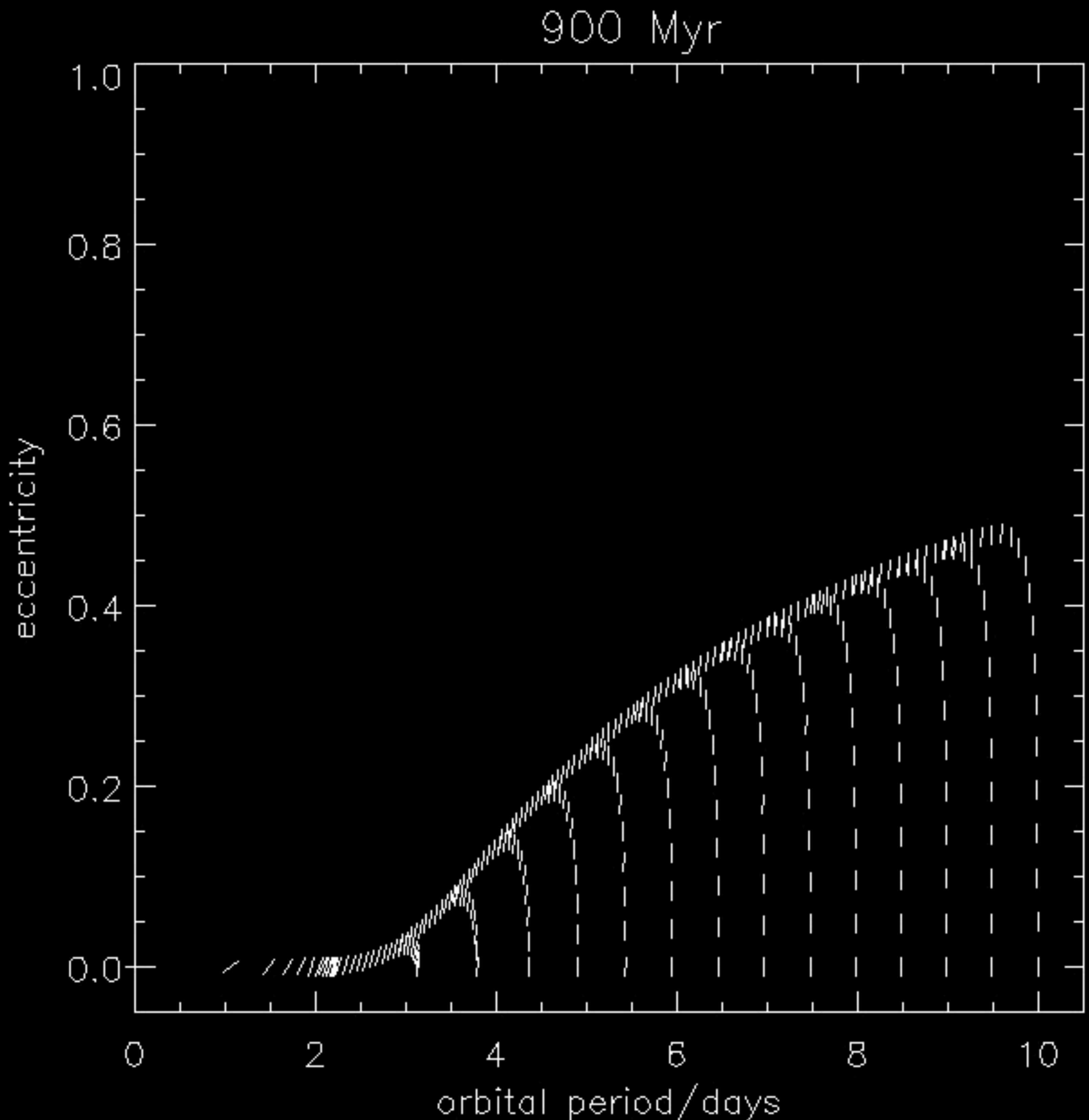


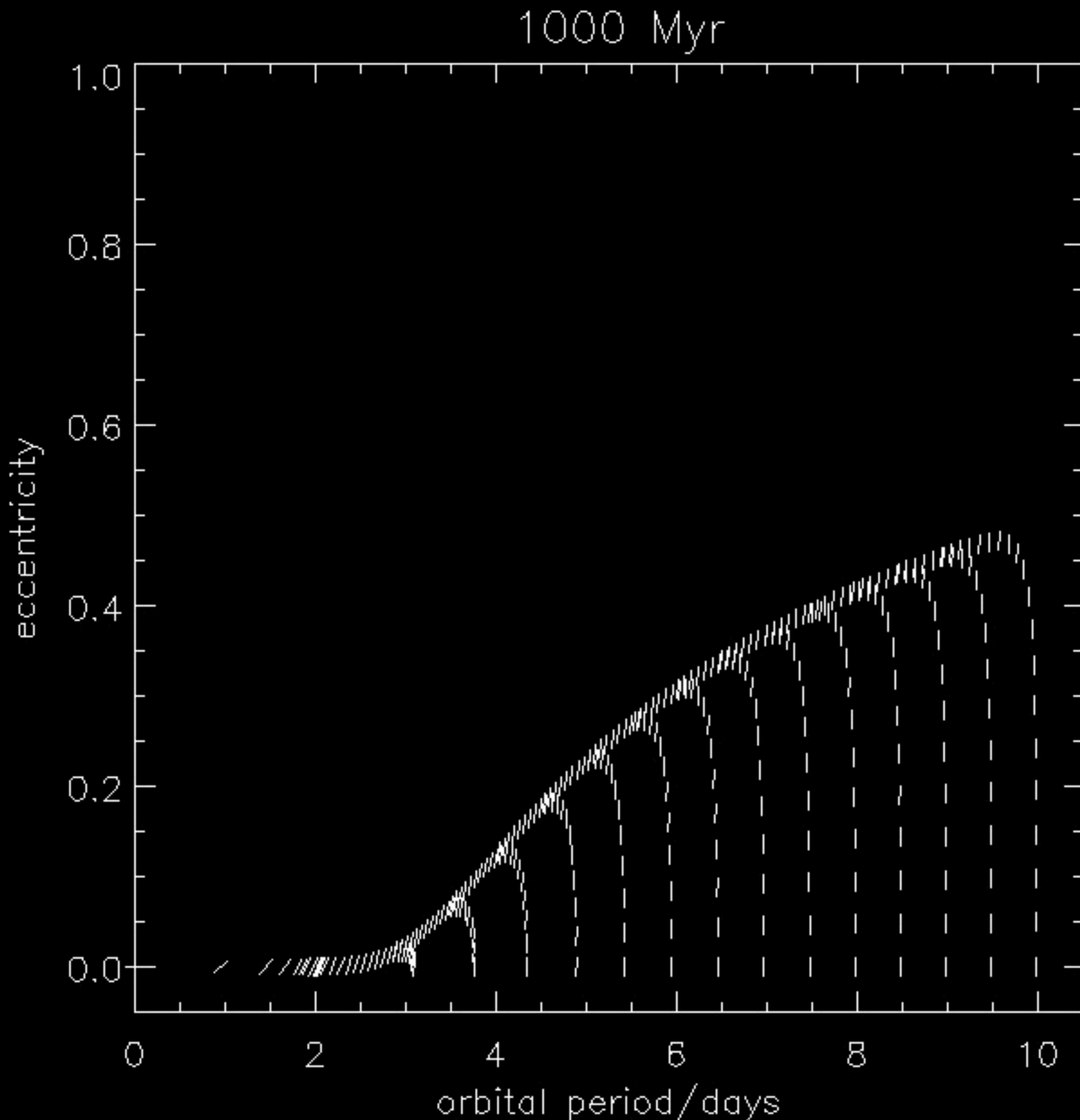
600 Myr
eccentricity
orbital period/days
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10

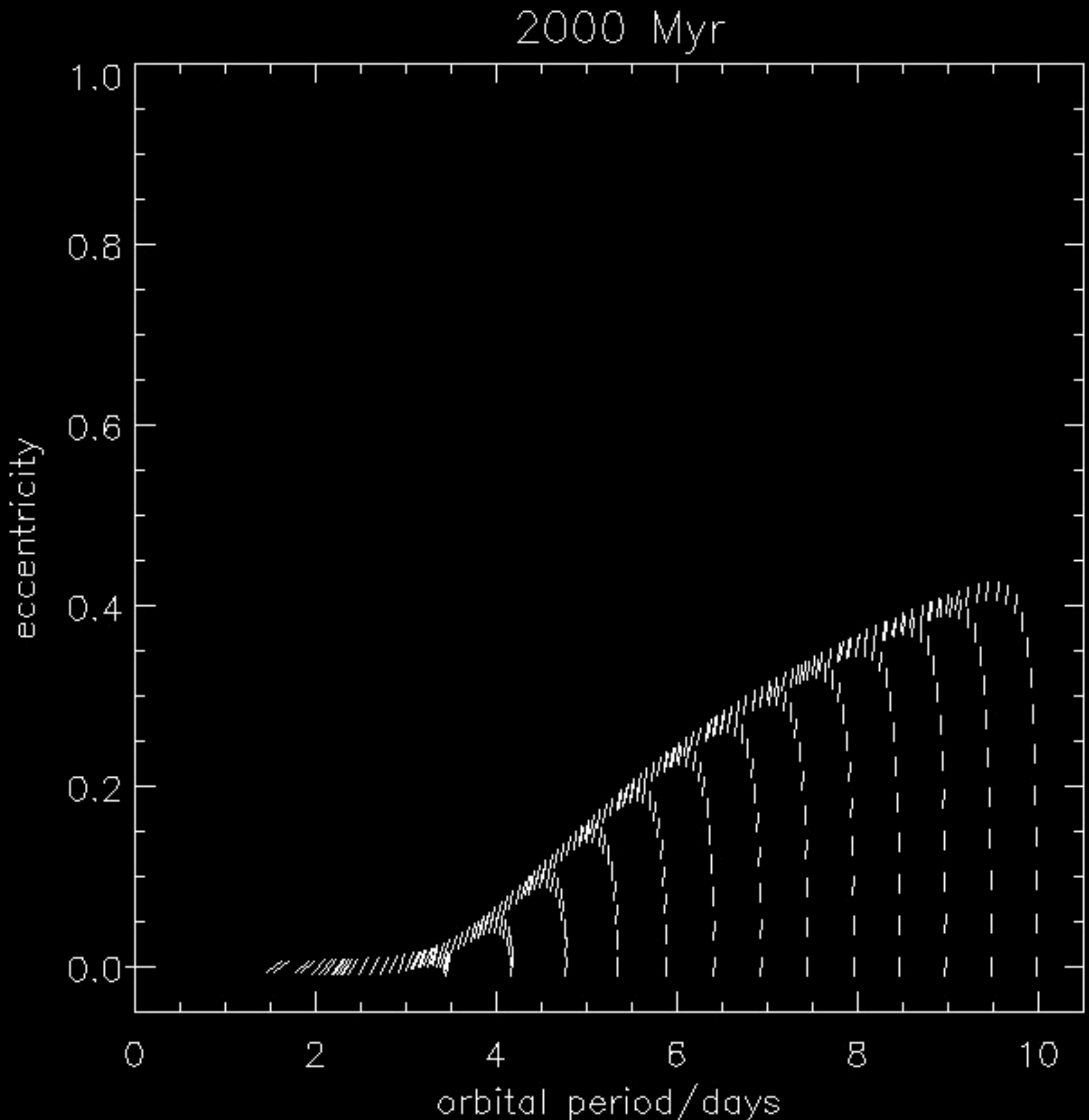

700 Myr
eccentricity
orbital period/days
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10

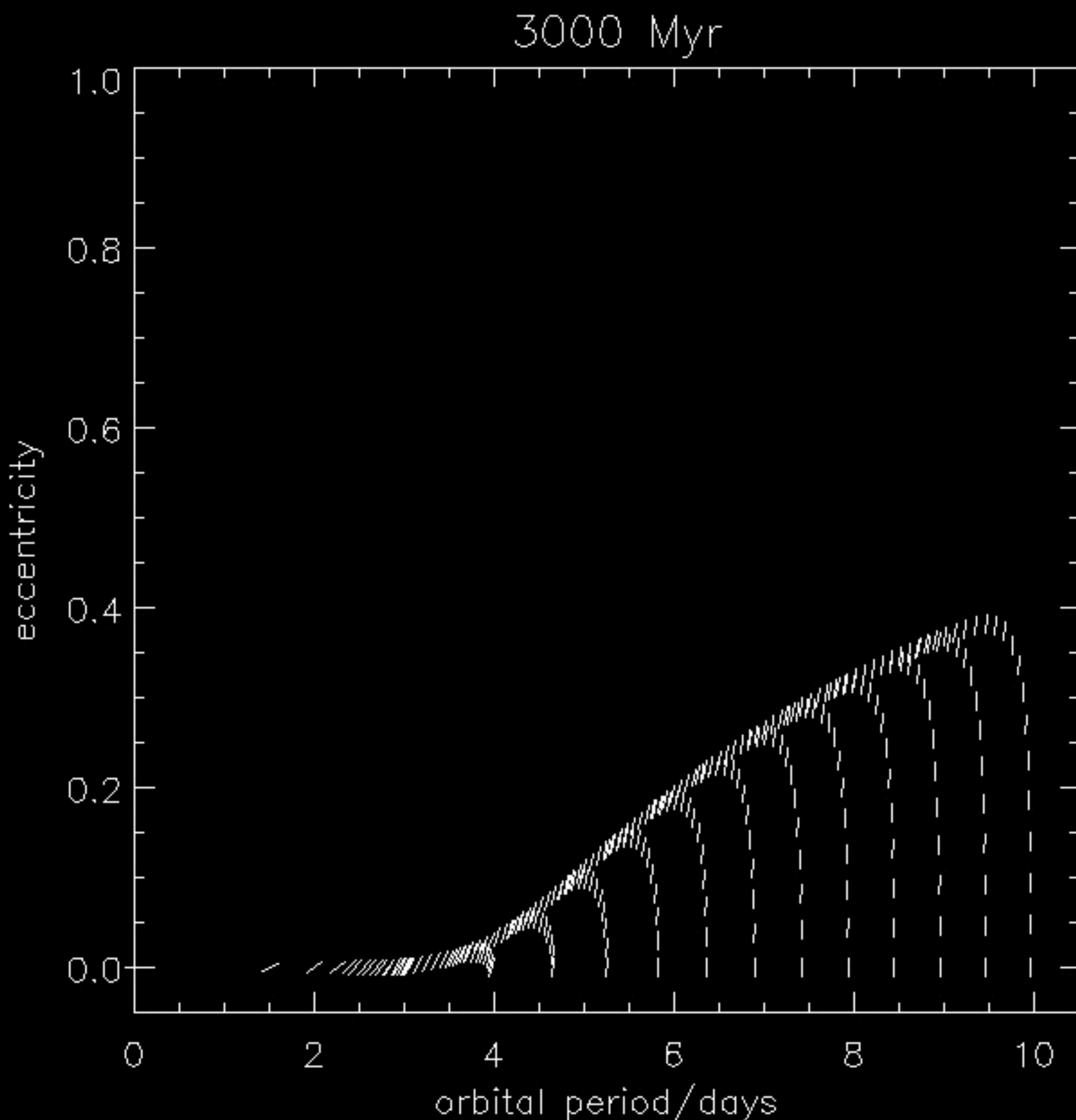


2011年12月2日金曜日

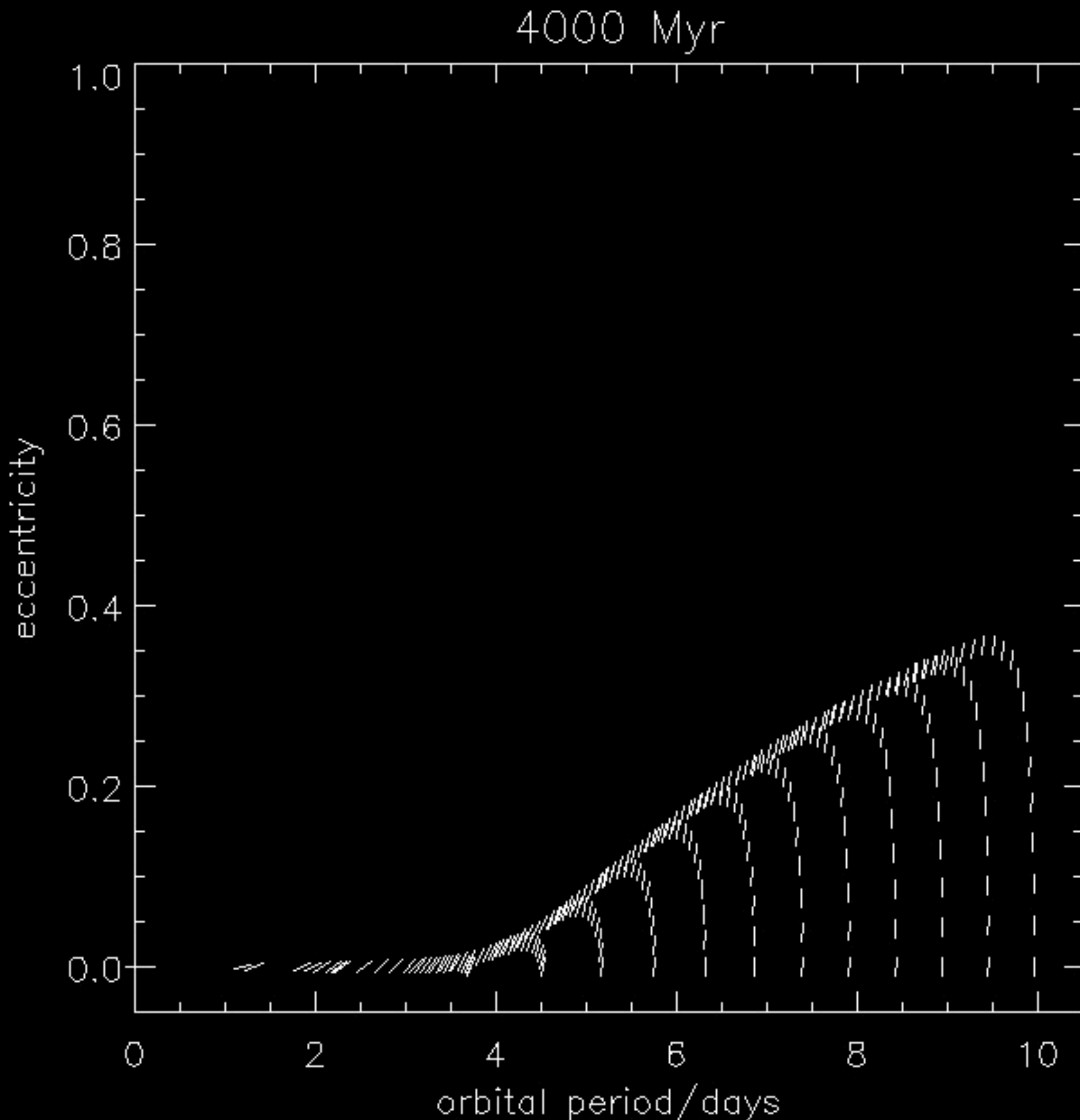

900 Myr
1.0
0.8
0.6
0.4
0.2
0.0
eccentricity
0
2
4
6
8
10
orbital period/days

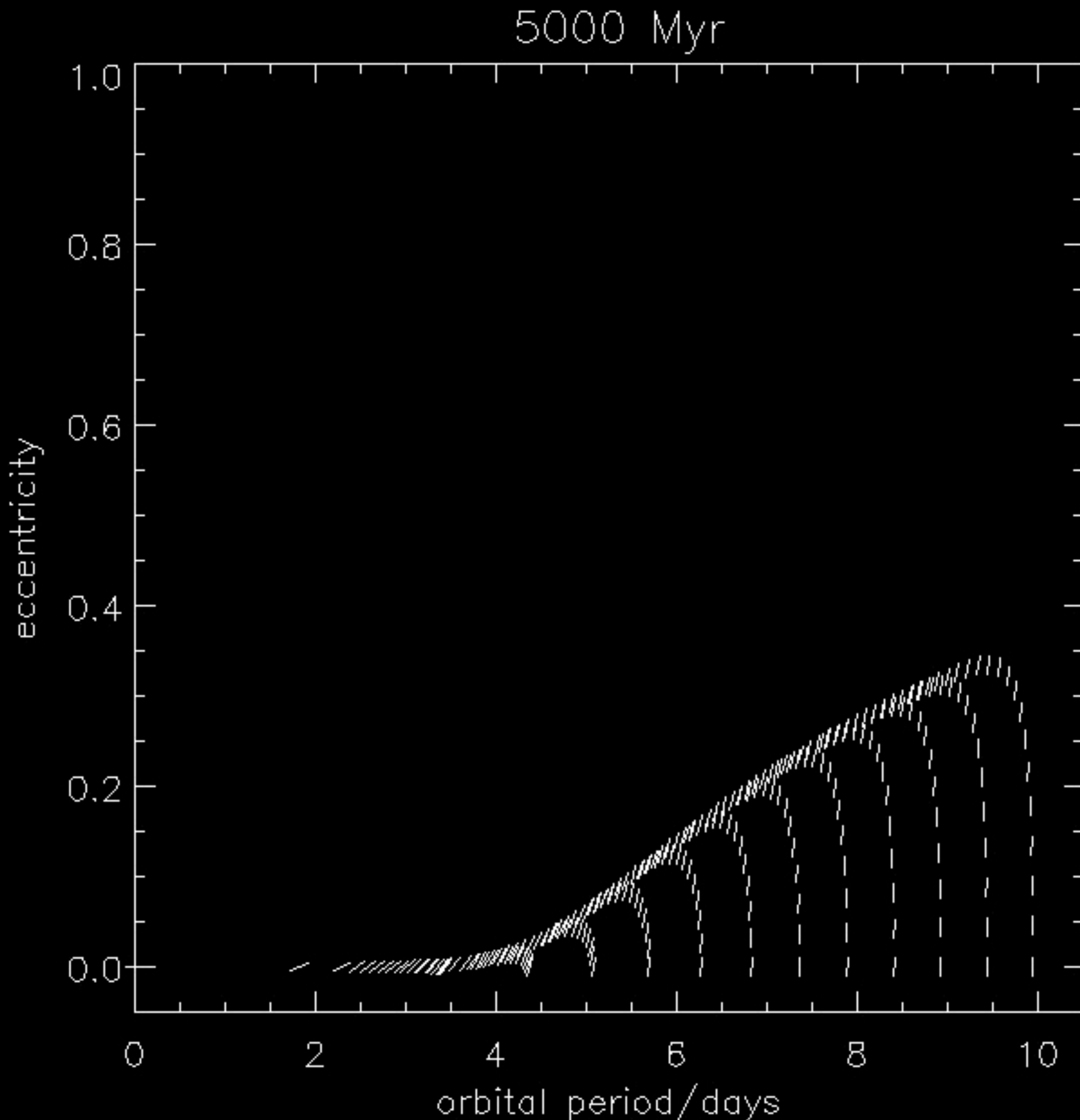

1000 Myr
eccentricity
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10
orbital period/days

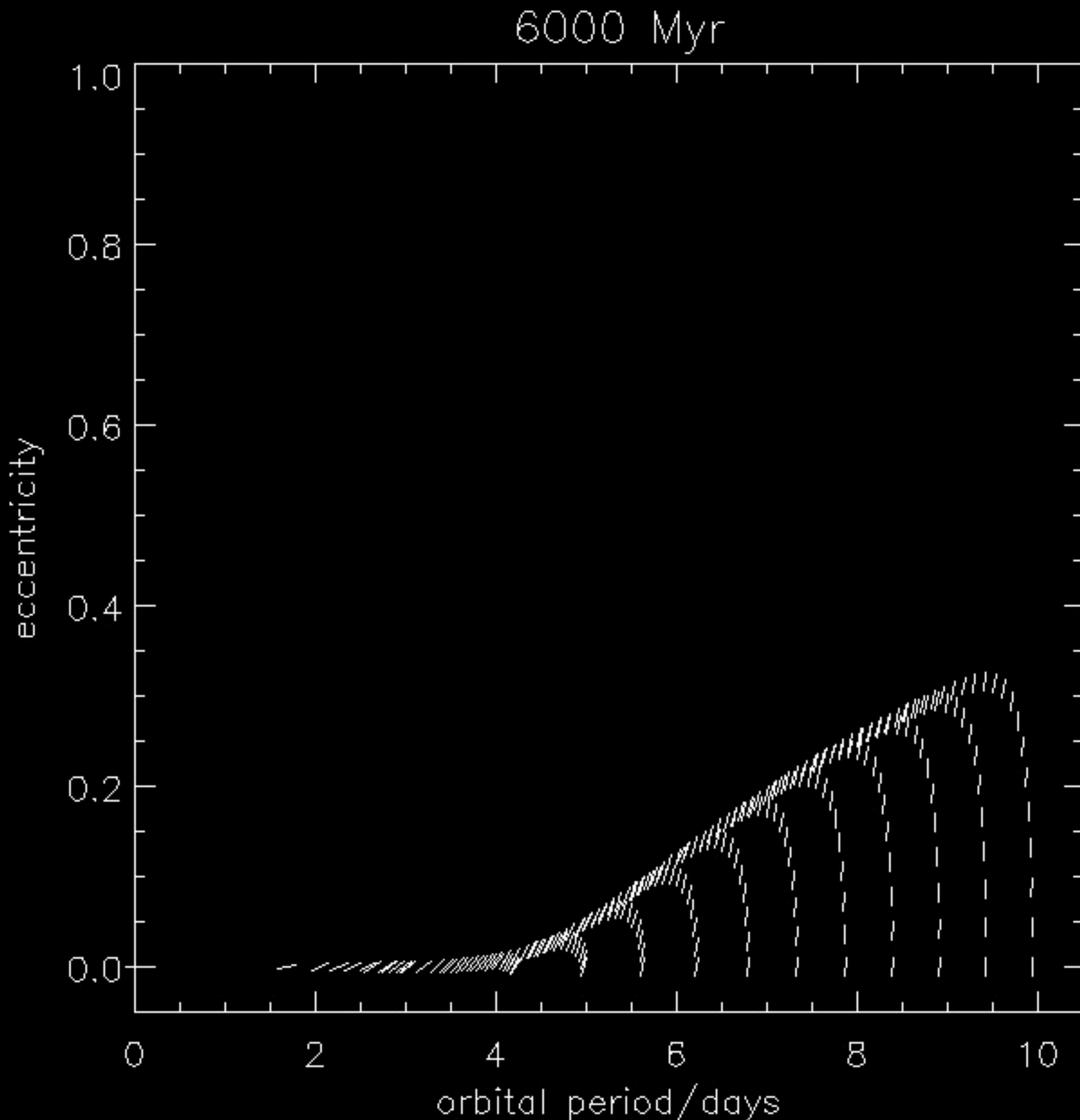

2000 Myr
1.0
0.8
0.6
0.4
0.2
0.0
eccentricity
0
2
4
6
8
10
orbital period/days

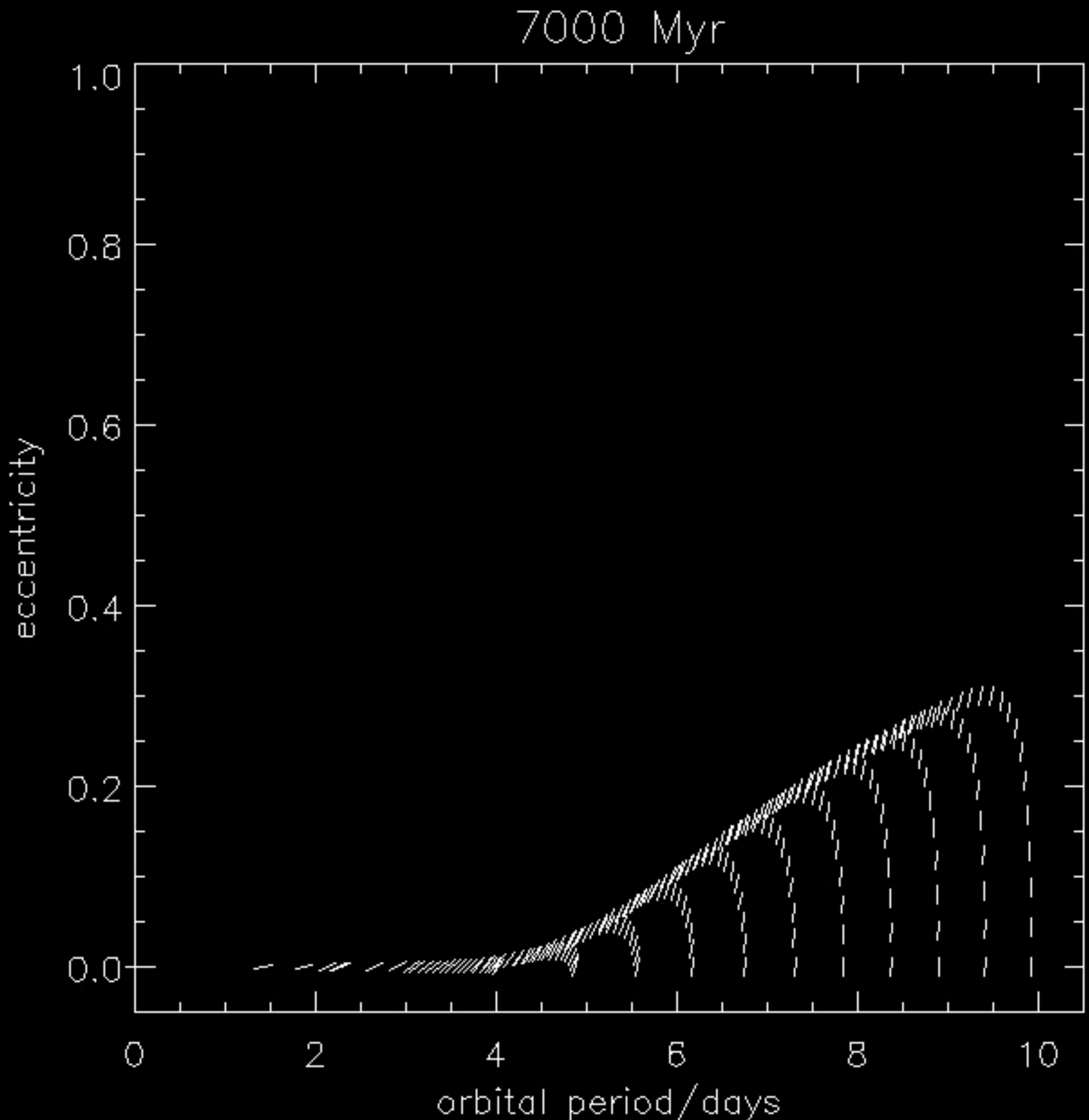


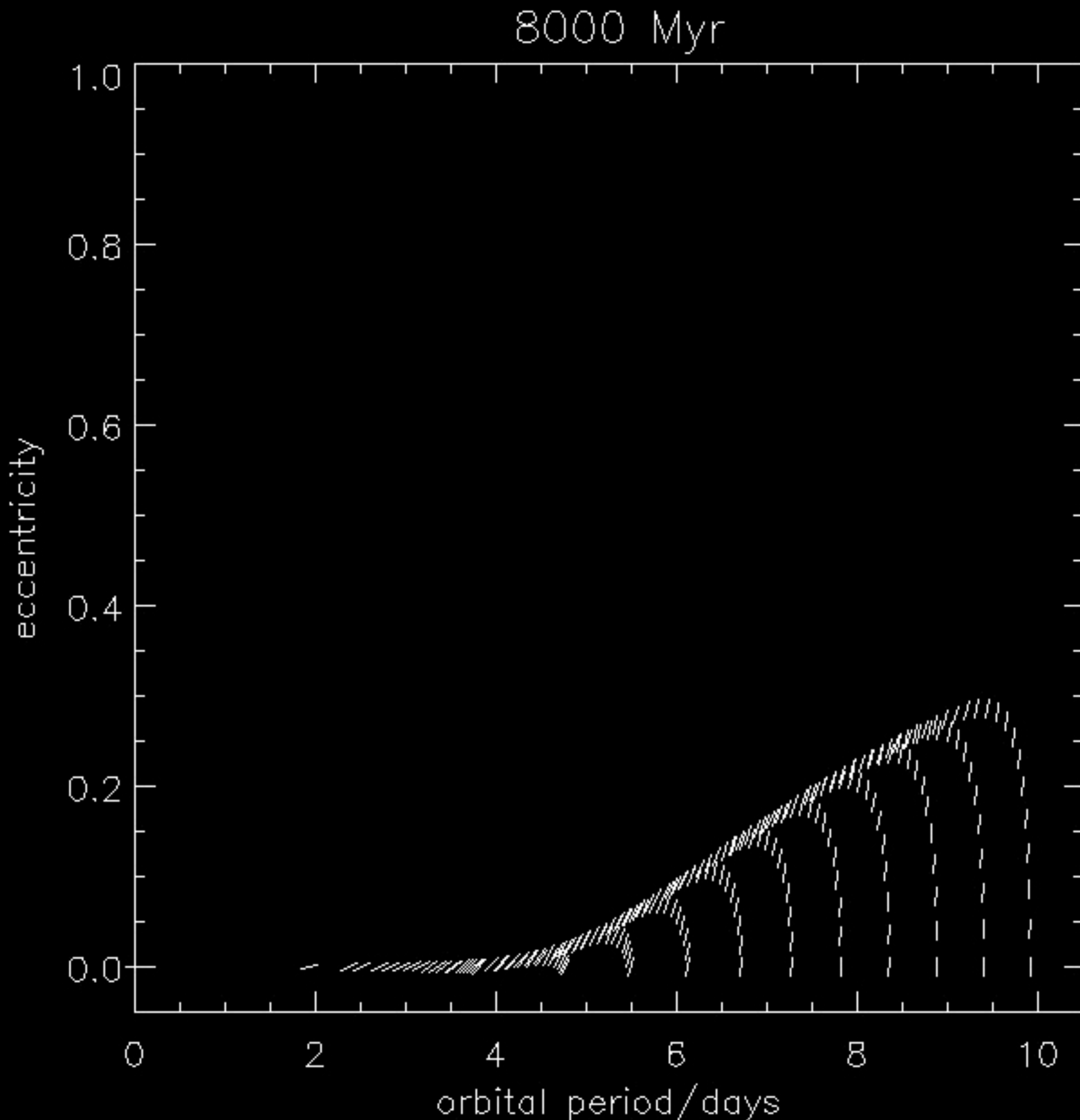
3000 Myr
eccentricity
orbital period/days
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10

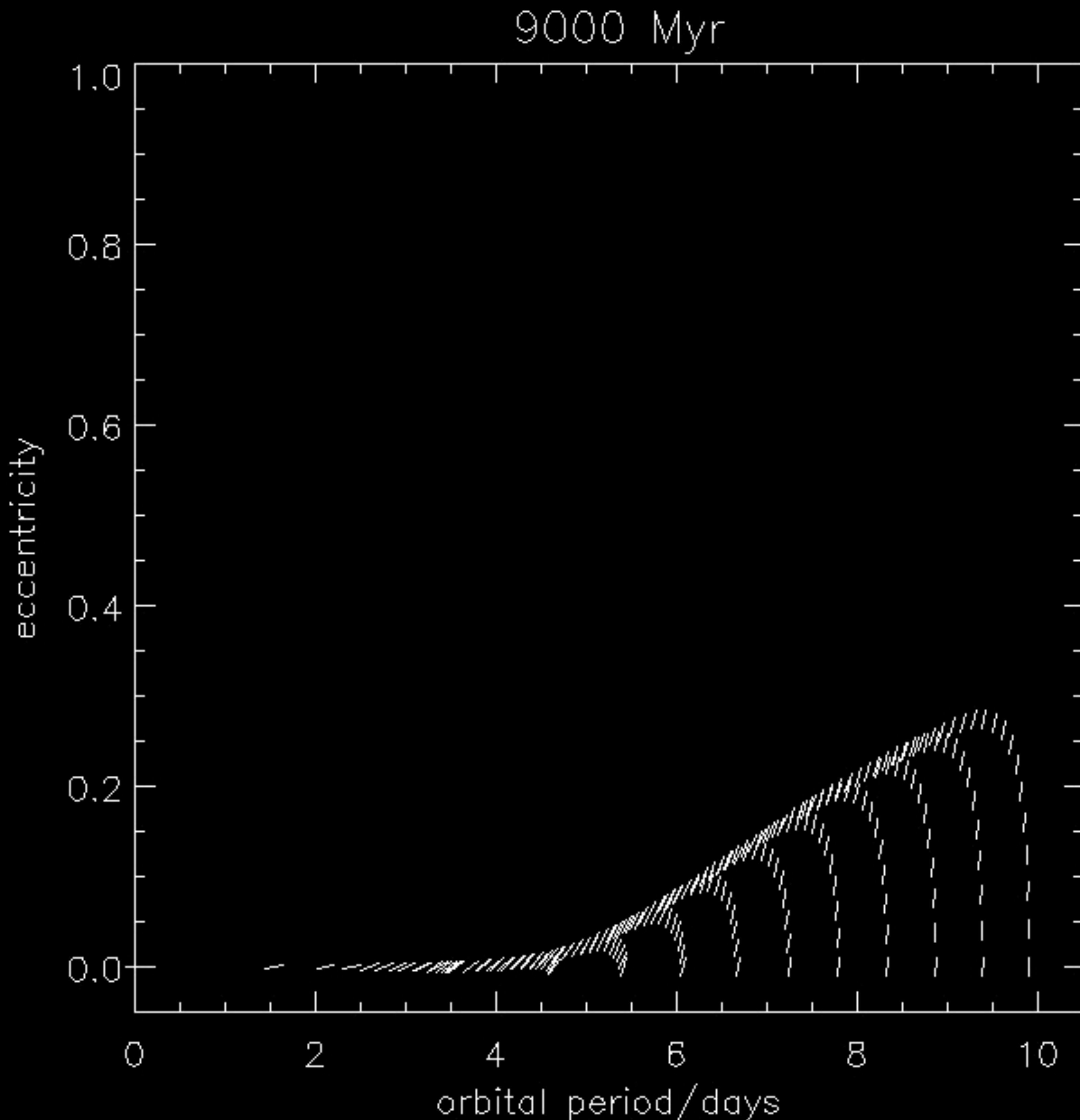

4000 Myr
1.0
0.8
0.6
0.4
0.2
0.0
eccentricity
0
2
4
6
8
10
orbital period/days

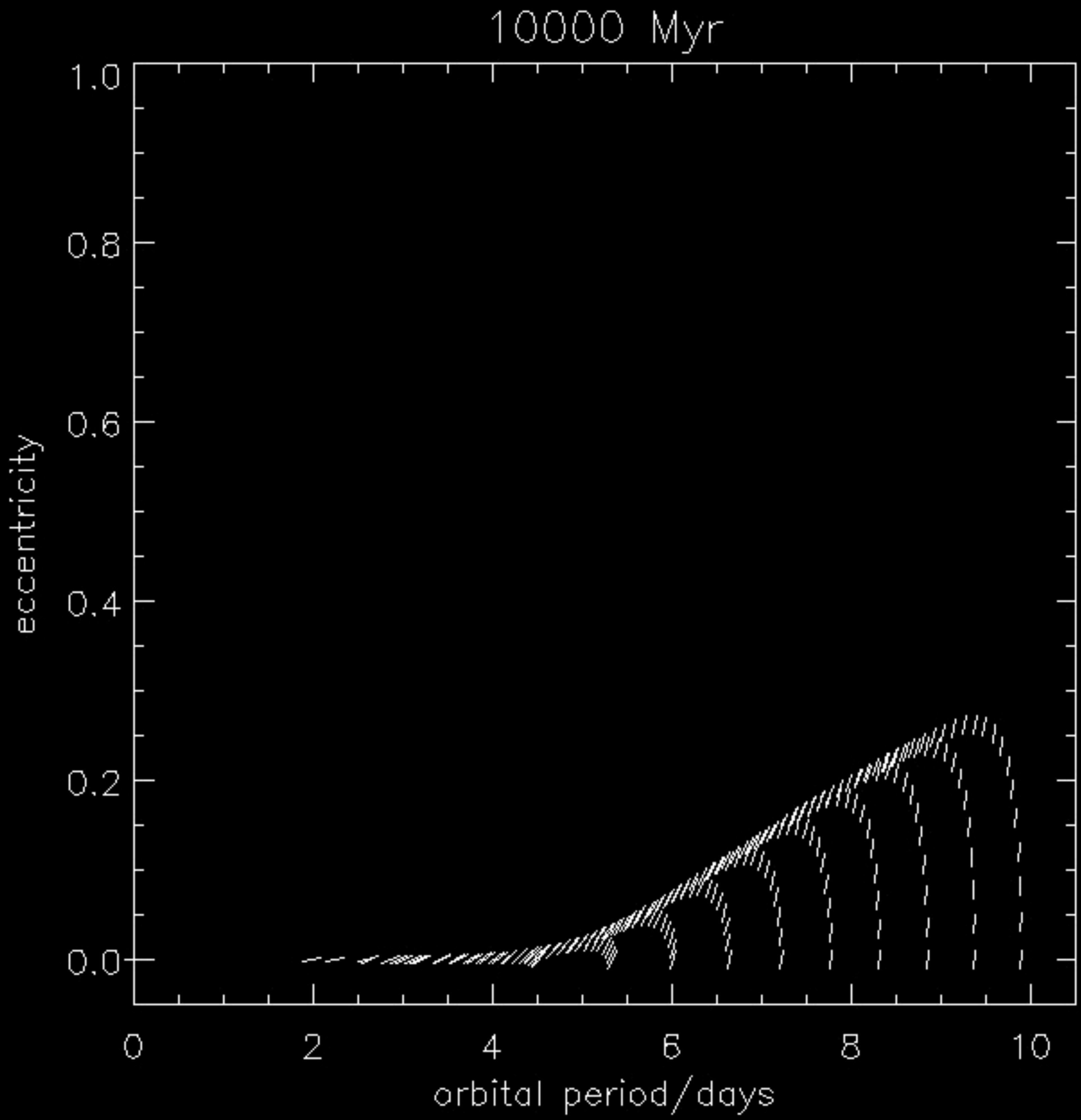

5000 Myr
1.0
0.8
0.6
0.4
0.2
0.0
eccentricity
0
2
4
6
8
10
orbital period/days


6000 Myr
eccentricity
orbital period/days
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10

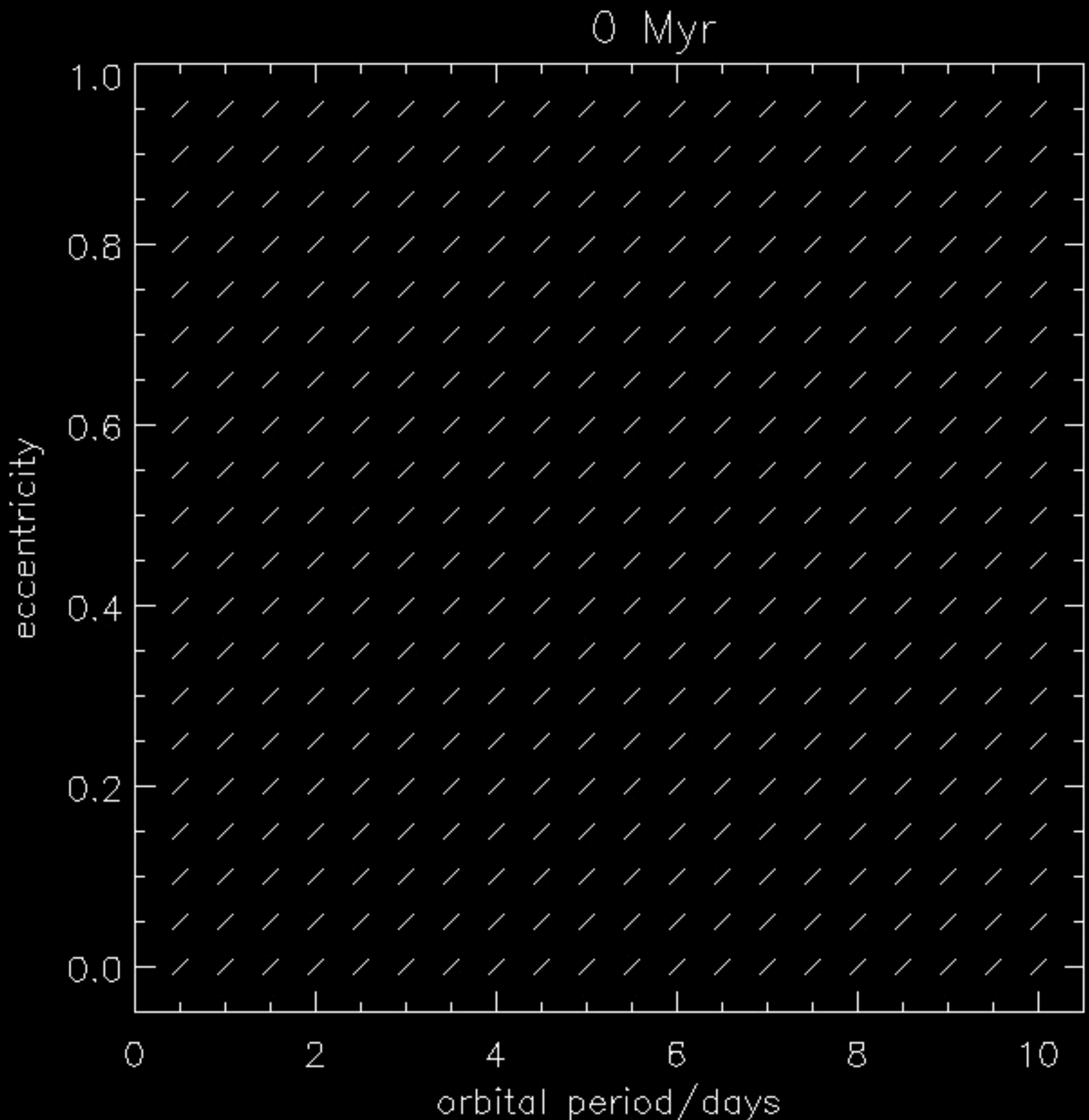

7000 Myr
1.0
0.8
0.6
0.4
0.2
0.0
eccentricity
0
2
4
6
8
10
orbital period/days

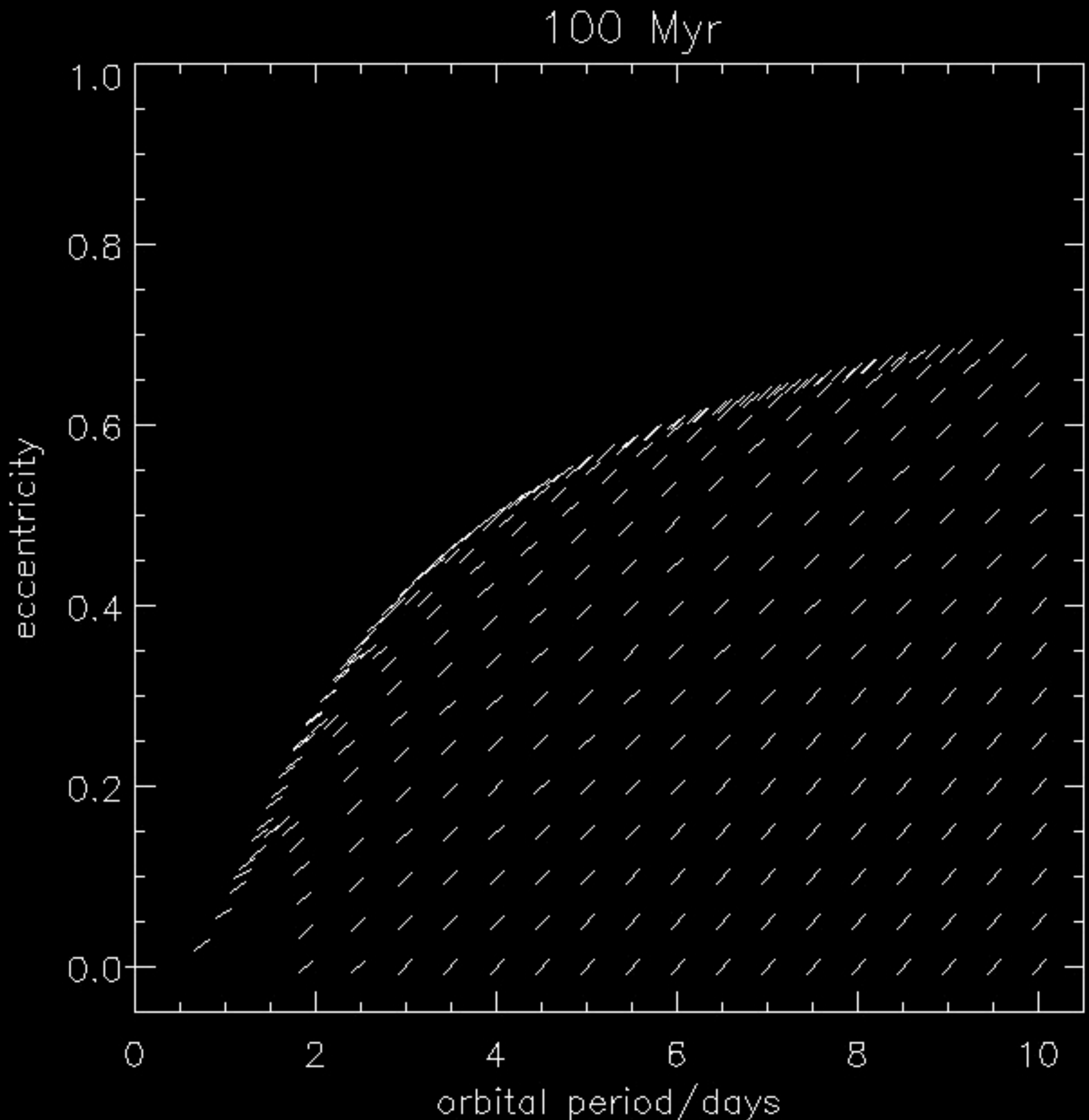

8000 Myr
eccentricity
orbital period/days
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10

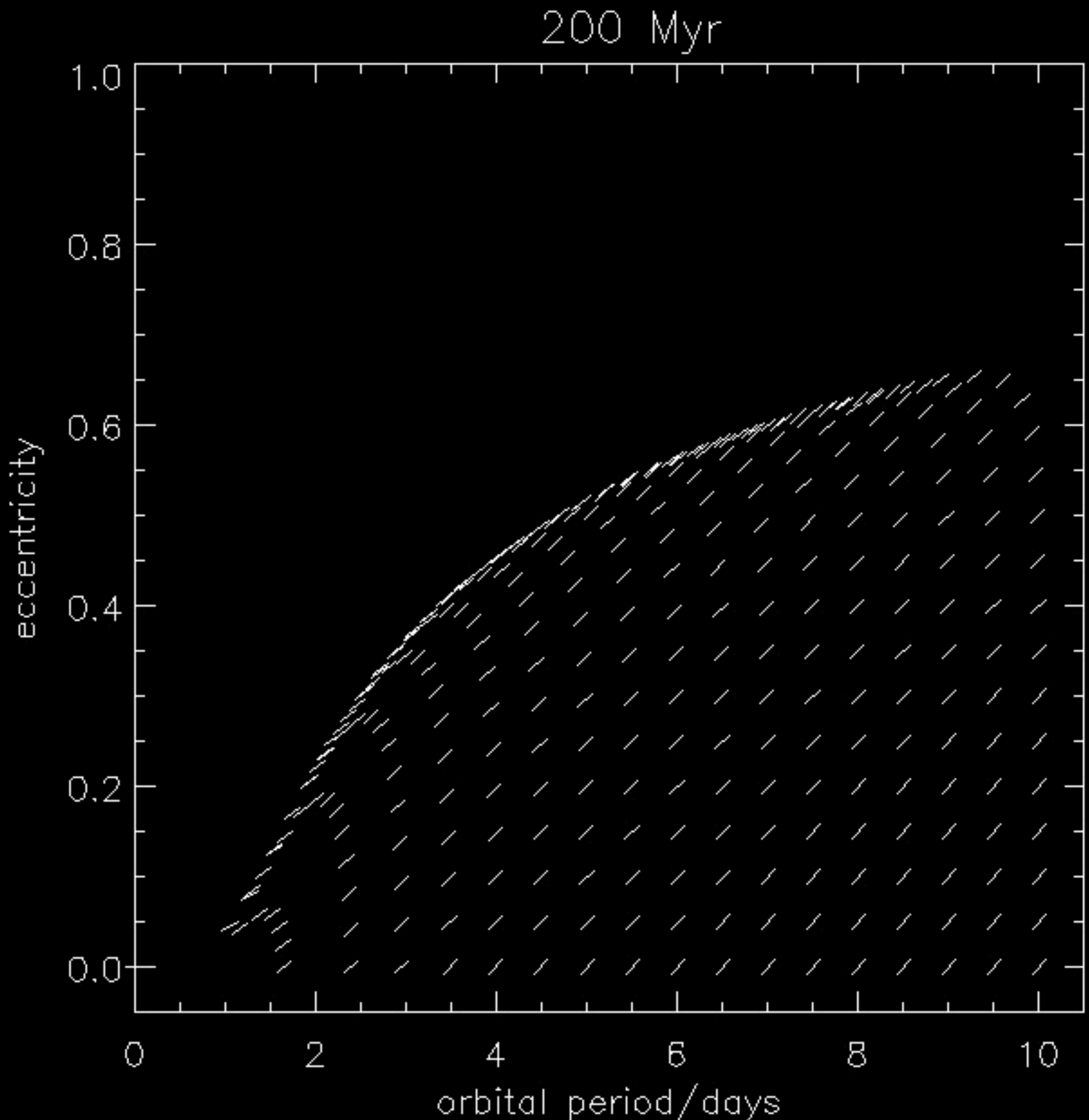

9000 Myr
1.0
0.8
0.6
0.4
0.2
0.0
eccentricity
0
2
4
6
8
10
orbital period/days

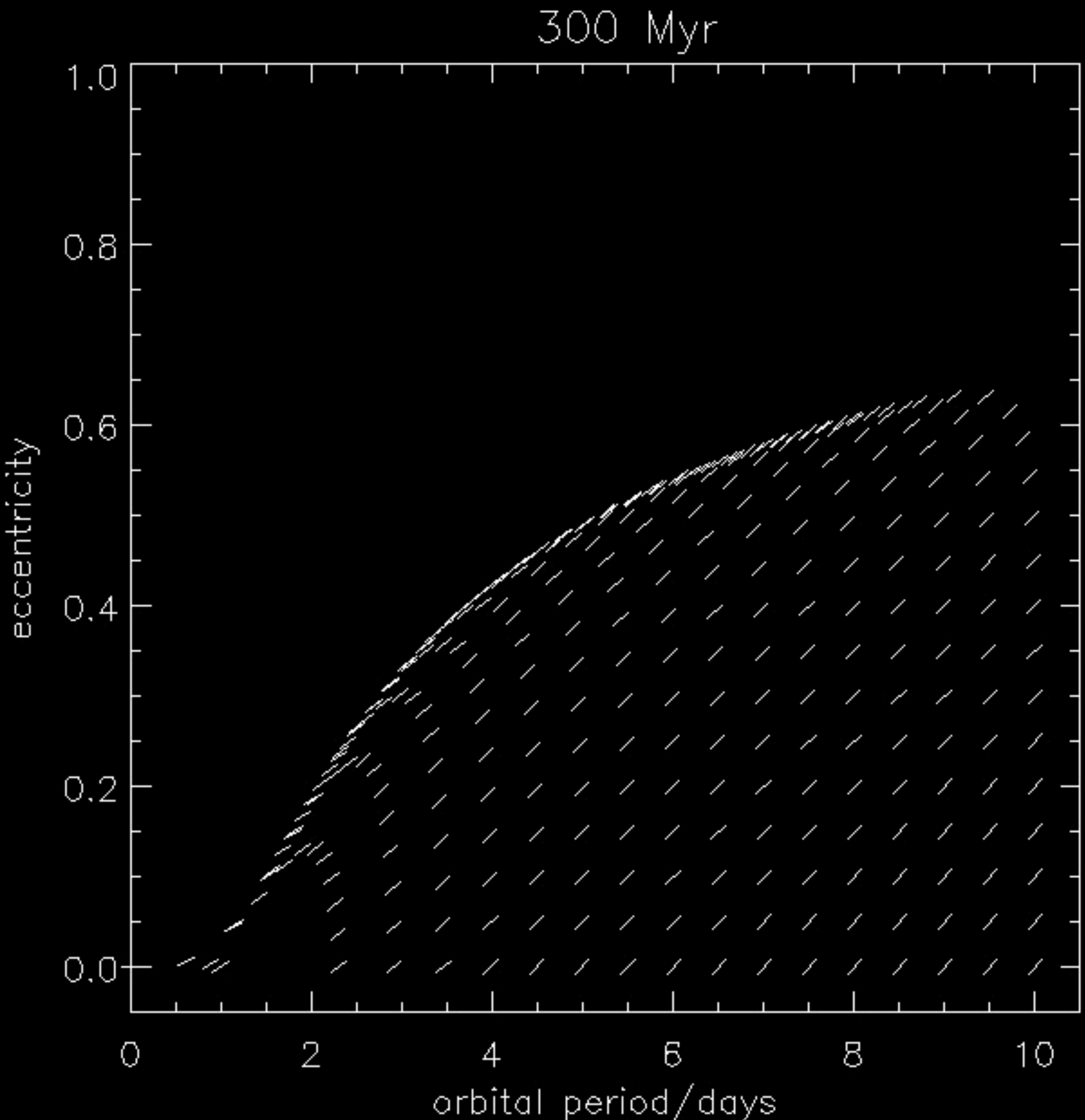

10000 Myr
eccentricity
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10
orbital period/days

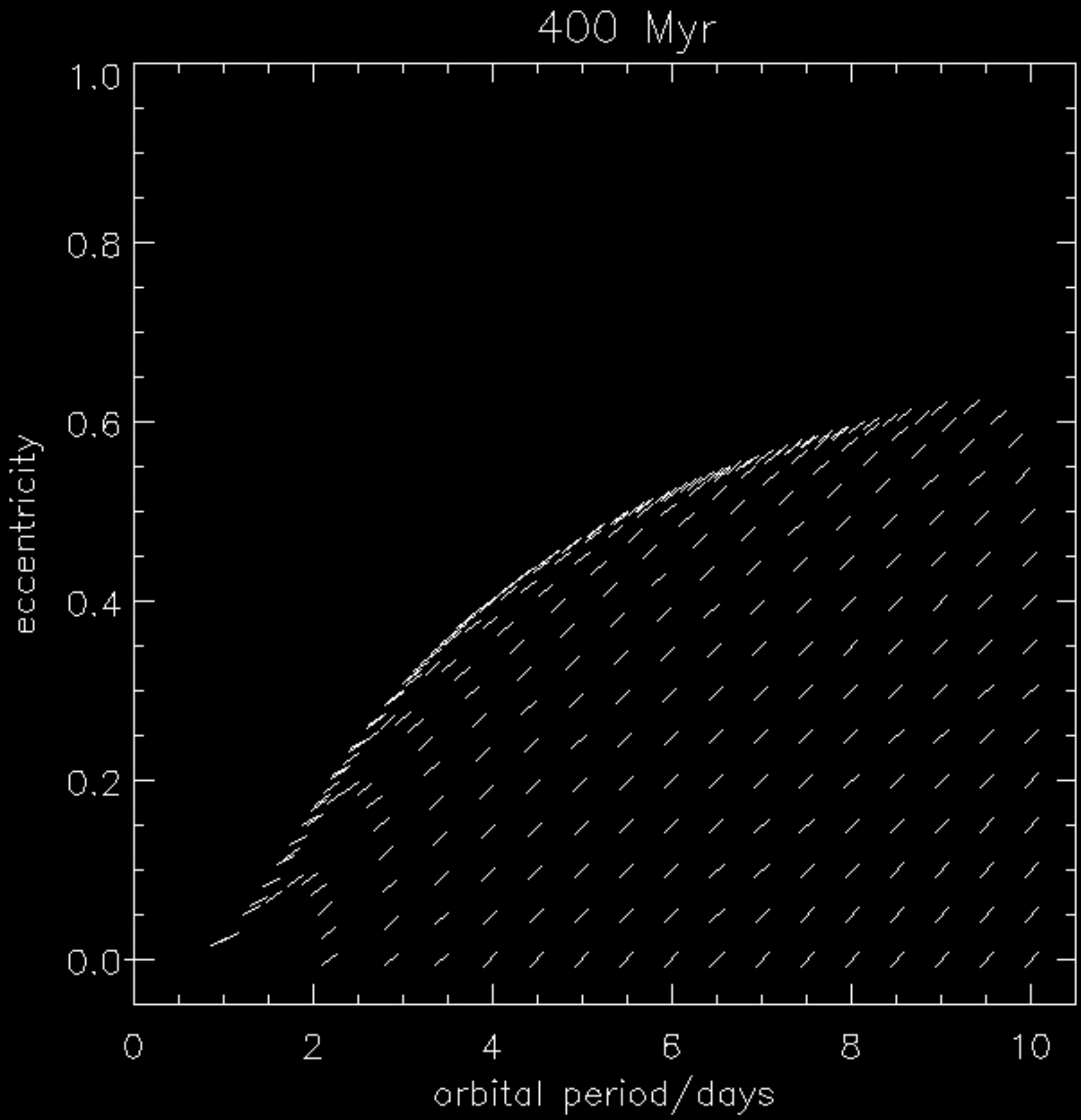
No tide in planet


0 Myr
1.0
0.8
0.6
0.4
0.2
0.0
eccentricity
0
2
4
6
8
10
orbital period/days

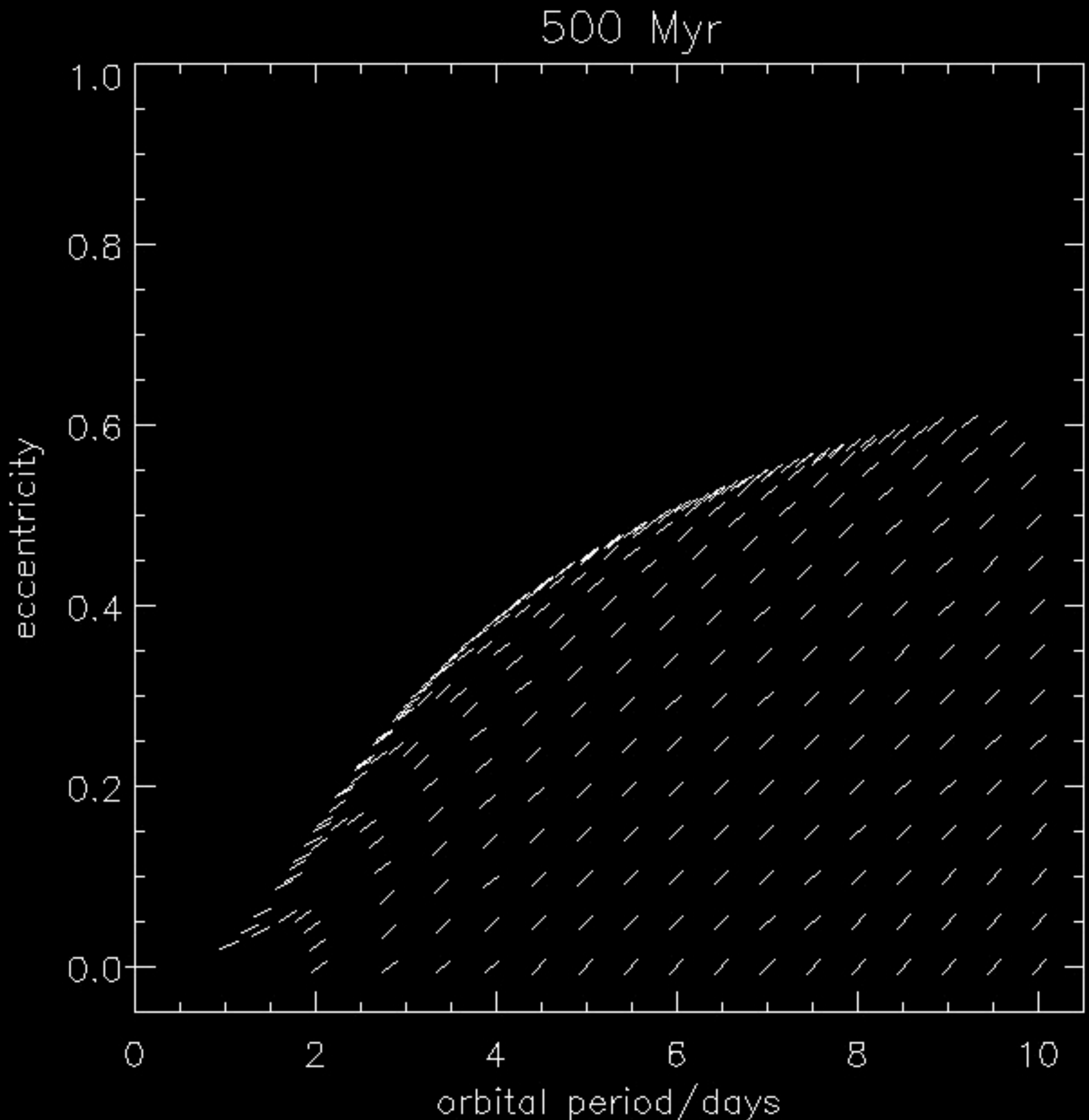

100 Myr
1.0
0.8
0.6
0.4
0.2
0.0
eccentricity
0
2
4
6
8
10
orbital period/days

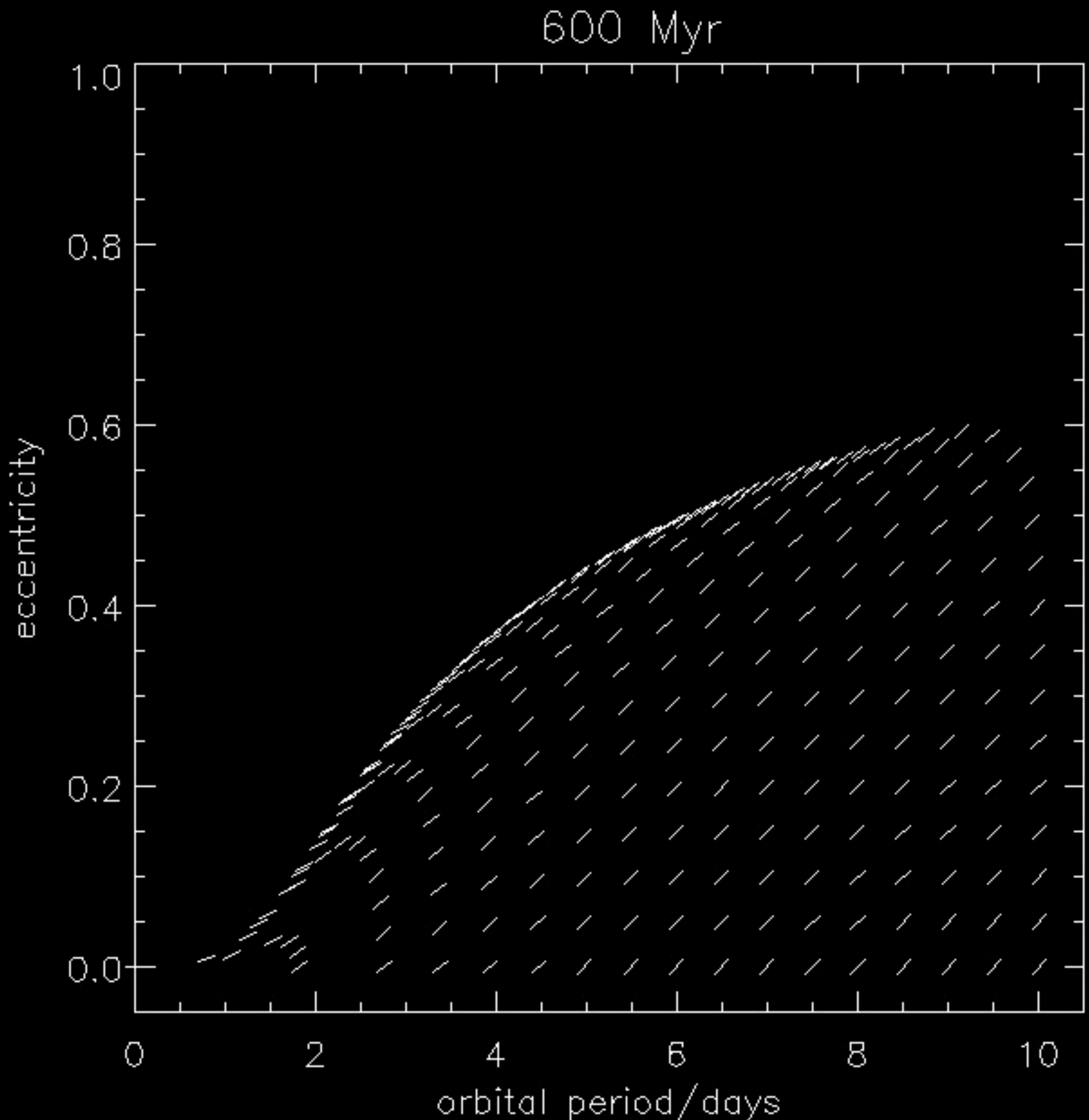

200 Myr
eccentricity
orbital period/days
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10

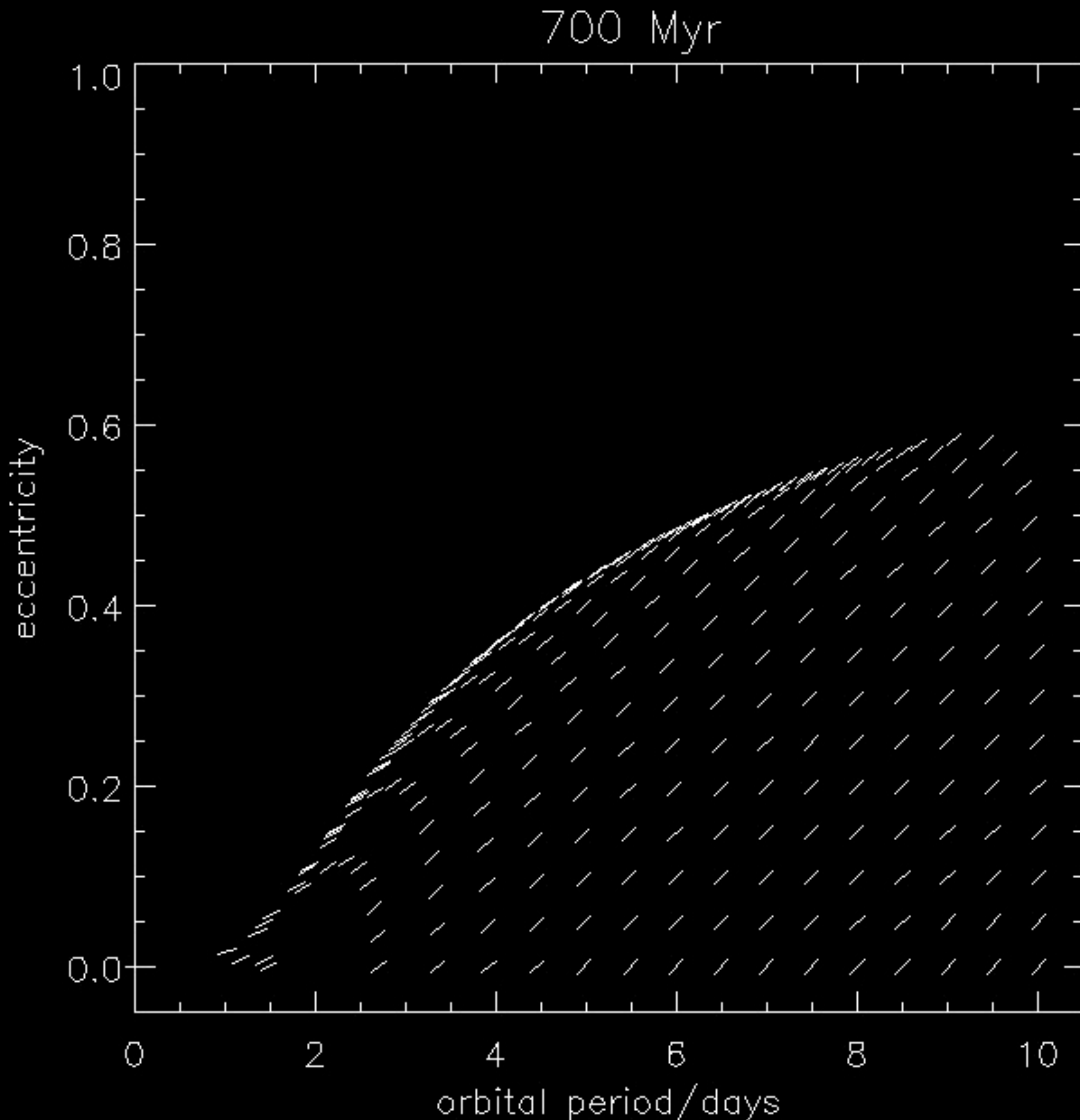

300 Myr
eccentricity
orbital period/days
0.0
0.2
0.4
0.6
0.8
1.0
0
2
4
6
8
10



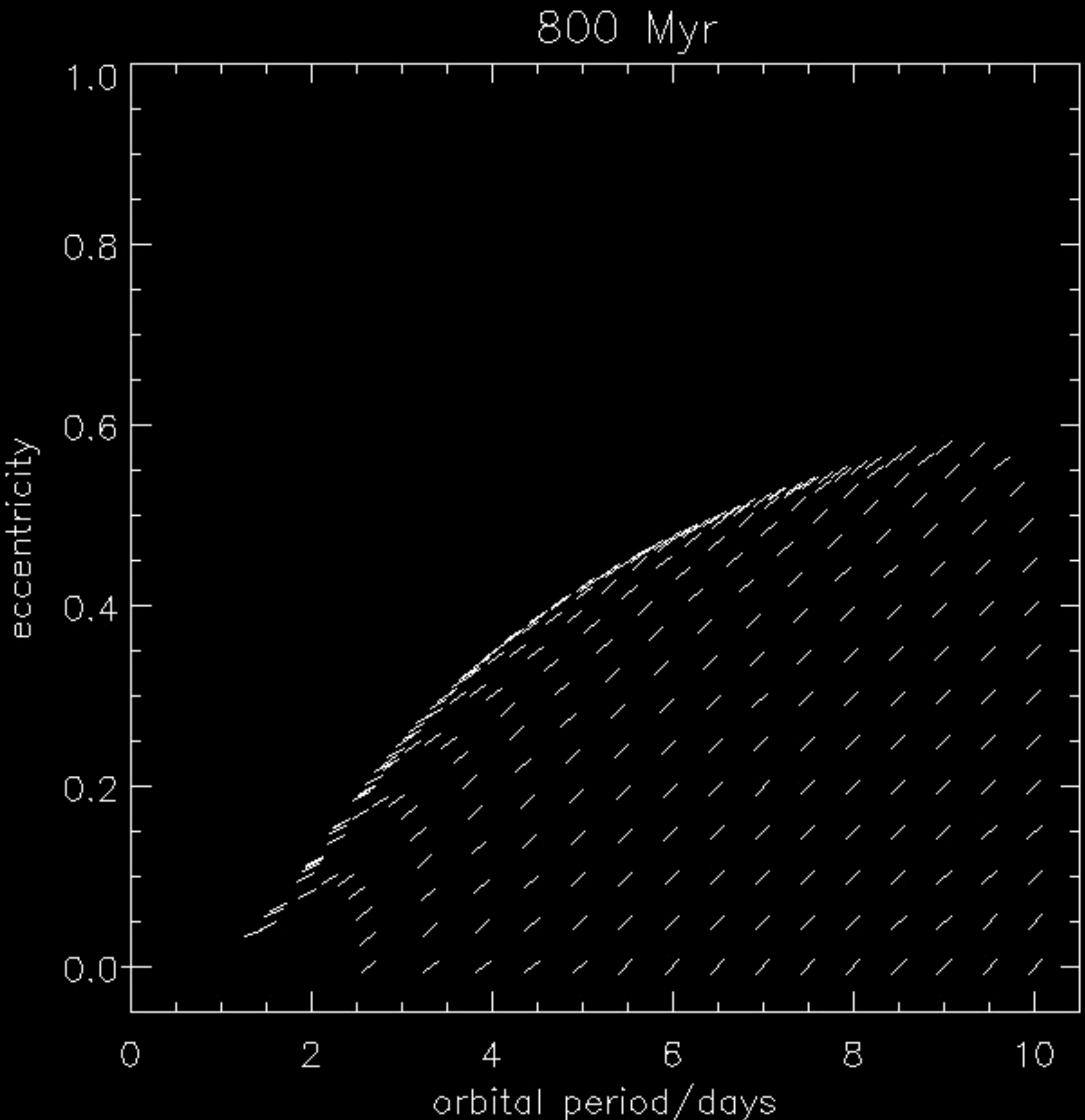
400 Myr
eccentricity
orbital period/days
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10

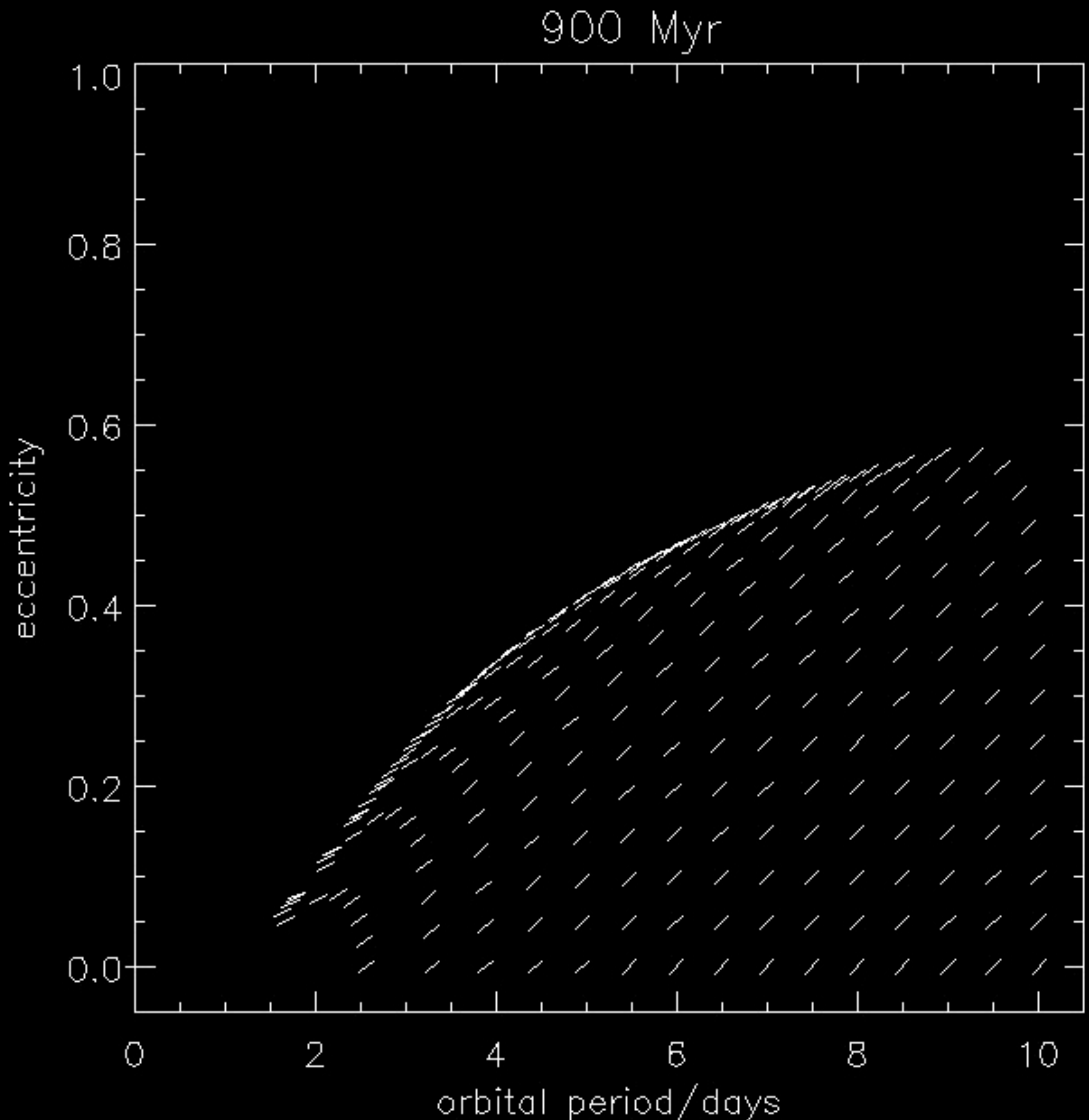

500 Myr
1.0
0.8
0.6
0.4
0.2
0.0
eccentricity
0
2
4
6
8
10
orbital period/days

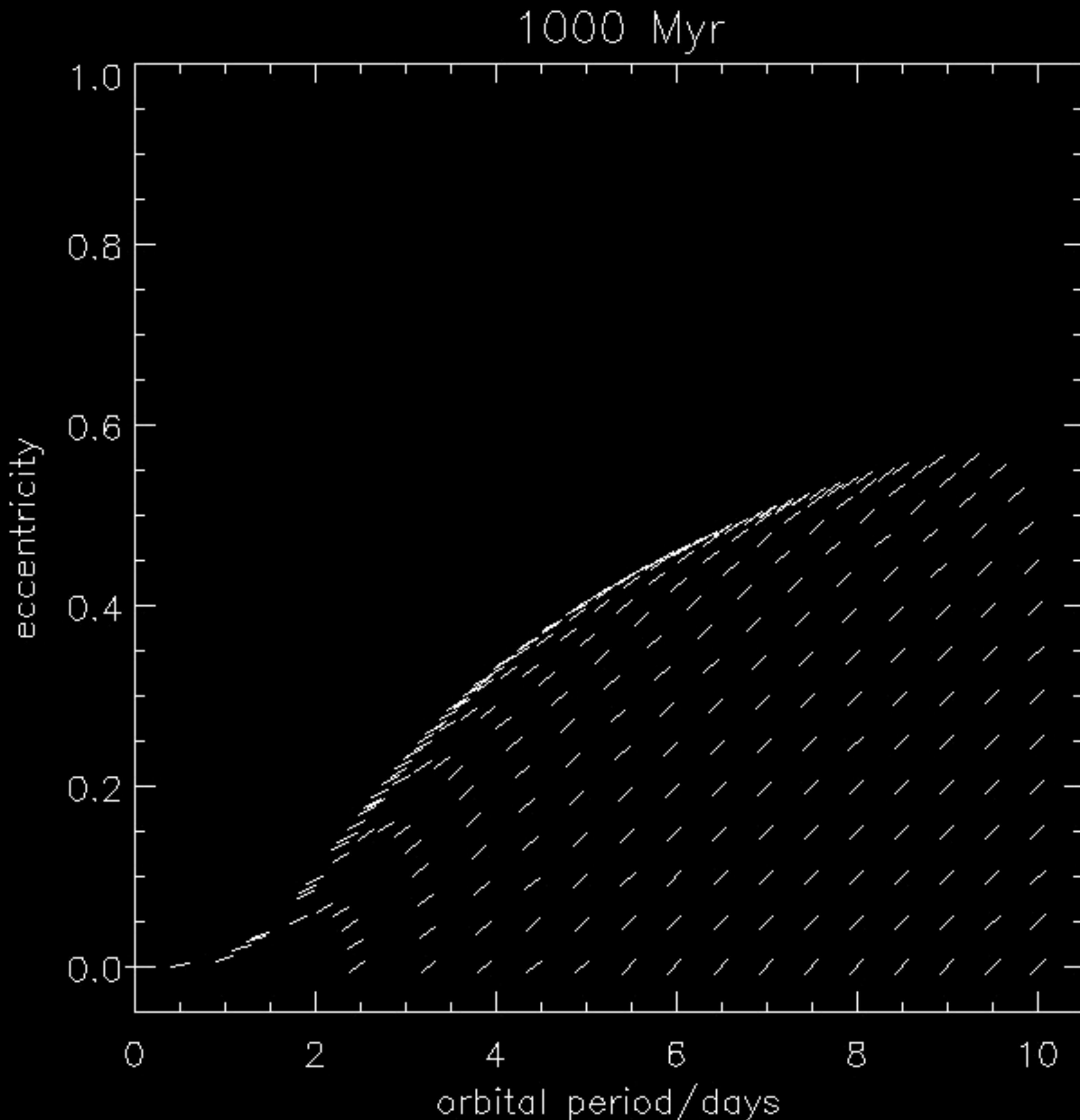

600 Myr
1.0
0.8
0.6
0.4
0.2
0.0
eccentricity
0
2
4
6
8
10
orbital period/days

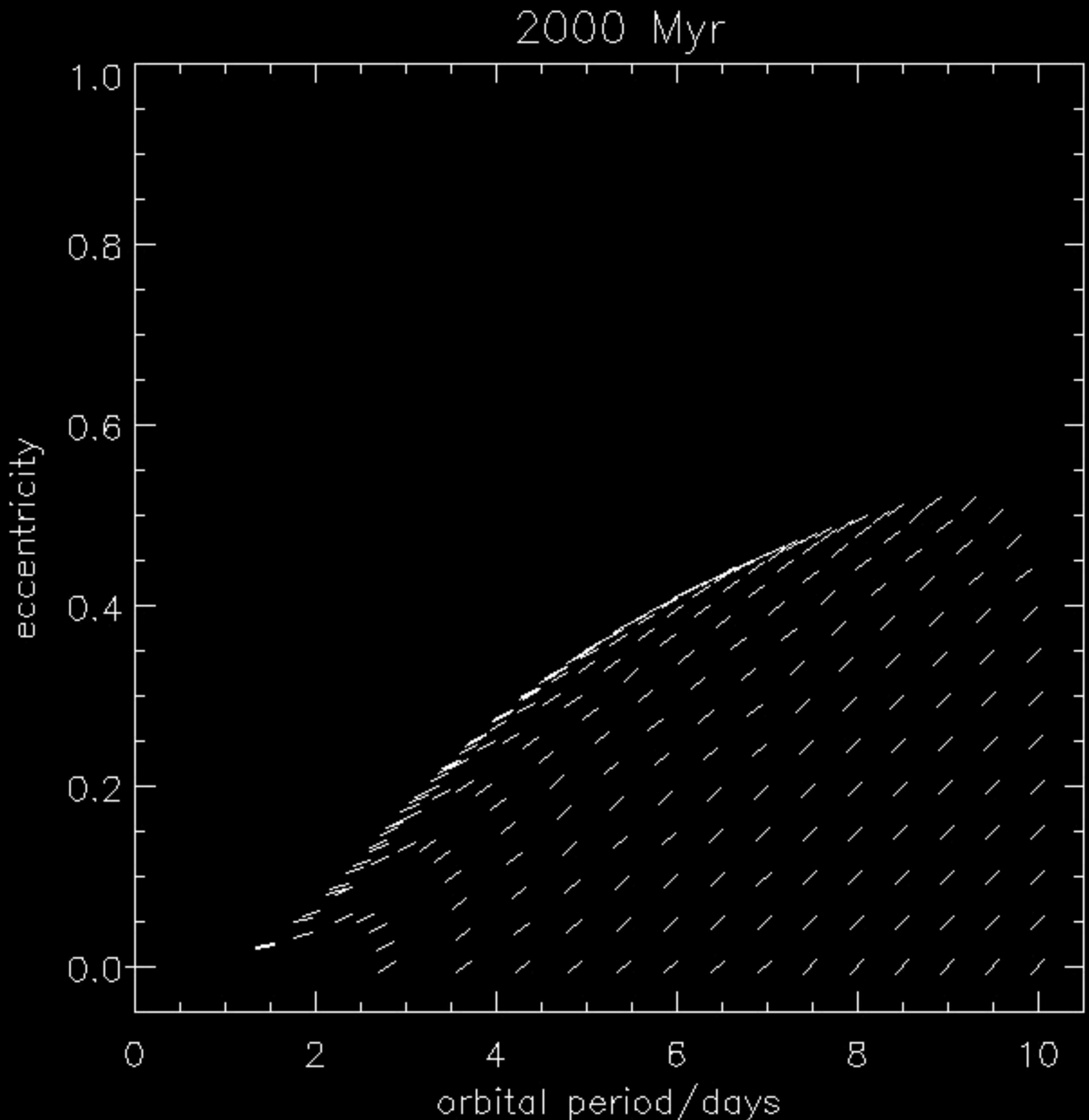


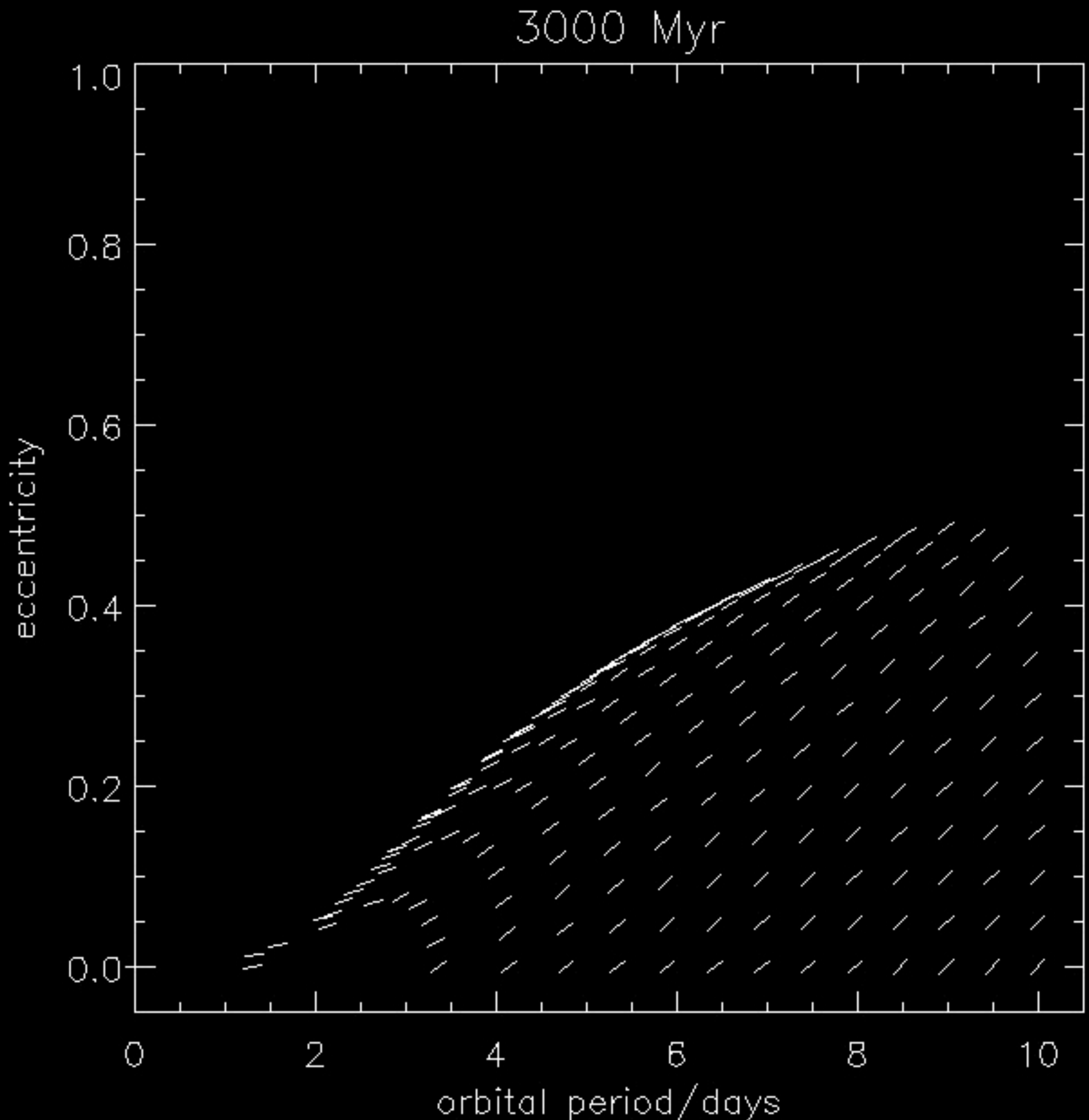
2011年12月2日金曜日

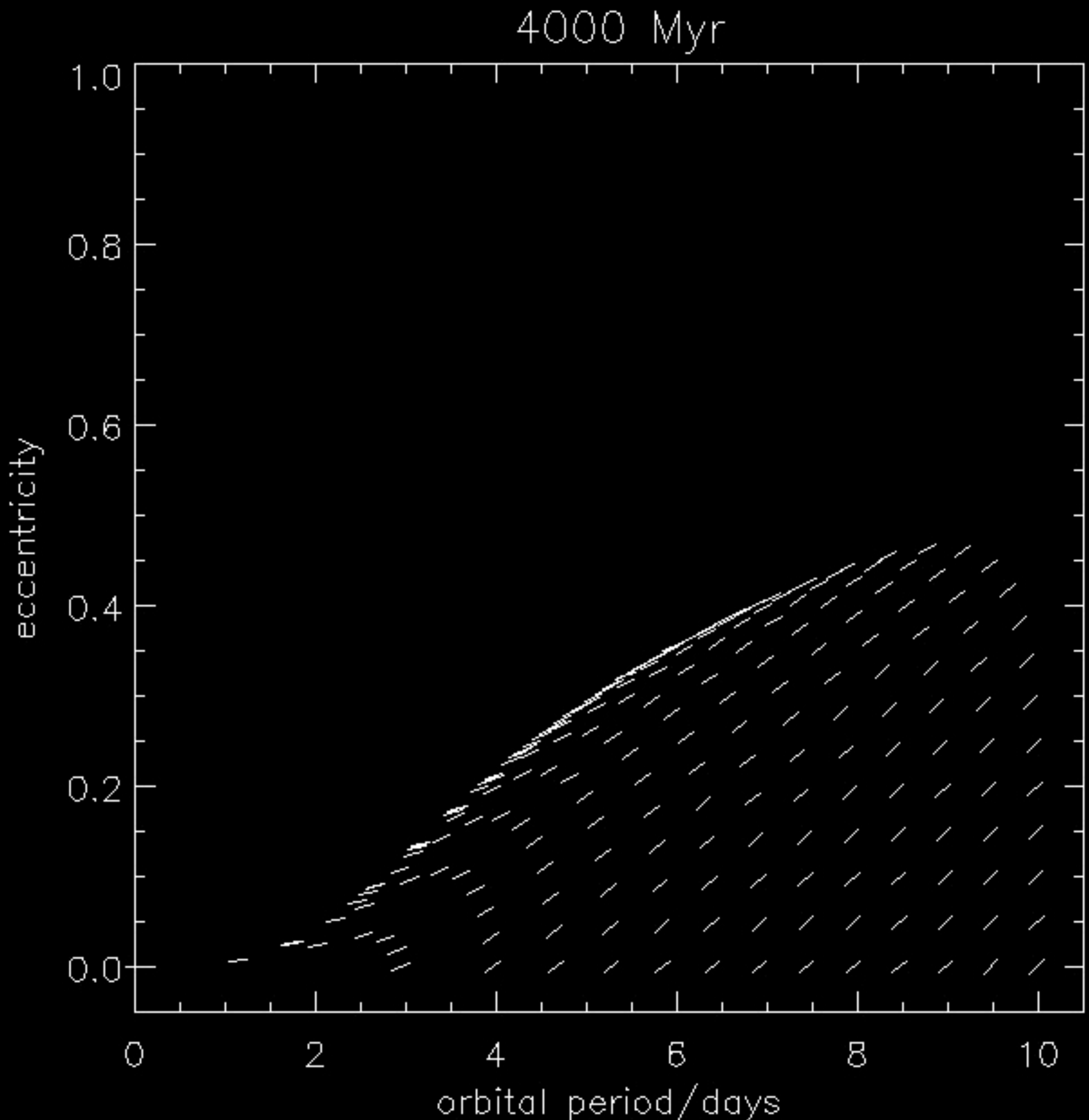

800 Myr
eccentricity
orbital period/days
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10

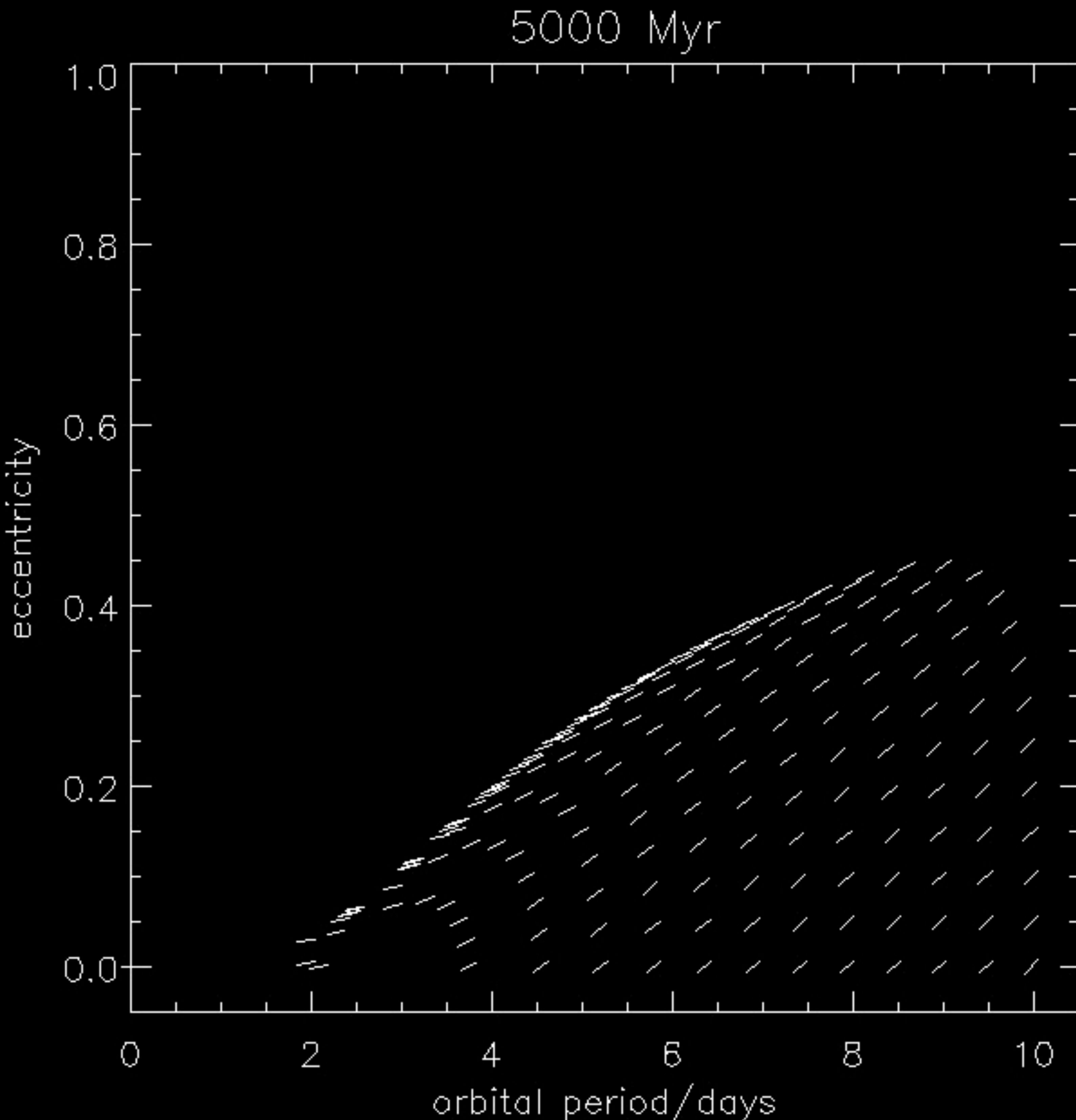

900 Myr
eccentricity
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10
orbital period/days


1000 Myr
eccentricity
orbital period/days
0.0
0.2
0.4
0.6
0.8
1.0
0
2
4
6
8
10

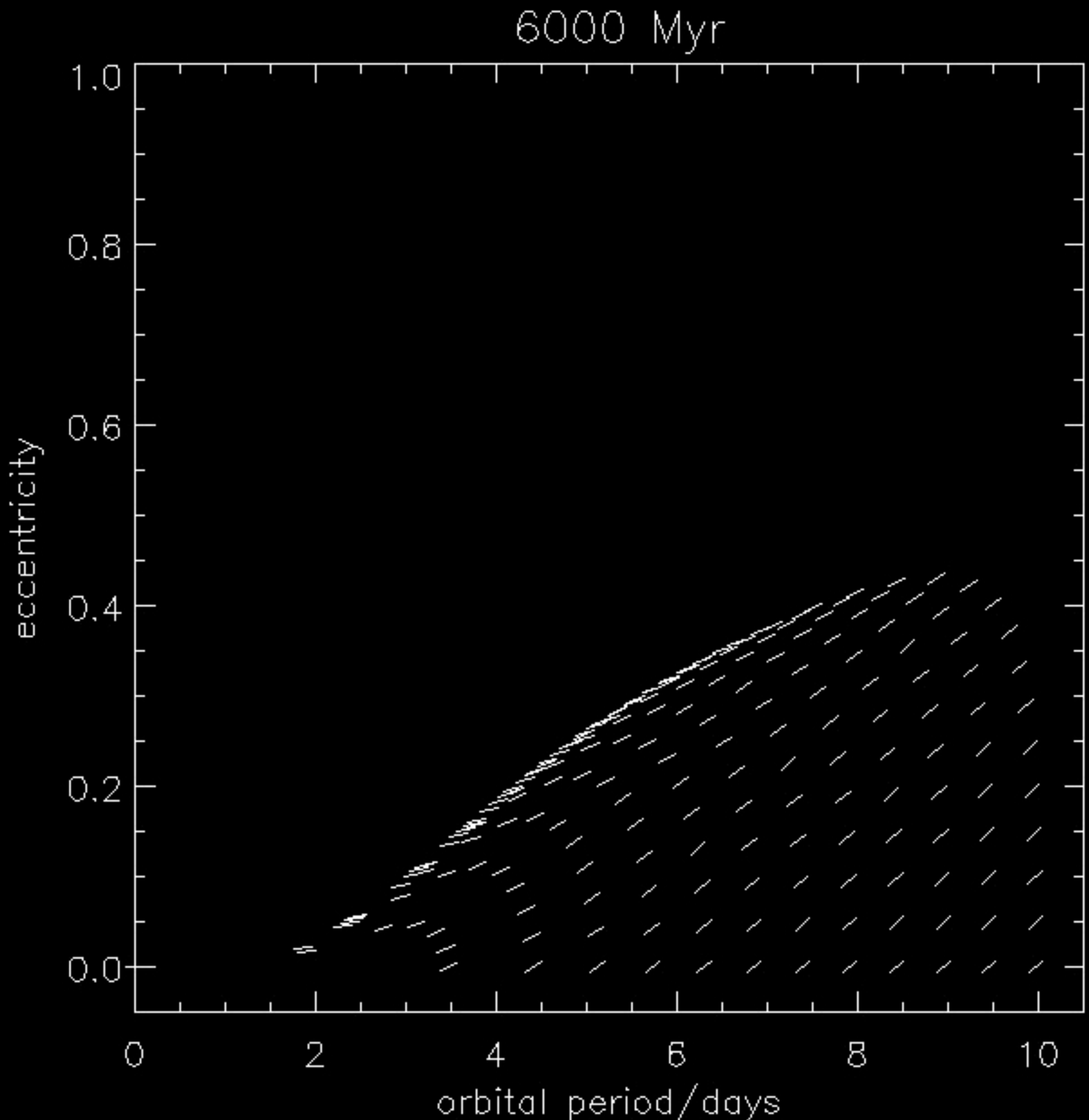

2000 Myr
1.0
0.8
0.6
0.4
0.2
0.0
eccentricity
0
2
4
6
8
10
orbital period/days

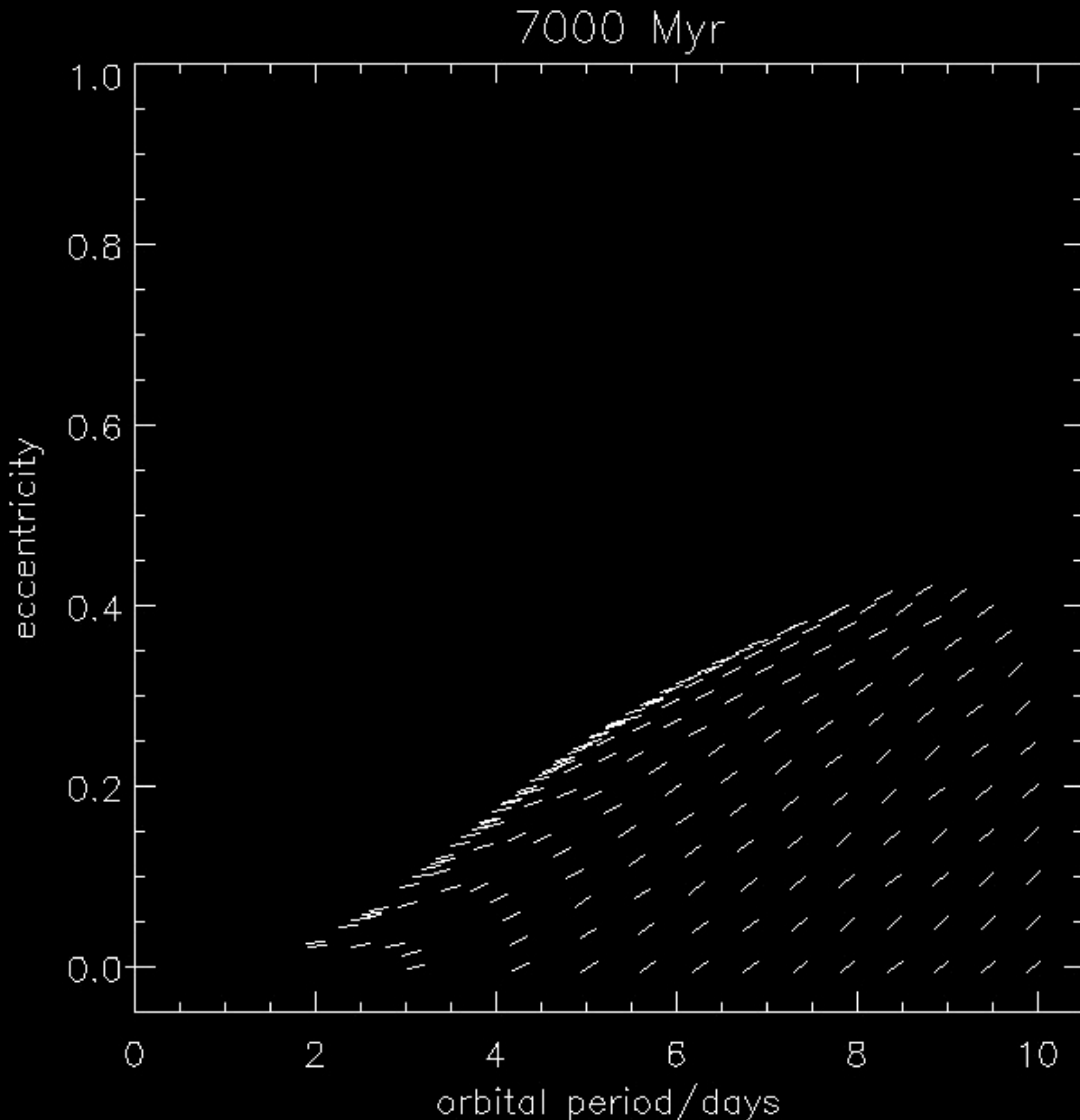

3000 Myr
eccentricity
orbital period/days
0.0
0.2
0.4
0.6
0.8
1.0
0
2
4
6
8
10

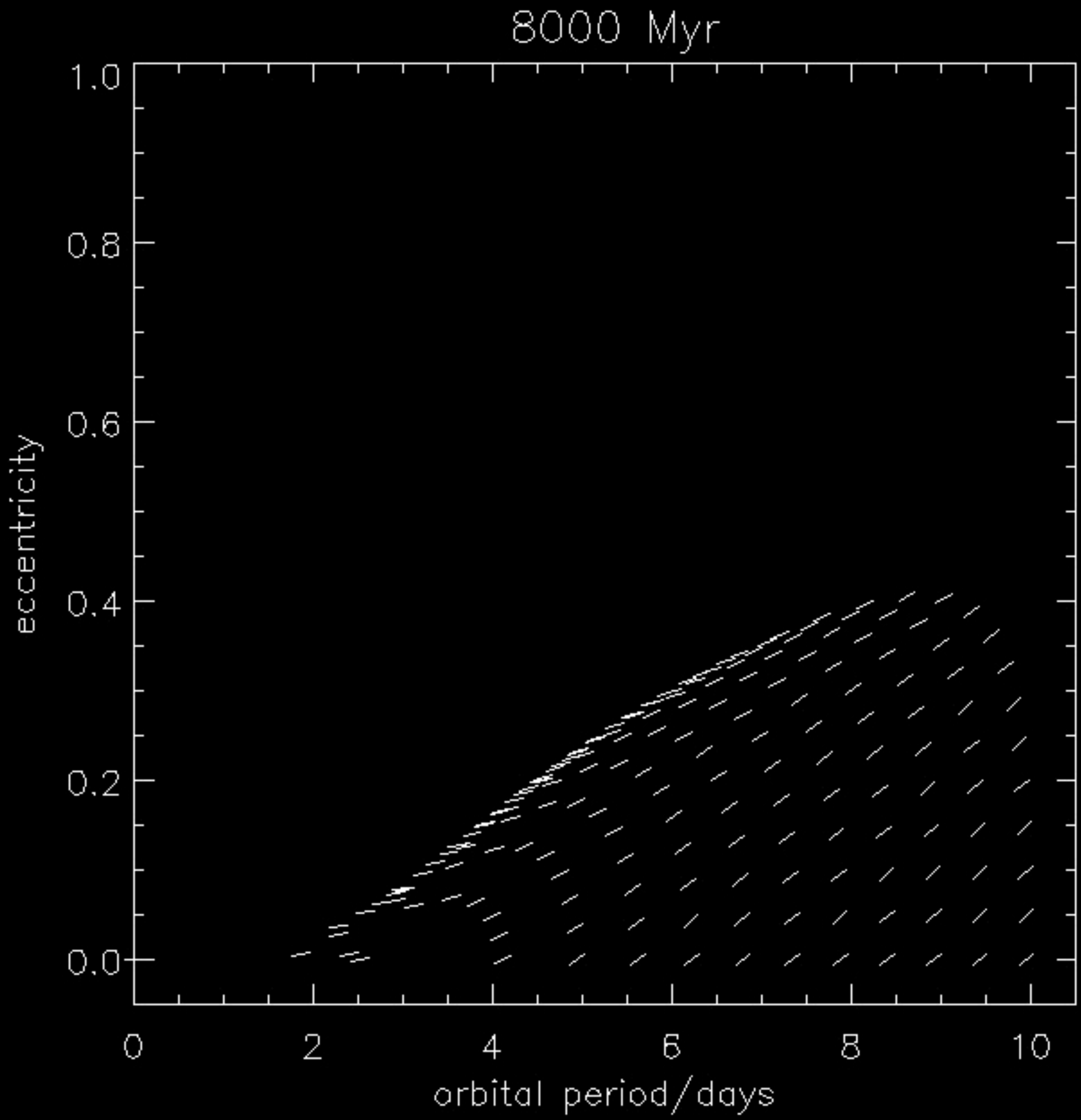

4000 Myr
eccentricity
orbital period/days
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10



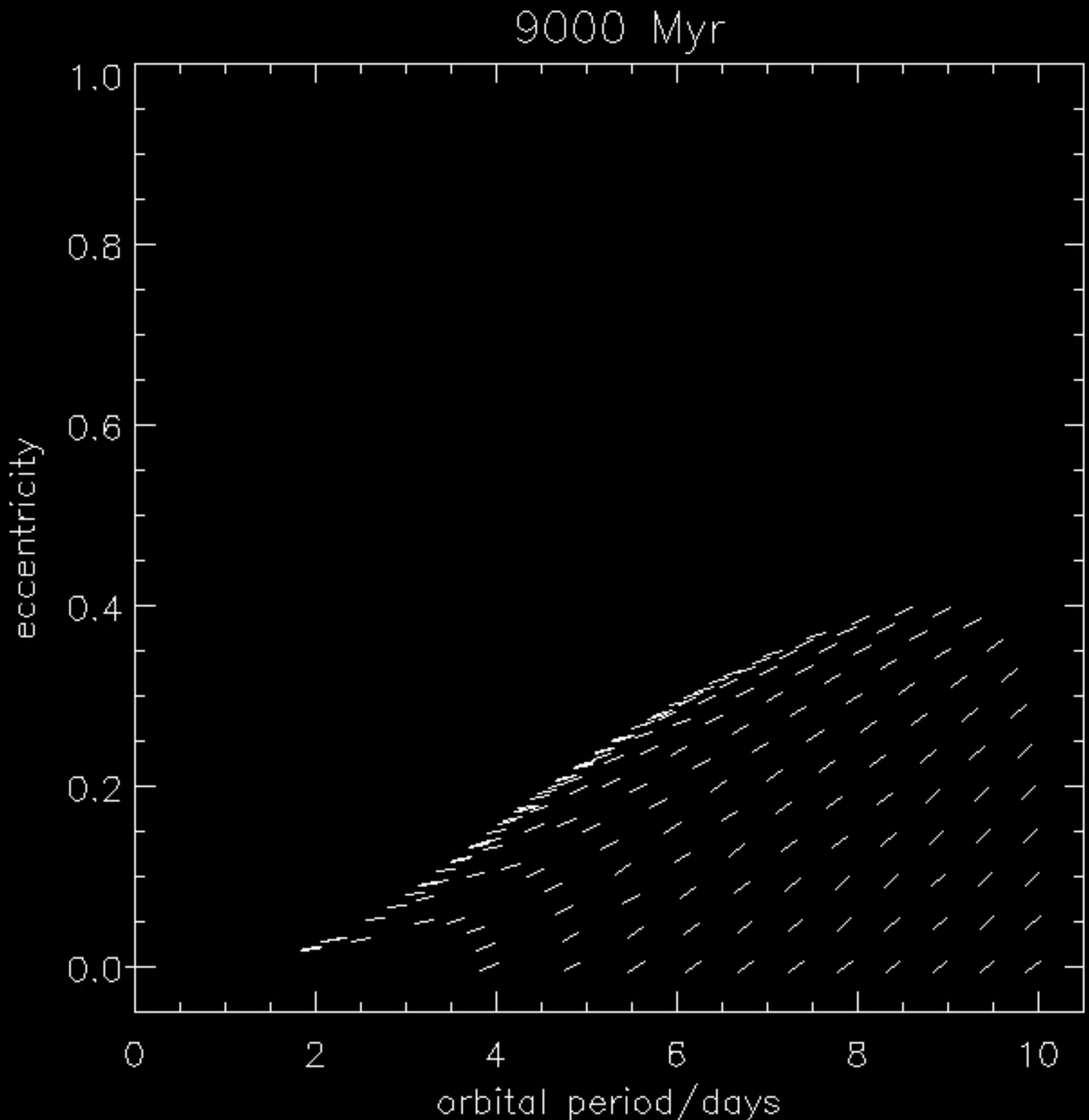
5000 Myr
eccentricity
orbital period/days
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10


6000 Myr
eccentricity
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10
orbital period/days

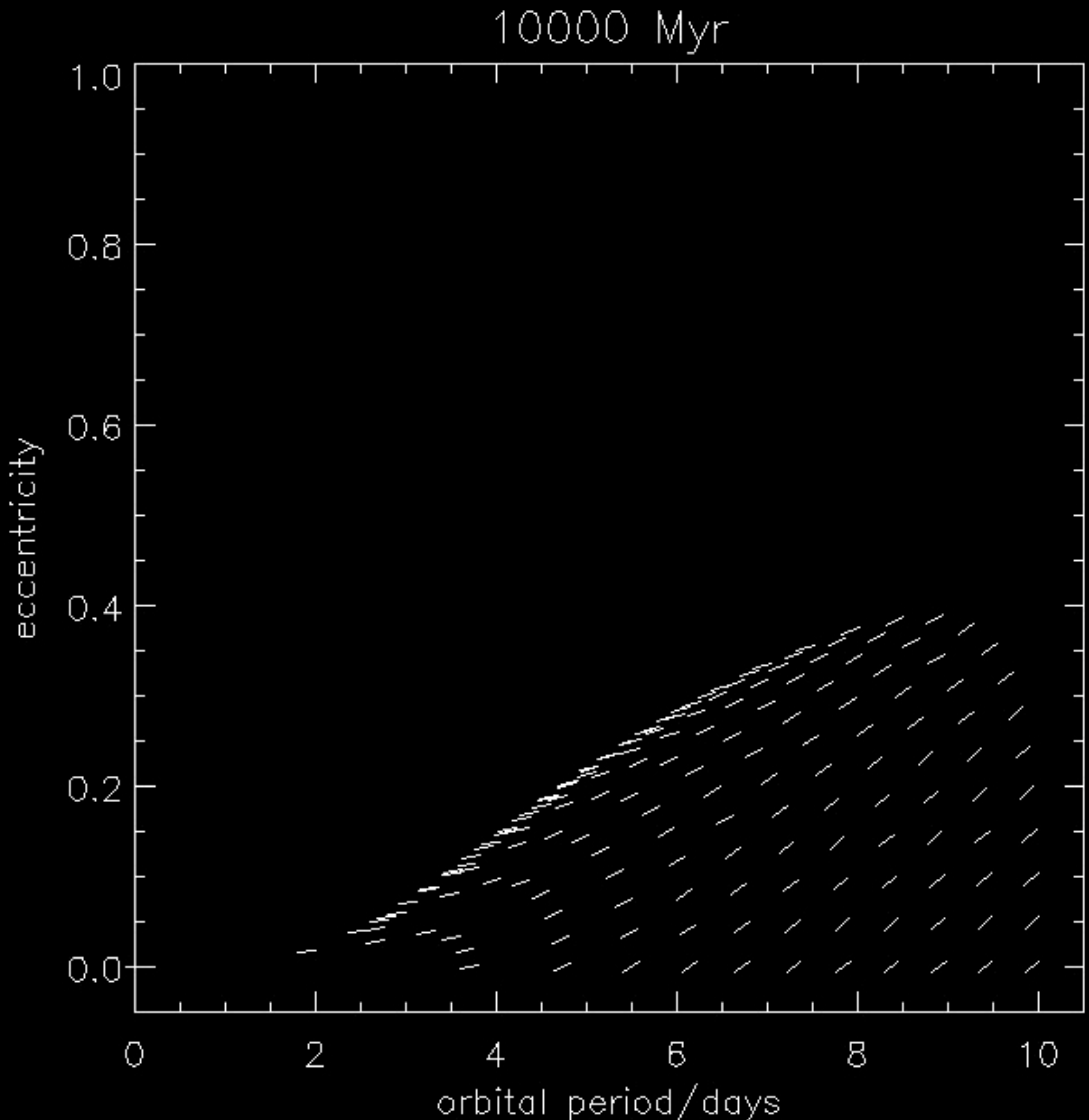

7000 Myr
1.0
0.8
0.6
0.4
0.2
0.0
eccentricity
0
2
4
6
8
10
orbital period/days



8000 Myr
eccentricity
orbital period/days
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10



9000 Myr
eccentricity
orbital period/days
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10


10000 Myr
eccentricity
orbital period/days
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10

Timescales (for small e)

- Circularization (using tide in planet)

$$3\,Q'_{\rm p}\,\frac{M_{\rm p}}{M_{\rm J}}\left(\frac{R_{\rm p}}{R_{\rm J}}\right)^{-5}\left(\frac{M_\star}{M_\odot}\right)^{2/3}\left(\frac{P}{\rm day}\right)^{13/3}\ {\rm yr}$$

- Orbital decay (inspiral)

$$0.02\,Q'_\star\,\frac{M_*}{M_{\rm p}}\left(\frac{\bar\rho}{\bar\rho_\odot}\right)^{5/3}\left(\frac{P}{\rm day}\right)^{13/3}\ {\rm yr}$$

- Spin-orbit alignment

$$0.07\,Q'_\star\,\frac{M_*}{M_{\rm p}}\left(\frac{\bar\rho}{\bar\rho_\odot}\right)^{5/3}\left(\frac{P}{\rm day}\right)^{13/3}\ {\rm yr}$$

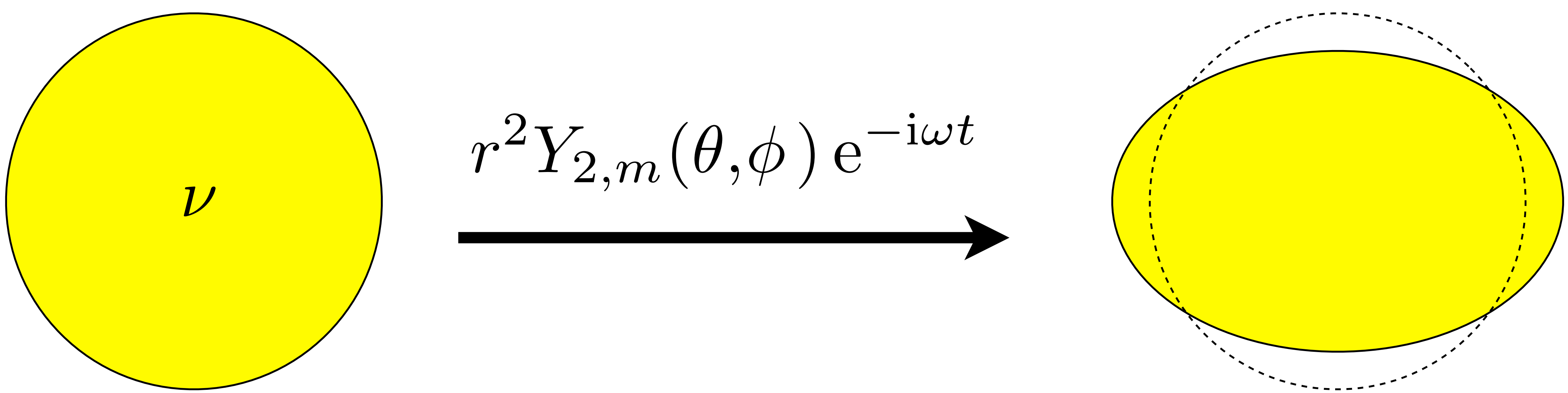
(but reduced if orbit has more angular momentum than stellar spin)

Tidal dissipation in rotating stars and giant planets

J-P Zahn’s categorization :

- “Equilibrium tide”

Dissipation associated with large-scale tidal bulge



- “Dynamical tide”

Dissipation associated with low-frequency waves

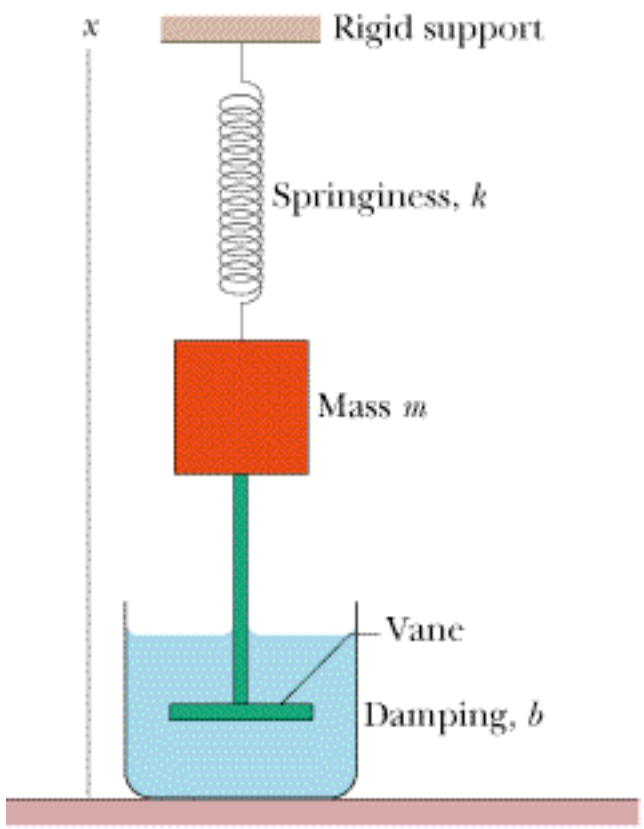
Analogy: forced harmonic oscillator

$$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = f\,\mathrm{e}^{-\mathrm{i}\omega t}$$

$$x = k\,\frac{f}{\omega_0^2}\,\mathrm{e}^{-\mathrm{i}\omega t}$$

$$\begin{aligned} k &= \left(1 - \frac{2\mathrm{i}\omega\gamma}{\omega_0^2} - \frac{\omega^2}{\omega_0^2}\right)^{-1} \\ &\approx \left(1 + \mathrm{i}\,Q^{-1}\right) \end{aligned}$$

$$[\,\omega,\gamma \ll \omega_0\,]$$



$$Q = \frac{\omega_0^2}{2\omega\gamma} = \frac{2\pi \times \text{maximum potential energy}}{\text{energy dissipated per cycle}} \gg 1$$

http://www.webassign.net/hrw/hrw7_15-15.gif

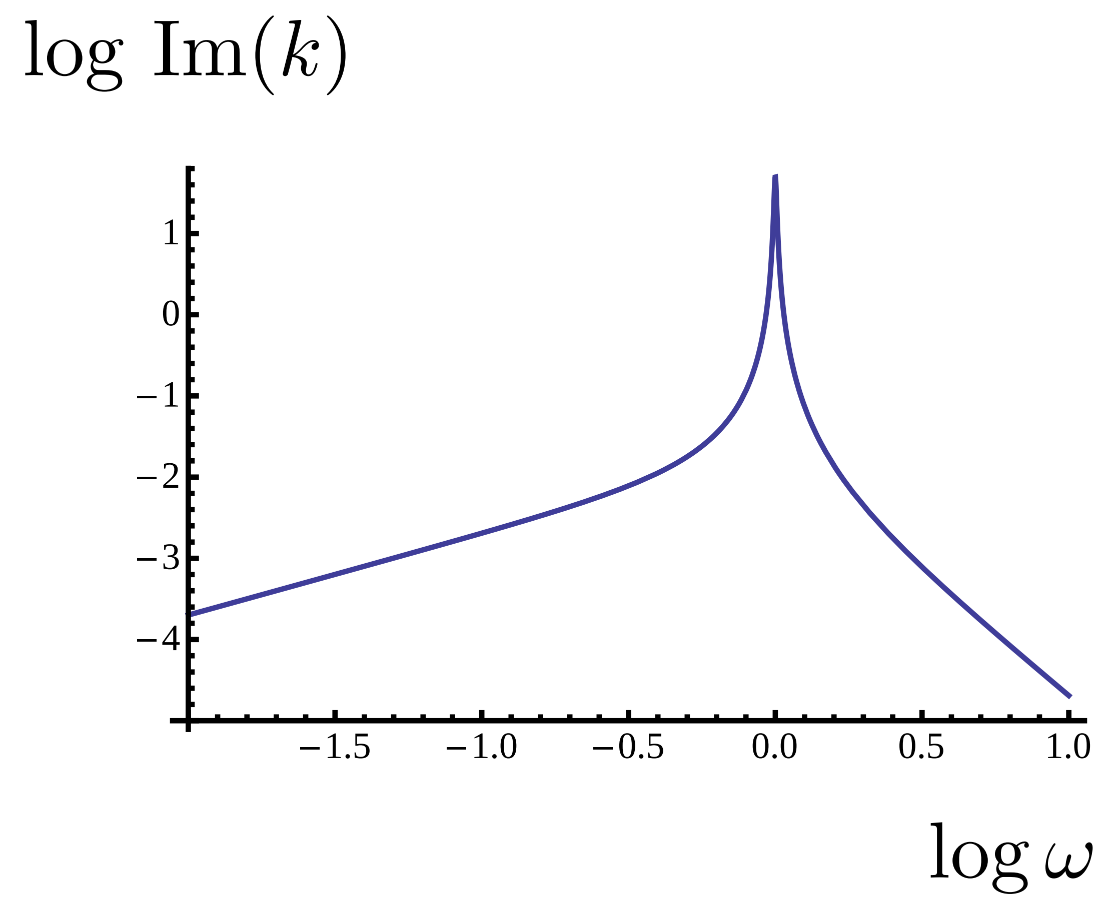
Analogy: forced harmonic oscillator

$$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = f\,\mathrm{e}^{-\mathrm{i}\omega t}$$

$$x = k\,\frac{f}{\omega_0^2}\,\mathrm{e}^{-\mathrm{i}\omega t}$$

$$\begin{aligned} k &= \left(1 - \frac{2\mathrm{i}\omega\gamma}{\omega_0^2} - \frac{\omega^2}{\omega_0^2}\right)^{-1} \\ &\approx \left(1 + \mathrm{i}\,Q^{-1}\right) \end{aligned}$$

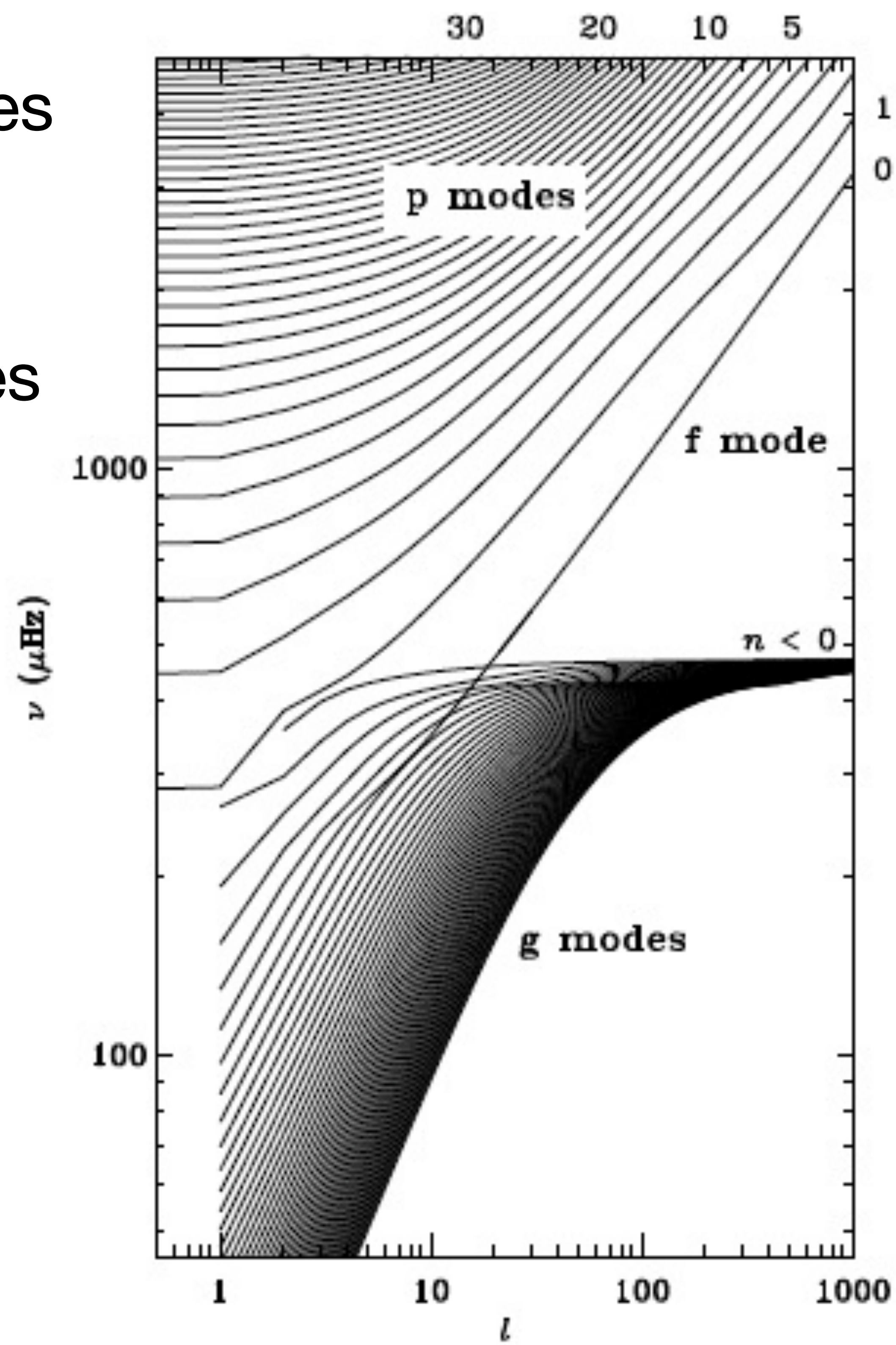
$$[\,\omega,\gamma \ll \omega_0\,]$$



$$Q = \frac{\omega_0^2}{2\omega\gamma} = \frac{2\pi \times \text{maximum potential energy}}{\text{energy dissipated per cycle}} \gg 1$$

Normal modes and tidal overlap integrals

- Assume the body has a complete orthogonal set of ideal oscillation modes
- Calculate weak damping rates
- Project tidal forcing on to normal modes
- Each responds as a forced damped harmonic oscillator



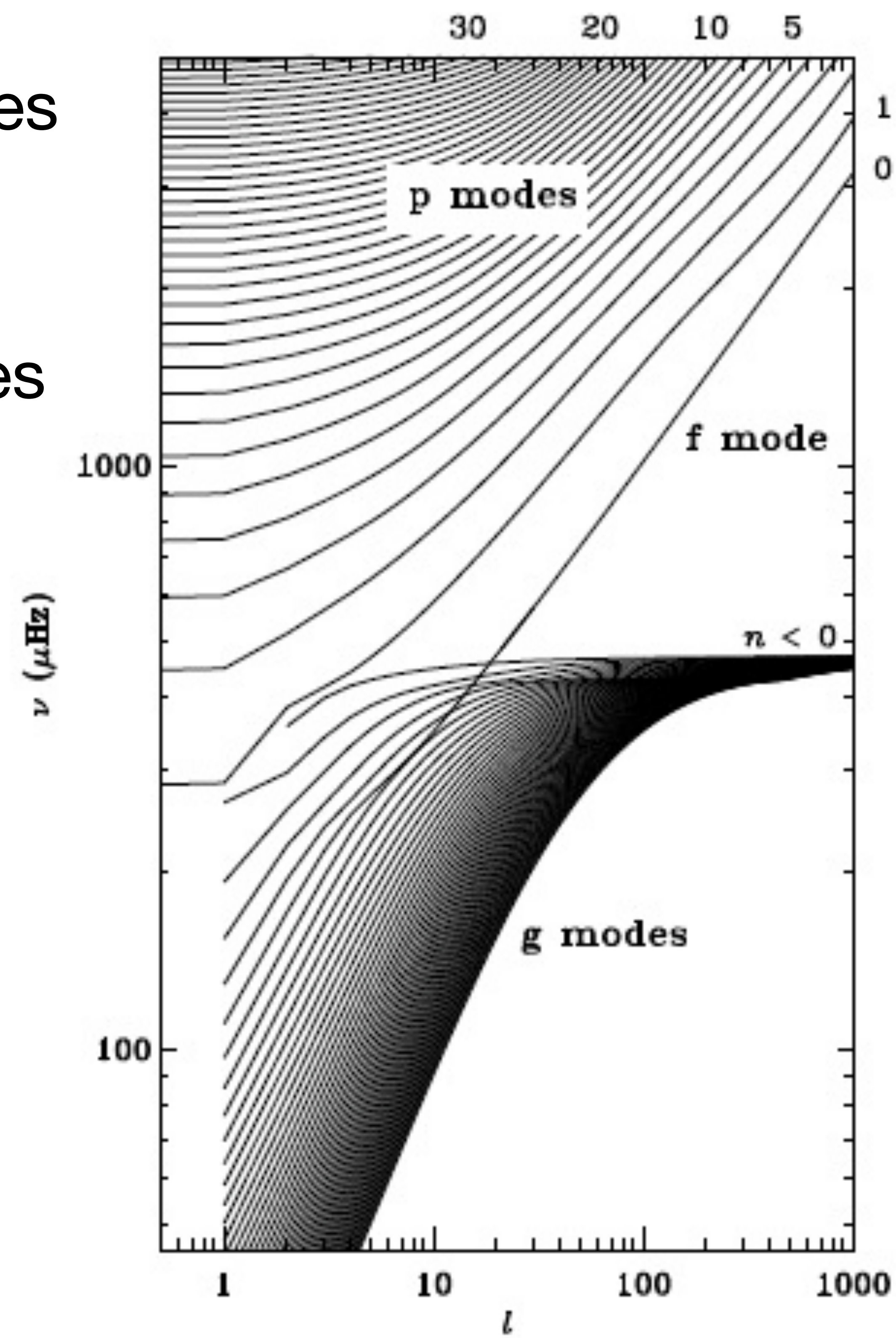
Christensen-Dalsgaard (2003)

Normal modes and tidal overlap integrals

- Assume the body has a complete orthogonal set of ideal oscillation modes
- Calculate weak damping rates
- Project tidal forcing on to normal modes
- Each responds as a forced damped harmonic oscillator

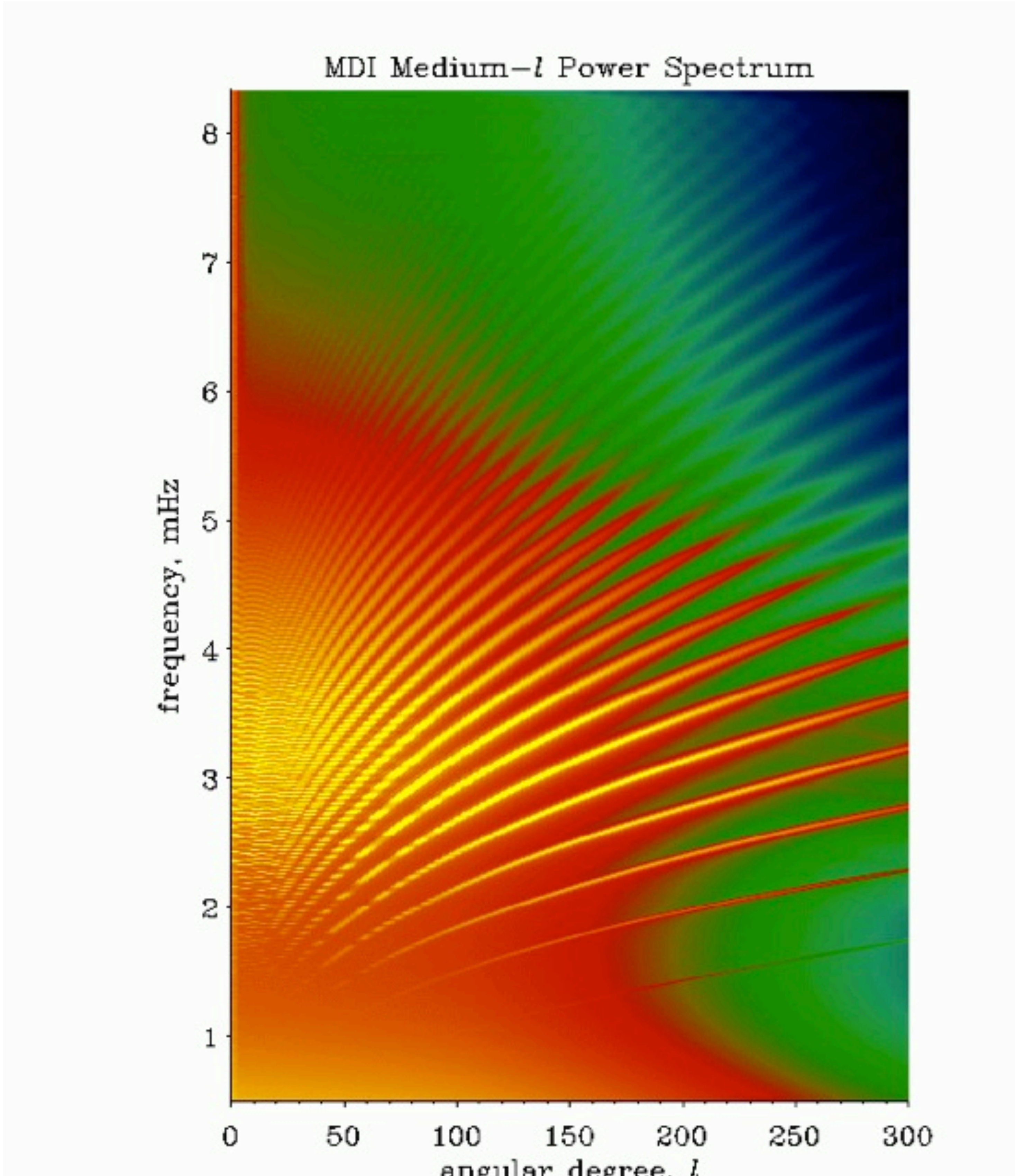
Cowling (1941): non-rotating star

- f, p and g modes
- most frequencies too high
- high-order g modes



Christensen-Dalsgaard (2003)

Normal modes observed in the Sun

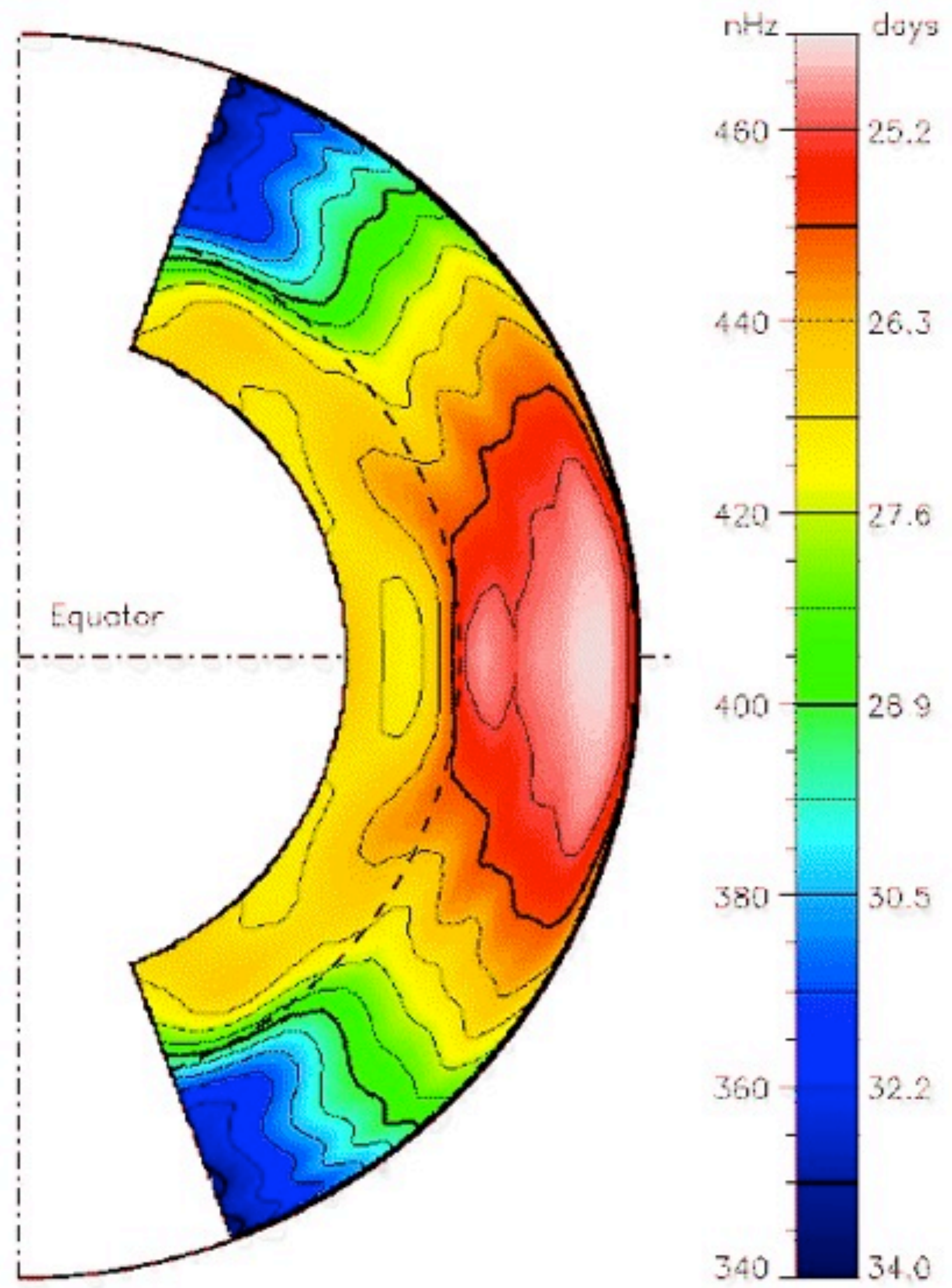


http://www.mps.mpg.de/projects/sun-climate/image/mdi005_s.gif

Oscillations with rotation

- No complete theory!

- Helioseismology:
  fast modes
  Ω → small correction



Thompson et al. (1996)

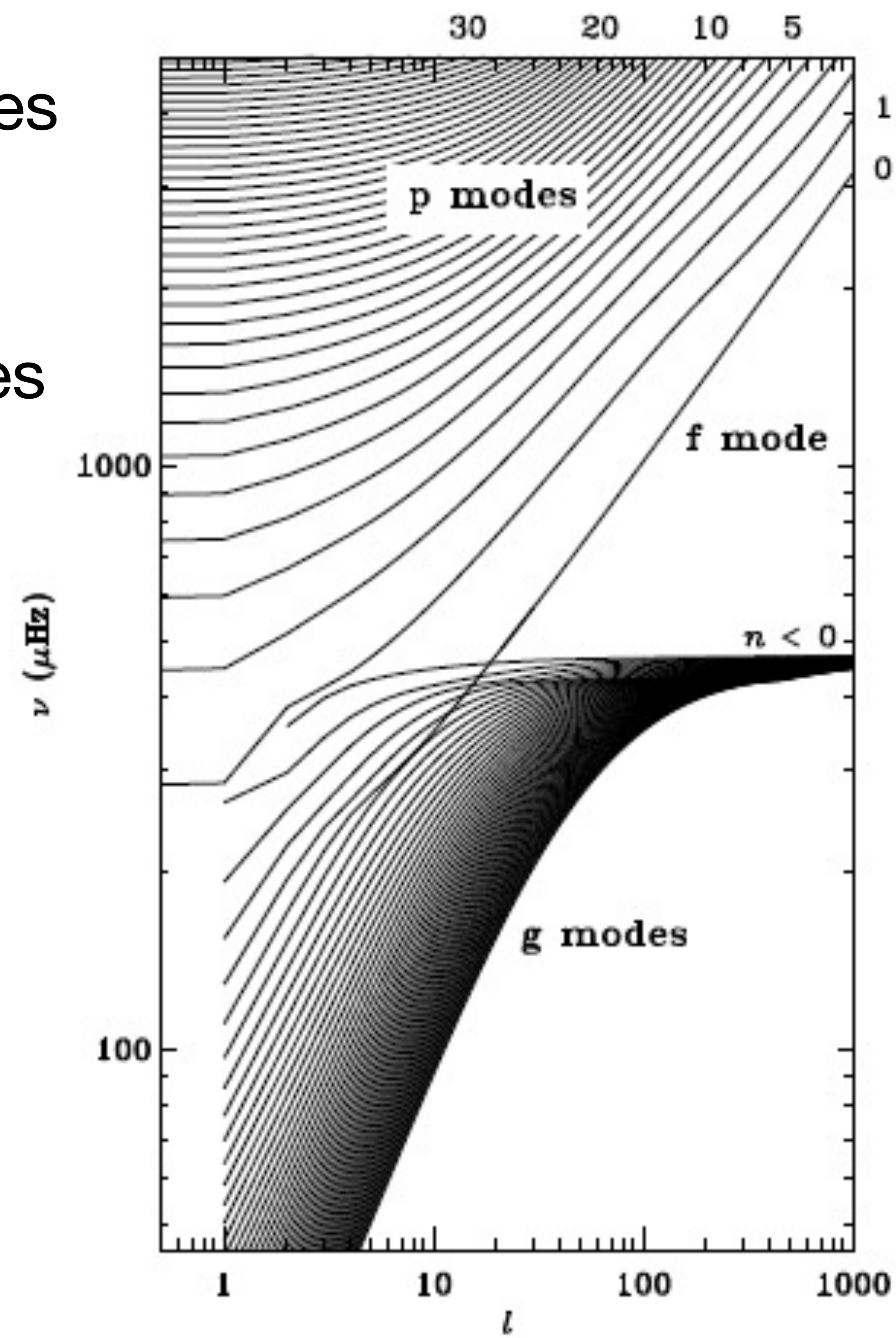
- Tides:
  slow modes
  Ω → radical alteration
  and new modes / waves

Normal modes and tidal overlap integrals

- Assume the body has a complete orthogonal set of ideal oscillation modes
- Calculate weak damping rates
- Project tidal forcing on to normal modes
- Each responds as a forced damped harmonic oscillator

Rotating stars and planets:

Approach may fail if waves don't reflect to form standing modes, or if normal modes can't be defined because of singularities



Christensen-Dalsgaard (2003)

Tidal forcing problem

Viscous uniformly rotating fluid

Tidal potential Ψ and linear response proportional to $\mathrm{e}^{-\mathrm{i}\omega t}$

$$-\mathrm{i}\omega u_i + 2\epsilon_{ijk}\Omega_j u_k = -\partial_i\Phi' - \partial_i\Psi - \frac{1}{\rho}\partial_i p' + \frac{\rho'}{\rho^2}\partial_i p + \frac{1}{\rho}\partial_j T_{ij}$$

$$-\mathrm{i}\omega\rho' + u_i\partial_i\rho = -\rho\partial_i u_i$$

$$-\mathrm{i}\omega p' + u_i\partial_i p = -\Gamma_1 p\partial_i u_i$$

$$T_{ij} = 2\mu S_{ij} + \mu_{\mathrm{b}}(\partial_k u_k)\delta_{ij}$$

$$S_{ij} = \partial_i u_j + \partial_j u_i - \tfrac{2}{3}(\partial_k u_k)\delta_{ij}$$

$$\partial_{ii}\Phi' = 4\pi G\rho'$$

Given $\Psi = \left(\frac{r}{R}\right)^l Y_{l,m}(\theta,\phi)$

find $\Phi' = k_{l,m}\left(\frac{r}{R}\right)^{-(l+1)} Y_{l,m}(\theta,\phi) + \cdots \qquad (r > R)$

Tidal forcing problem

Energy dissipation rate

$$D = \frac{1}{2} \int \left(2\mu S_{ij}^* S_{ij} + \mu_\mathrm{b} |\partial_i u_i|^2\right) \mathrm{d}V$$

Energy input rate

$$-\frac{1}{2}\omega \,\mathrm{Im} \int \rho' \Psi^* \,\mathrm{d}V = \mathrm{Im}(k_{l,m}) \frac{(2l+1)R}{8\pi G} = D$$

Tidal torque

$$T = \frac{m}{\omega} D$$

$T,\ D \Leftrightarrow$ rate of tidal evolution

Complications:

differential rotation, thermal diffusion, convection,

magnetic fields, nonlinearity, ...

From Goldreich (1963)

rigidity and Q the specific dissipation function

$$Q = \frac{2\pi E^*}{\oint (dE/dt)dt}, \quad (2)$$

where E^* is the peak energy stored in the system during a cycle and $\oint (dE/dt)dt$ is the energy dissipated over a complete cycle. **Q will in general vary with the frequency and amplitude of the tide and the size of the sphere in addition to its composition.**

//

cycle per second to one cycle per year (8). In this case ϵ_1 must be the dominant term and the sign of $\overline{(de/dt)}_p$ is the same as the sign of $2\omega - 3n$. **While this constant behaviour of Q with frequency may not be true for all planets (especially not the major ones)** it is still likely that the ϵ_1 term is dominant because of its relatively large coefficient. If this ϵ_1 term is dominant, we have $\overline{(de/dt)}_p > 0$ for all satellites

//

small amplitude will have a phase lag which increases when its peak is reinforcing the peak of the tide of major amplitude. This non-linear behaviour cannot be treated in detail since very little is known about the response of the planets to tidal forces, except for the Earth. **In our discussions we shall use the language of linear tidal theory, but we must keep in mind that our numbers are really only parametric fits to a non-linear problem.**

http://bellerophonchimera.files.wordpress.com/
2008/07/solar-maximum-september-1-20011.gif

Nonlinearity of tides in fluid bodies

- Equilibrium tidal amplitude

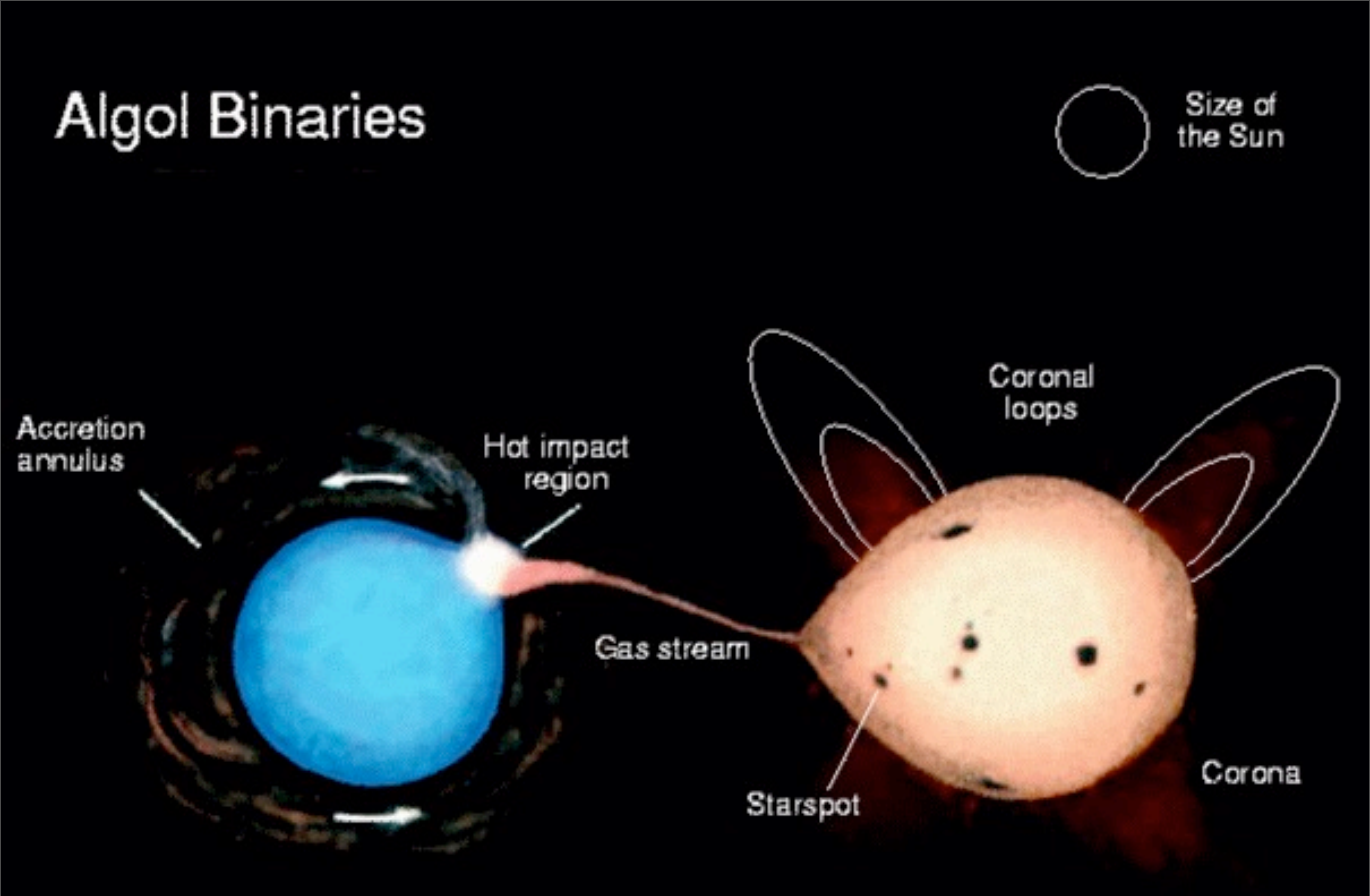
$$\frac{\xi}{R_1} \sim \frac{M_2}{M_1}\left(\frac{R_1}{a}\right)^3 \qquad \sim 1 \quad \text{for} \quad R_1 \sim \left(\frac{M_1}{M_2}\right)^{1/3} a \qquad \rightarrow$$

- Nonlinear breakdown through secondary instabilities when

$$\frac{\xi}{R_1} \sim \left(\frac{\nu}{R_1^2\omega}\right)^{1/2} \ll 1 \;\; ? \qquad \text{or} \qquad \frac{\xi}{R_1} \sim \frac{\nu}{R_1^2\omega} \;\; ?$$

- Internal wave nonlinearity

$$\sim \frac{\xi_{\rm wave}}{\lambda}$$



http://www2.astro.psu.edu/mrichards/research/binarypict.gif

Nonlinearity of tides in fluid bodies

- Equilibrium tidal amplitude

$$\frac{\xi}{R_1} \sim \frac{M_2}{M_1}\left(\frac{R_1}{a}\right)^3 \qquad \sim 1 \quad \text{for} \quad R_1 \sim \left(\frac{M_1}{M_2}\right)^{1/3} a$$

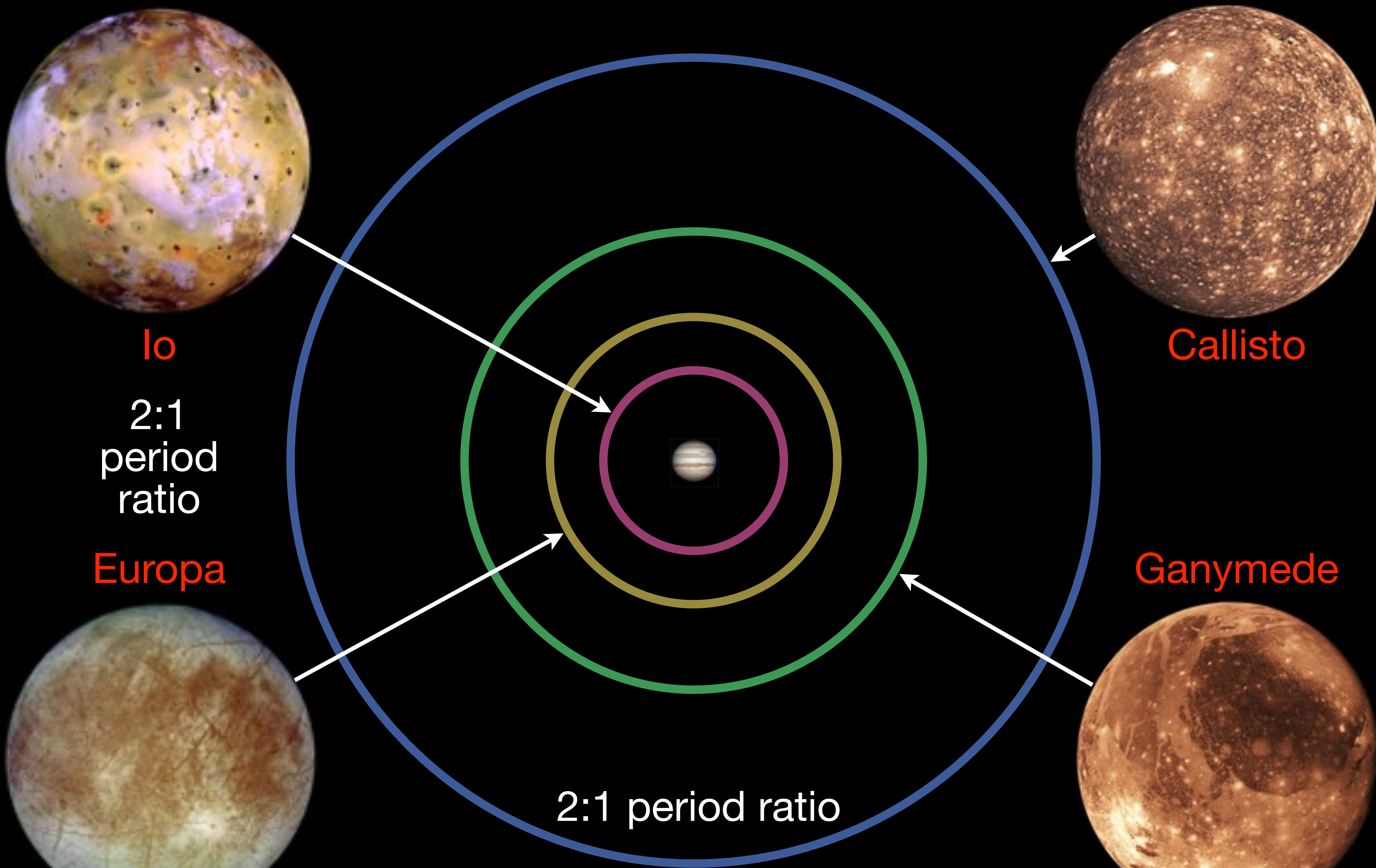
- Nonlinear breakdown through secondary instabilities when

$$\frac{\xi}{R_1} \sim \left(\frac{\nu}{R_1^2\omega}\right)^{1/2} \ll 1 \ \ ? \qquad \text{or} \qquad \frac{\xi}{R_1} \sim \frac{\nu}{R_1^2\omega} \ \ ?$$

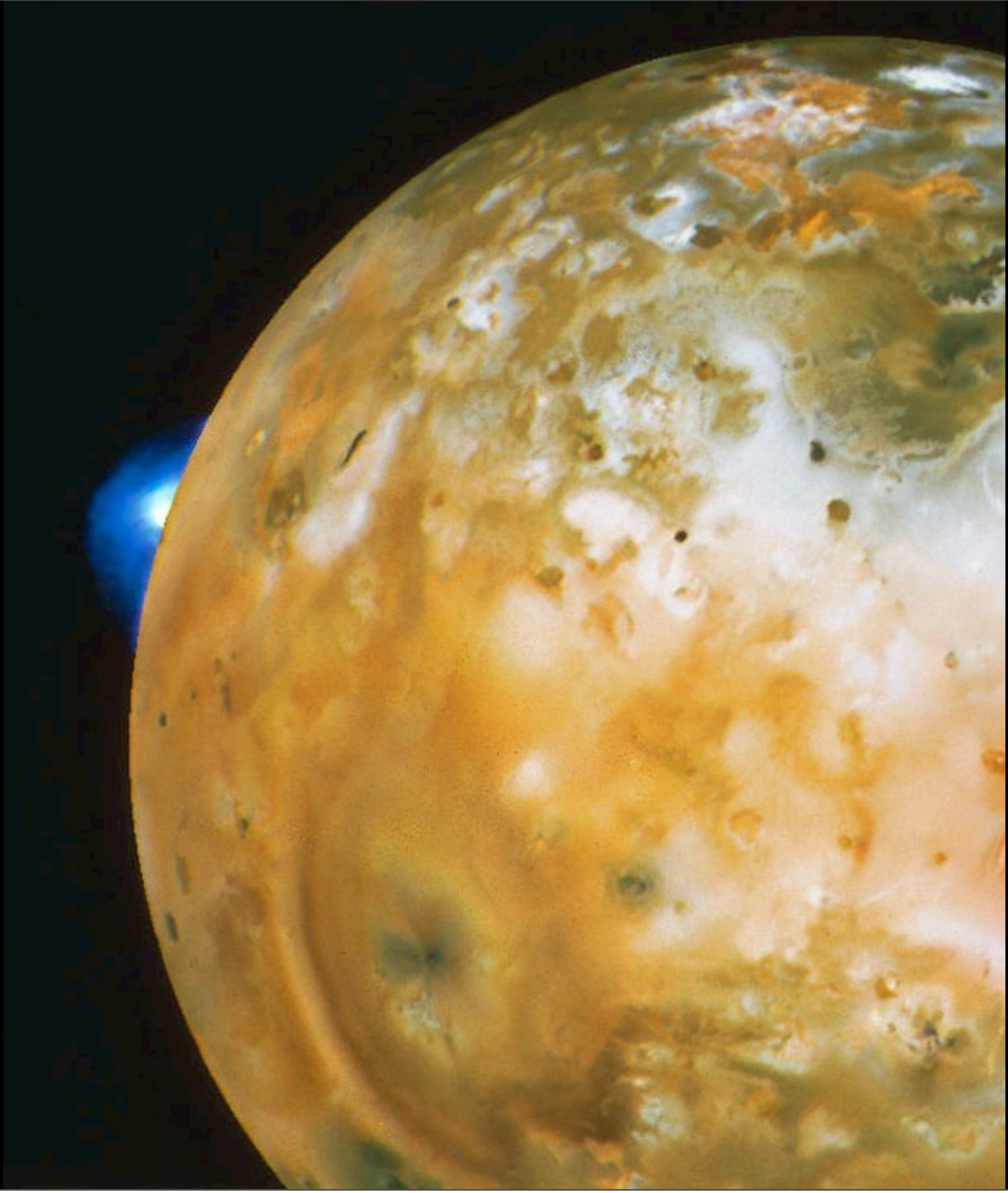
- Internal wave nonlinearity

$$\sim \frac{\xi_{\rm wave}}{\lambda}$$

Galilean moons of Jupiter



http://starryskies.com/The_sky/events/lunar-2003/planets.moons.jpg



NASA

Galilean moons of Jupiter

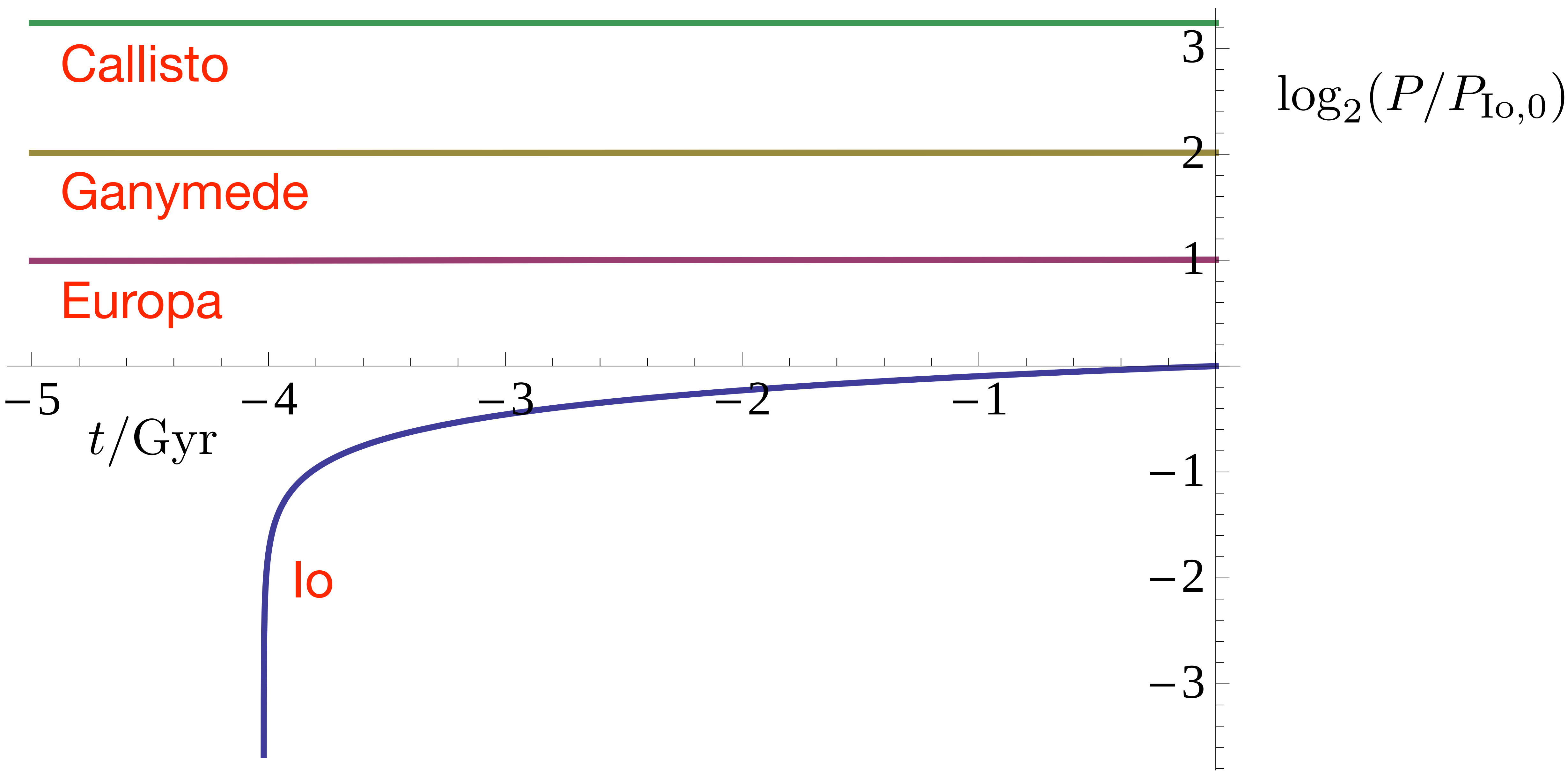
Assembly and maintenance of Laplace resonance:

- $2 \times 10^5 < Q'_\mathrm{J} < 8 \times 10^6$

(Goldreich 1965,
Yoder & Peale 1981)

Naive backward tidal evolution of Galilean satellites

- $Q'_{\mathrm{J}} = 10^6$



Galilean moons of Jupiter

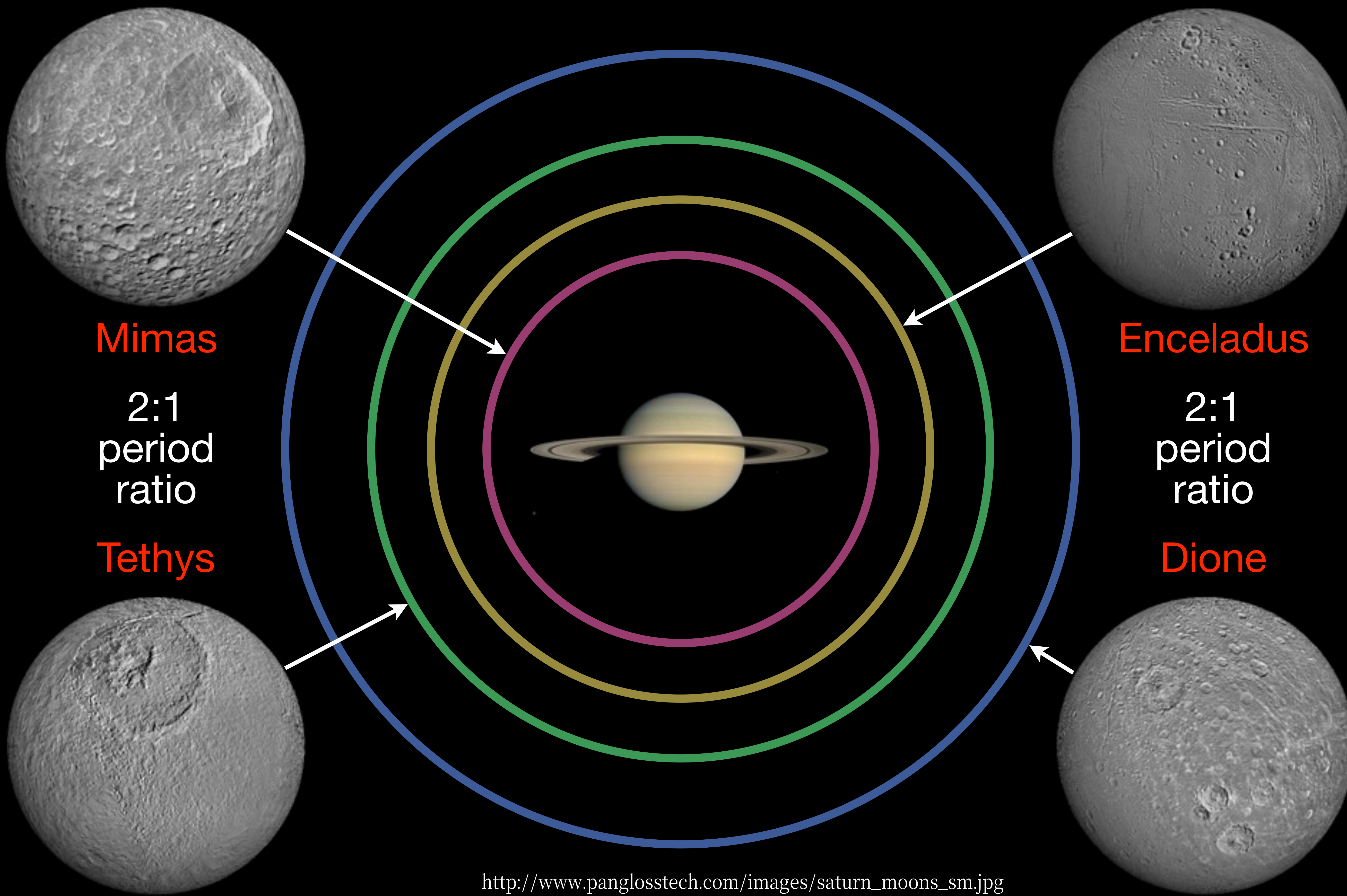
Lainey et al. (2009)

Table 1 | A selection of secular mean-motion accelerations published for the three inner Galilean moons

Ref.	Secular mean-motion acceleration ($\dot{n}/n$) (10^{-10} yr^{-1})		
	Io	Europa	Ganymede
9	$+3.3 \pm 0.5$	$+2.7 \pm 0.7$	$+1.5 \pm 0.6$
10	-0.074 ± 0.087	-0.082 ± 0.097	-0.098 ± 0.153
24	$+4.54 \pm 0.95$	$+5.6 \pm 5.7$	$+2.8 \pm 2.0$
25	$+2.27 \pm 0.70$	-0.67 ± 0.80	$+1.06 \pm 1.00$
26	$+3.6 \pm 1.0$	—	—
This paper	$+0.14 \pm 0.01$	-0.43 ± 0.10	-1.57 ± 0.27

$$Q'_{\mathrm{J,Io,now}} = 1.4 \times 10^5$$

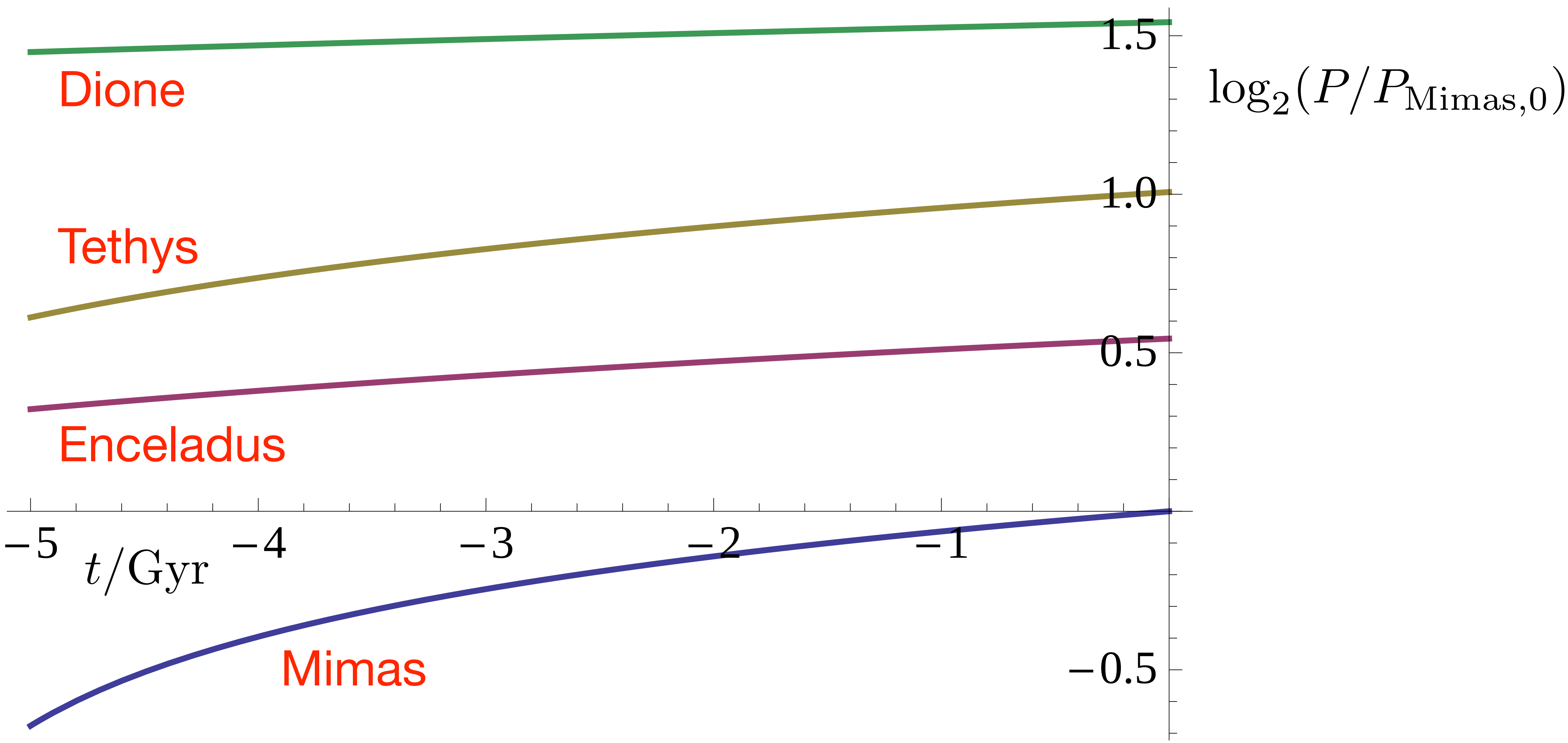
Inner moons of Saturn



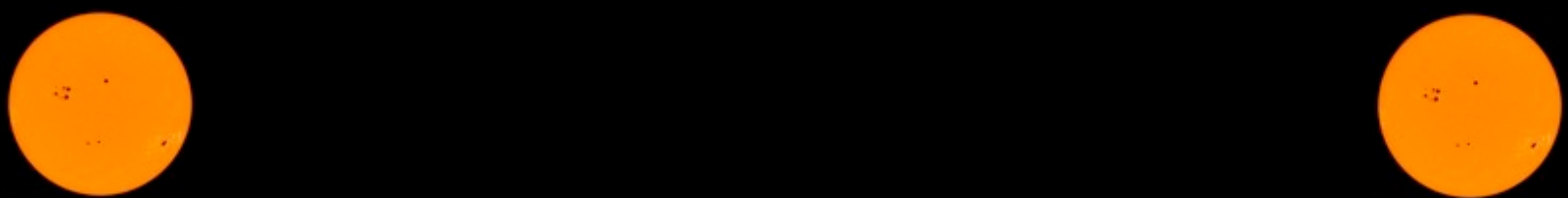
http://www.panglosstech.com/images/saturn_moons_sm.jpg

Naive backward tidal evolution of inner moons of Saturn

- $Q'_\mathrm{S} = 10^5$



Solar-type binary stars



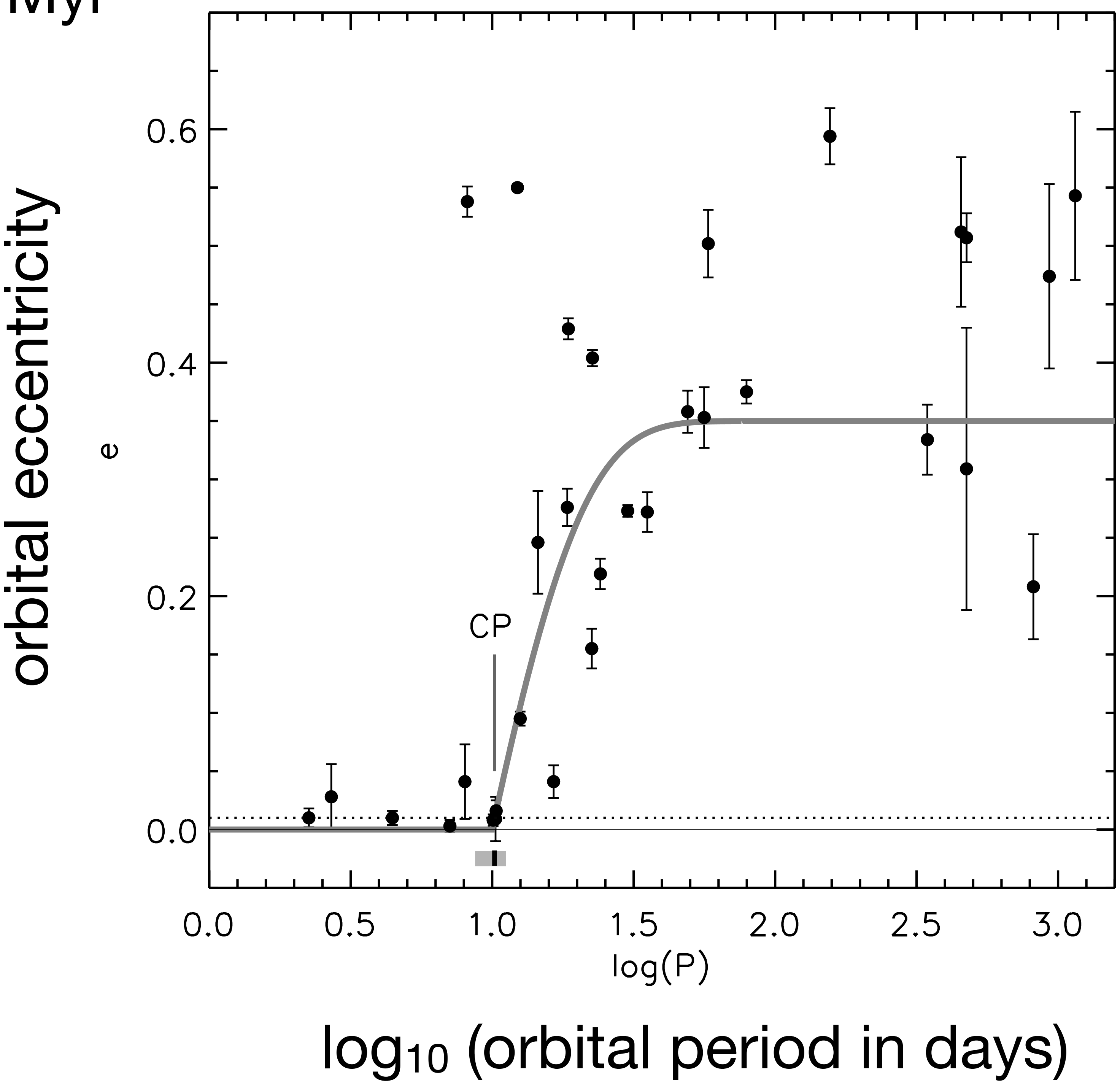
http://bellerophonchimera.files.wordpress.com/
2008/07/solar-maximum-september-1-20011.gif


NASA

Solar-type binary stars

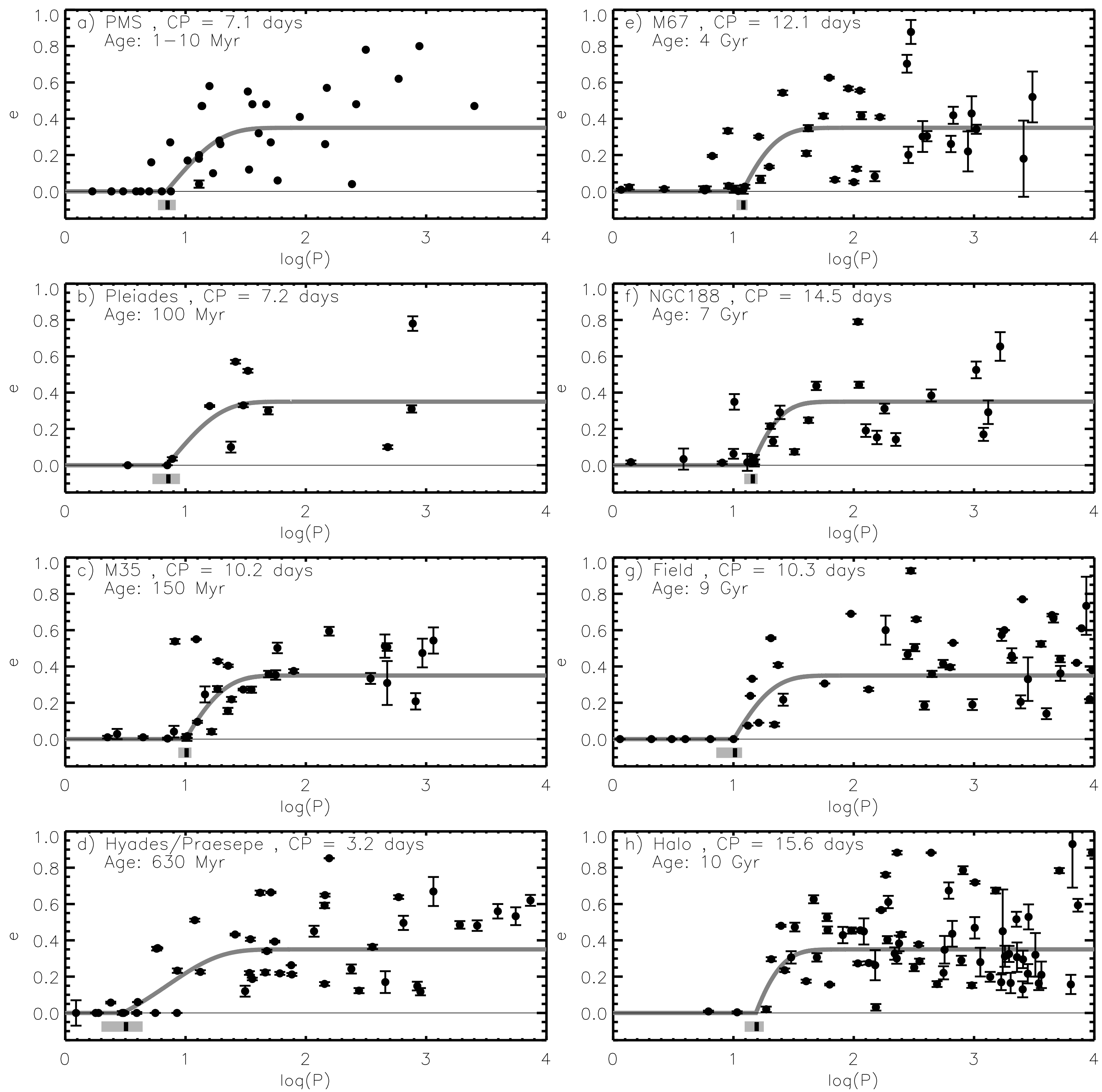
Meibom & Mathieu (2005)

M35, 150 Myr



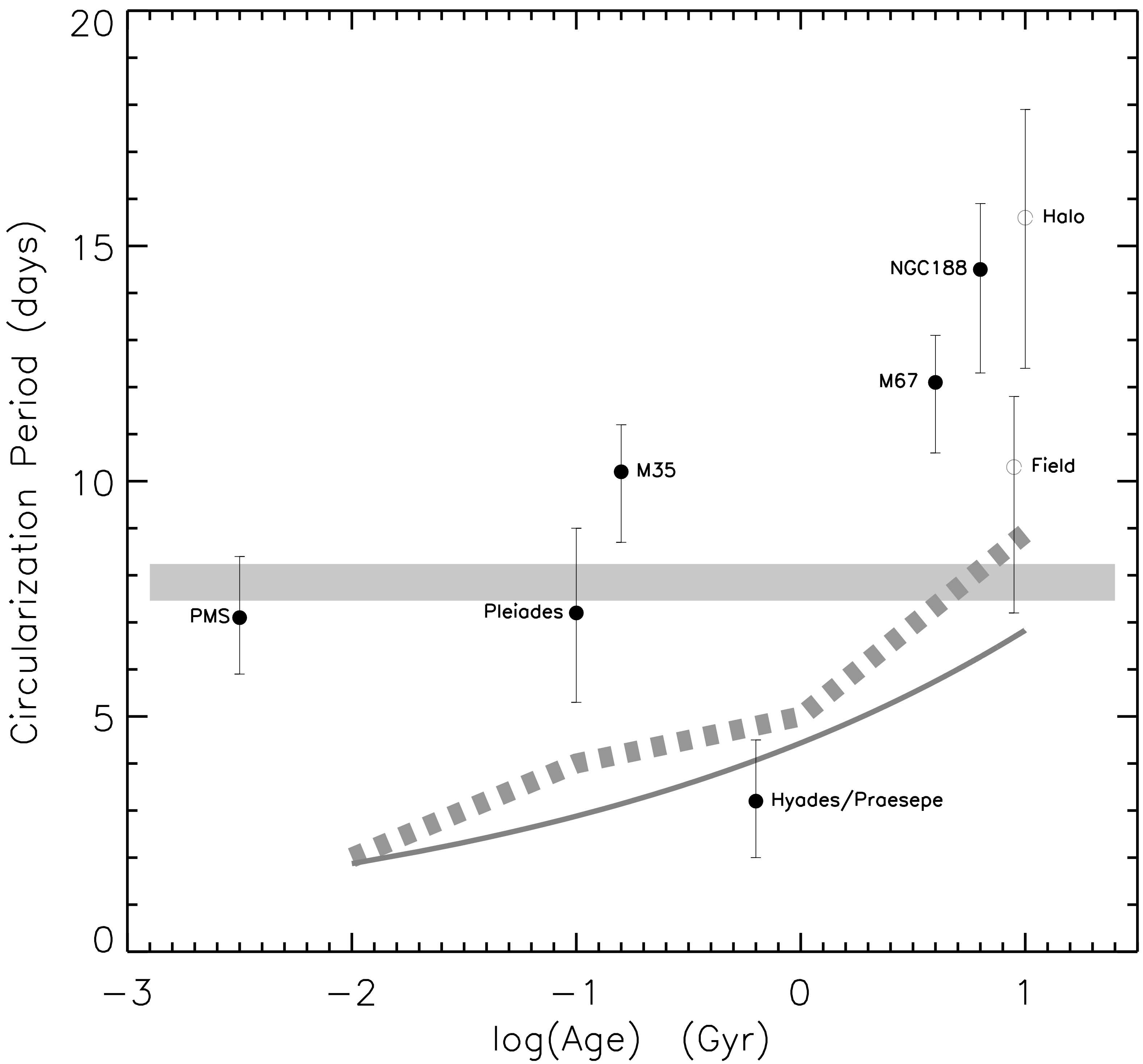
Solar-type binary stars

Meibom & Mathieu (2005)



Solar-type binary stars

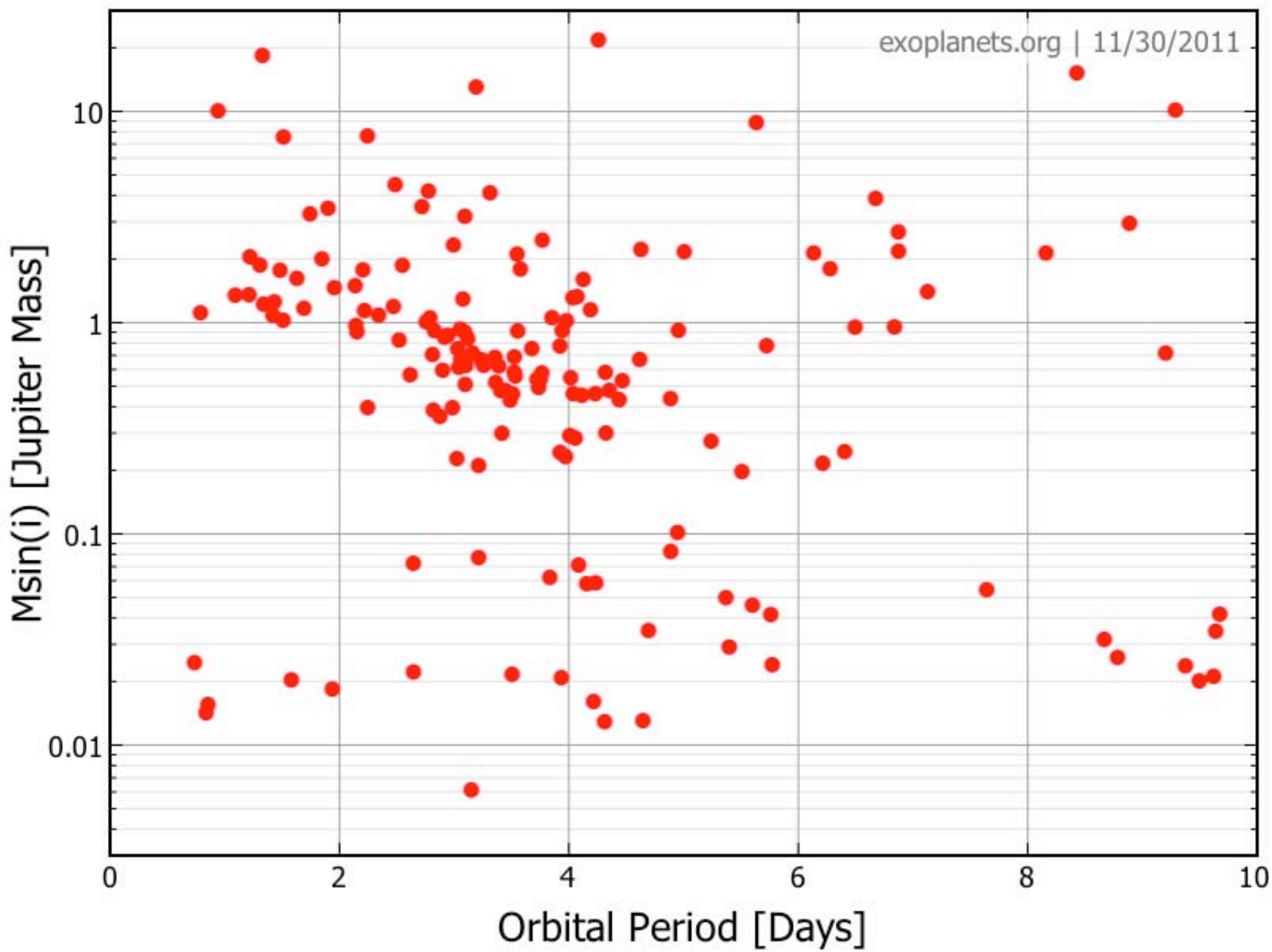
Meibom & Mathieu (2005)



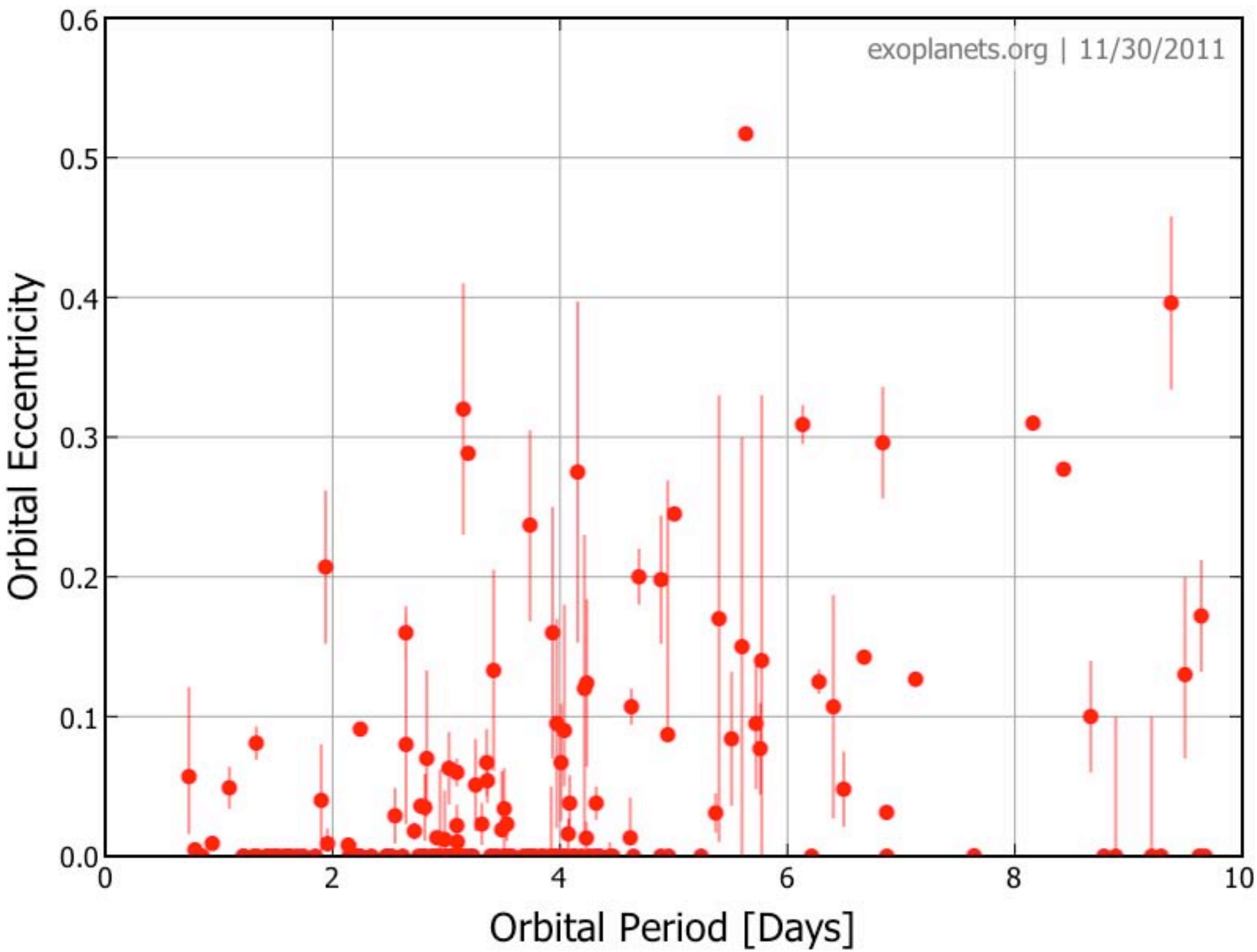
Hot Jupiters

http://bellerophonchimera.files.wordpress.com/
2008/07/solar-maximum-september-1-20011.gif

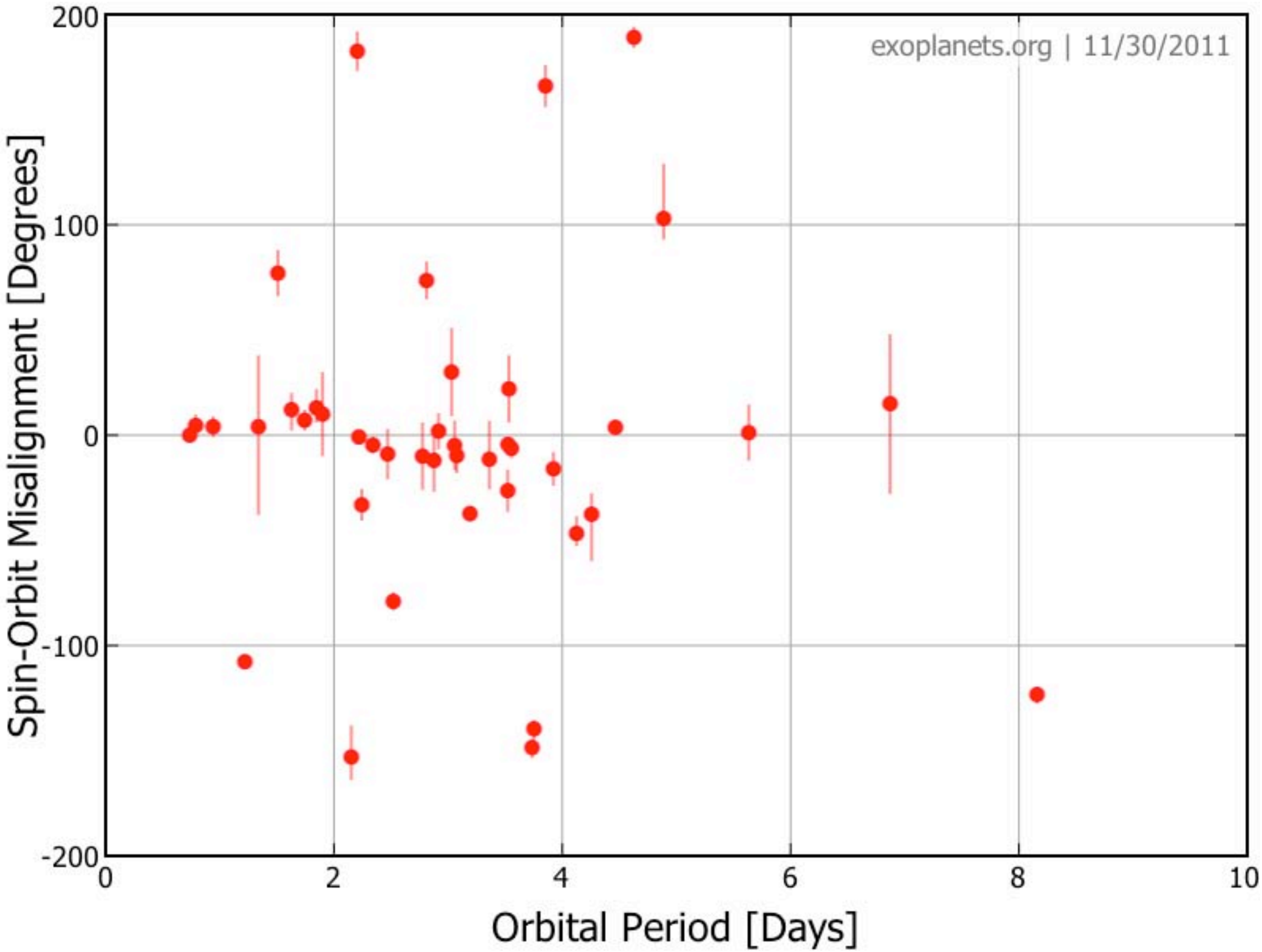
Short-period extrasolar planets



Short-period extrasolar planets



Short-period extrasolar planets



SOME KEY OBSERVATIONS

Very short orbital periods

- why do planets not spiral in?

Some eccentric orbits among these

- why are orbits not circularized?

(Mis)alignment of stellar spin and orbit

- clue to origin of hot Jupiters?

Radii of planets

- internal structure and heating

WASP-19 b

Hebb& 10

$P = 0.789$ day

$a = 0.0163$ AU

$e = 0.005 \pm 0.004$

$\lambda = 5 \pm 5°$

$M_P = 1.11 \pm 0.04\ M_J$

$R_P = 1.39 \pm 0.03\ R_J$

$= 0.56\ R_H$

$M_S = 0.93 \pm 0.02\ M_\odot$ (G8V)

$R_S = 0.99 \pm 0.02\ R_\odot$

$P_S = 10.5 \pm 0.2$ day

$t = 0.5\text{–}0.6$ Gyr ?

$t_e = 0.0003\ (Q'_P/10^6)$ Gyr

$t_a = 0.0076\ (Q'_S/10^6)$ Gyr

WASP-18 b

Hellier& 09

$P = 0.941$ day

$a = 0.020$ AU

$e = 0.009 \pm 0.003$

$\lambda = 4 \pm 5°$

$M_P = 10.1 \pm 0.3\ M_J$

$R_P = 1.11 \pm 0.06\ R_J$

$M_S = 1.22 \pm 0.03\ M_\odot$ (F6V)

$R_S = 1.22 \pm 0.06\ R_\odot$

$P_S \approx 6$ day ?

$t = 0.5-1.5$ Gyr

$t_e = 0.017\ (Q'_P/10^6)$ Gyr or $0.0019\ (Q'_S/10^6)$ Gyr

$t_a = 0.0010\ (Q'_S/10^6)$ Gyr

WASP-12 b

Hebb& 09

$P = 1.091$ day

$a = 0.023$ AU

$M_P = 1.35 \pm 0.05\ M_J$

$R_P = 1.79 \pm 0.09\ R_J$

$= 0.53\ R_H$

(See Li et al. 2010)

$e = 0.049 \pm 0.015$

$M_S = 1.28 \pm 0.05\ M_\odot$ (F6V)

$R_S = 1.63 \pm 0.08\ R_\odot$

$P_S > 20$ day ?

$t \approx 2$ Gyr

$t_e = 0.0004\ (Q'_P/10^6)$ Gyr

$t_a = 0.0050\ (Q'_S/10^6)$ Gyr

OGLE-TR-56 b

Konacki& 03, Torres& 08

$P = 1.212$ day

$e = 0$

$a = 0.024$ AU

$M_P = 1.39 \pm 0.18\ M_J$

$M_S = 1.23 \pm 0.07\ M_\odot$

$R_P = 1.36 \pm 0.09\ R_J$

$R_S = 1.36 \pm 0.09\ R_\odot$

$t = 1.9 - 4.2$ Gyr

$t_e = 0.002\ (Q'_P/10^6)$ Gyr

$t_a = 0.013\ (Q'_S/10^6)$ Gyr

HD 41004 B b

Zucker& 04

P = 1.328 day

e = 0.08 ± 0.01

a = 0.018 AU

$M_P \sin i = 18 \pm 1\ M_J$

$M_S = 0.4 \pm 0.04\ M_\odot$ (M2.5V)

$t_e \approx 0.1\ (Q'_P/10^6)$ Gyr or $0.1\ (Q'_S/10^6)$ Gyr

$t_a \approx 0.05\ (Q'_S/10^6)$ Gyr

GJ 436 b

Butler& 04, Torres& 08

P = 2.644 day

a = 0.029 AU

e = 0.16 ± 0.05

M_P = 0.073 ± 0.003 M_J

R_P = 0.377 ± 0.009 R_J

M_S = 0.45 ± 0.01 $M_\odot$ (G8V)

R_S = 0.46 ± 0.01 $R_\odot$

t = 1 – 10 Gyr

$t_e = 1.2\,(Q'_P/10^6)$ Gyr

$t_a = 110\,(Q'_S/10^6)$ Gyr

XO-3 b

Johns-Krull& 08, Winn& 09

$P = 3.192$ day

$a = 0.048$ AU

$M_P = 13.3 \pm 0.6\ M_J$

$R_P = 1.22\ R_J$

$e = 0.288 \pm 0.004$

$\lambda = 37.3 \pm 3.7°$

$M_S = 1.41 \pm 0.08\ M_\odot$ (F5V)

$R_S = 2.13 \pm 0.21\ R_\odot$

$P_S < 3.73 \pm 0.23$ day ?

$t_e = 0.025\ (Q'_S/10^6)$ Gyr

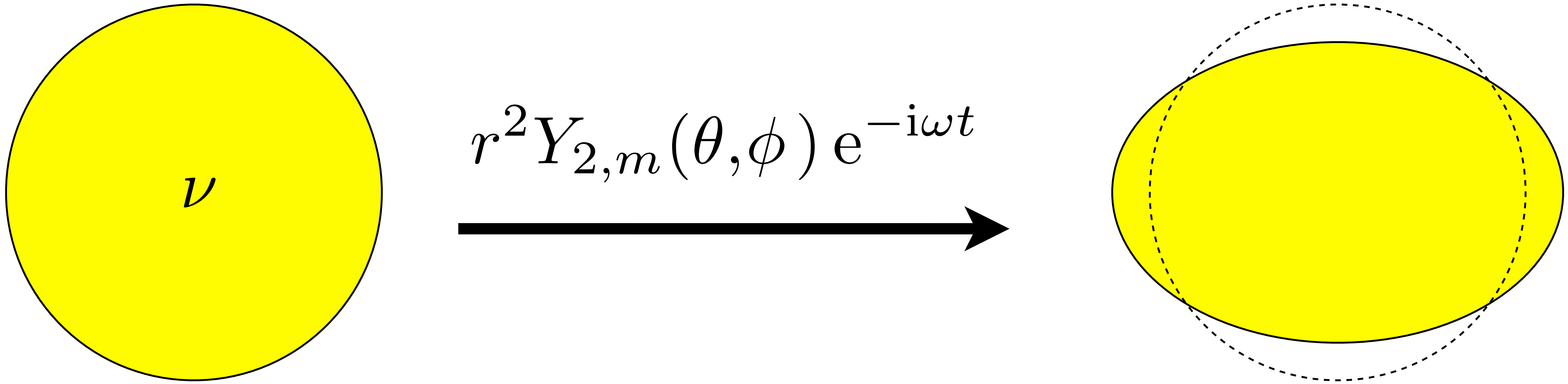
$t_a = 0.014\ (Q'_S/10^6)$ Gyr

Tidal dissipation in rotating stars and giant planets

J-P Zahn's categorization :

- "Equilibrium tide"

Dissipation associated with large-scale tidal bulge



- "Dynamical tide"

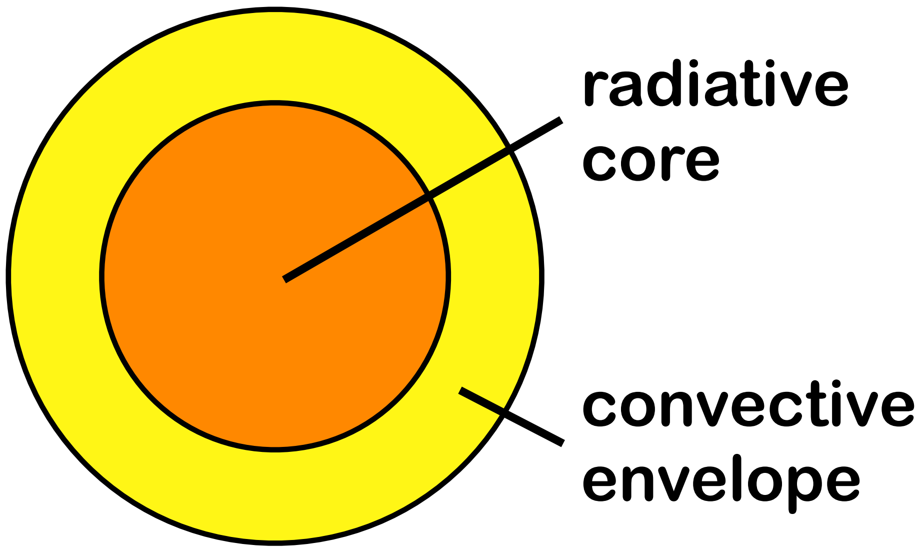
Dissipation associated with low-frequency waves

Tidal Q of solar-type stars and giant planets

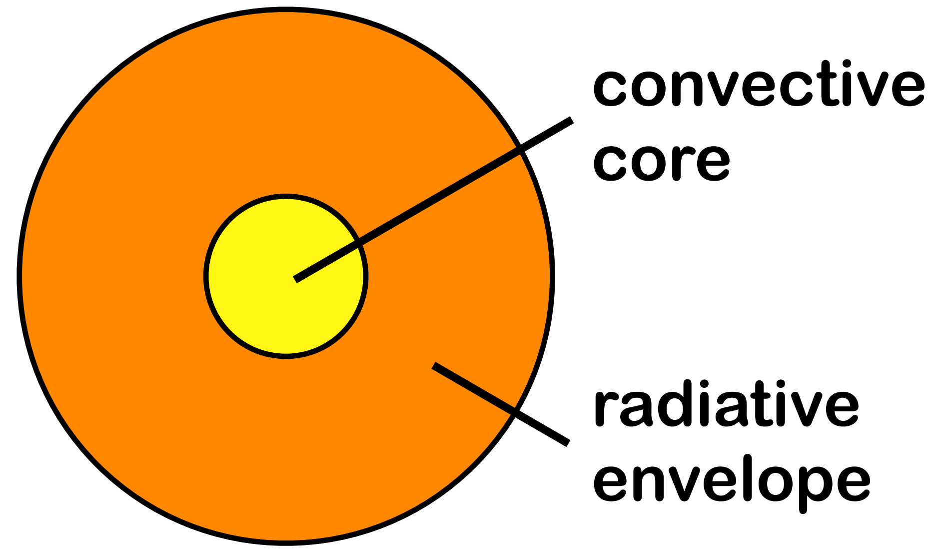
No simple answer!

- Q (or $k_{l,m}(\omega)$) is a response function, not a simple number
- Fluid dynamical calculations are still exploratory
- Planetary interior models are uncertain

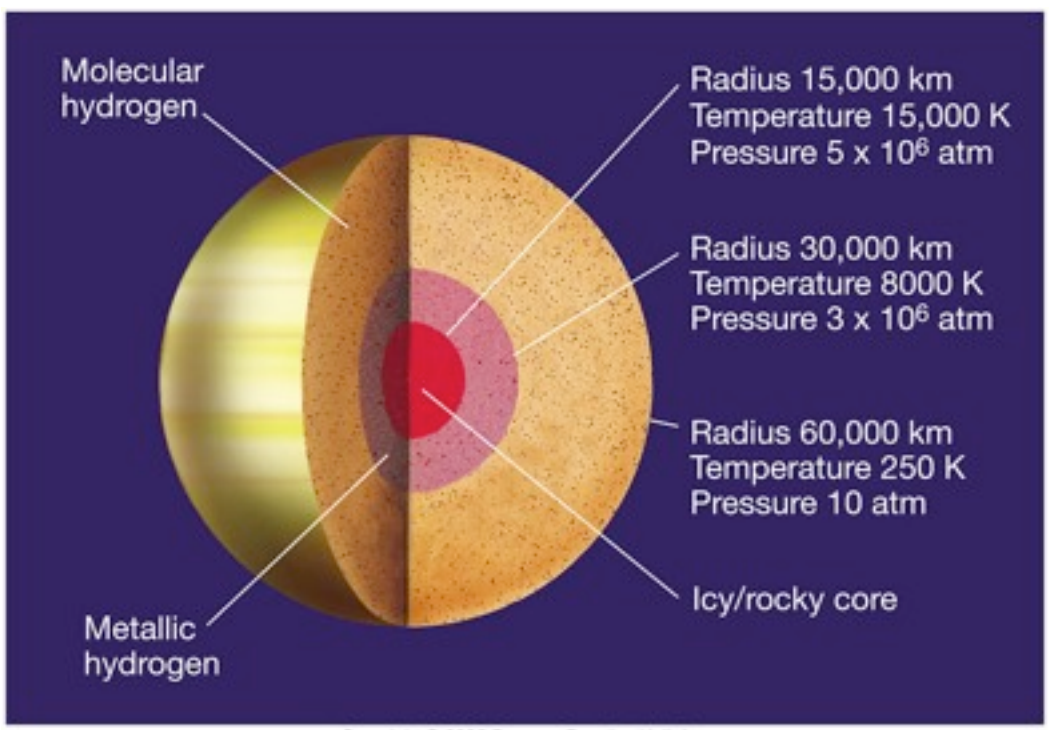
http://poorrichard.files.wordpress.com/2010/03/minefield1.jpg



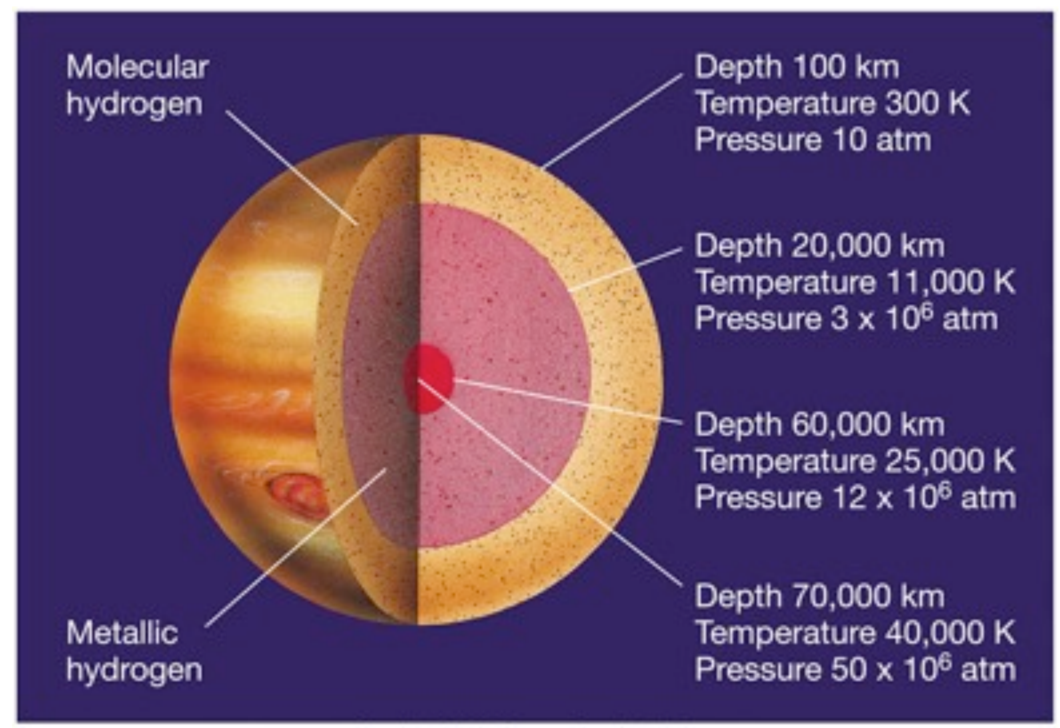
late-type star



early-type star



Saturn



Jupiter

Chaisson and McMillan, 2005

Tidal dissipation in rotating stars and giant planets

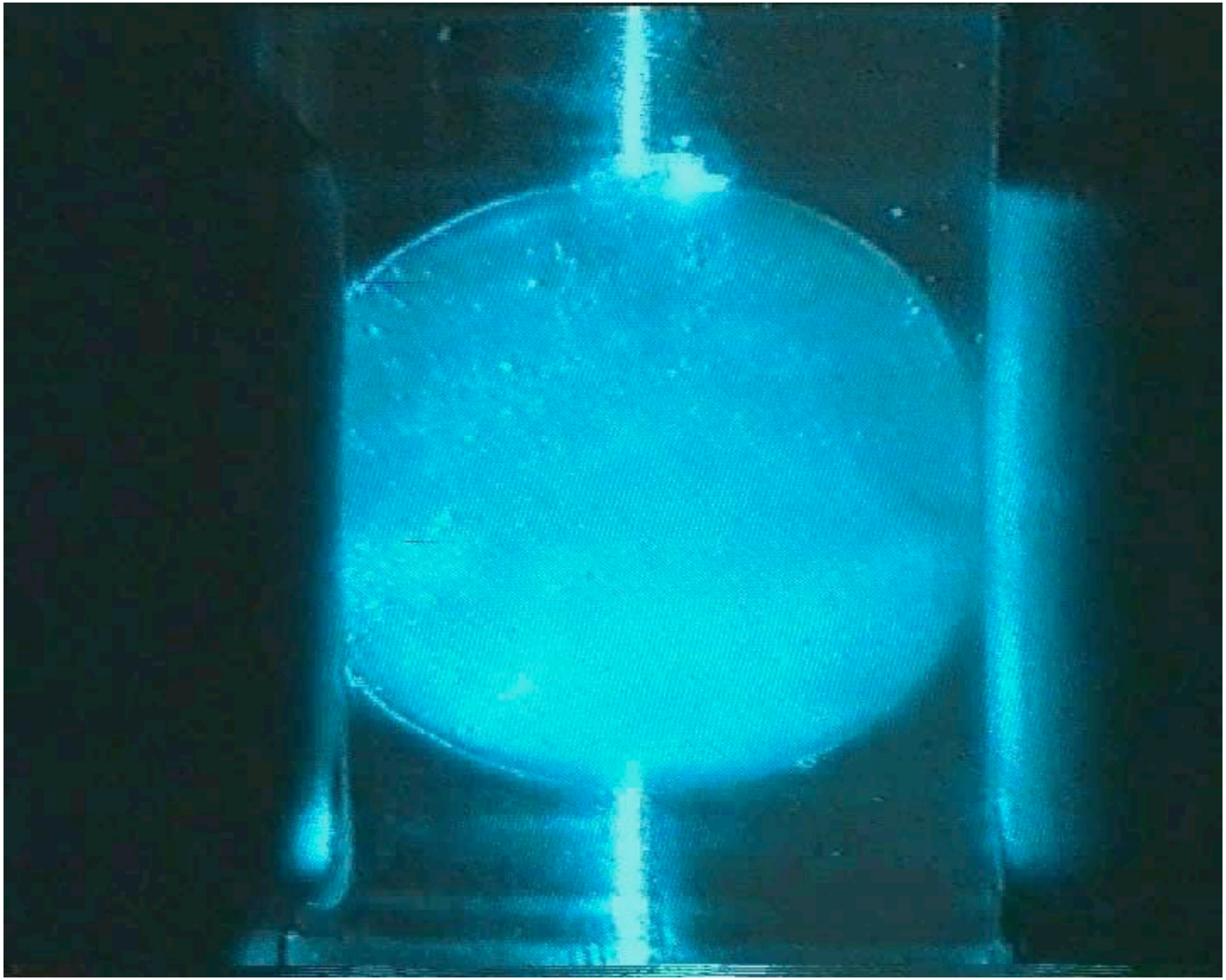
"Equilibrium tide"

- solid regions (viscoelastic, etc.)
- convective regions (turbulent "viscosity")
- other physics (phase transitions, helium separation)
- nonlinear breakdown (elliptical instability, etc.) ➜

"Dynamical tide"

- inertial waves in convective regions
- inertia-gravity waves in radiative regions

Elliptical instability in a deformed rotating sphere



Lacaze, Le Gal & Le Dizès (2004)

Tidal dissipation in rotating stars and giant planets

"Equilibrium tide"

- solid regions (viscoelastic, etc.)
- convective regions (turbulent "viscosity")
- other physics (phase transitions, helium separation)
- nonlinear breakdown (elliptical instability, etc.)

"Dynamical tide"

- inertial waves in convective regions
- inertia-gravity waves in radiative regions

Low-frequency waves in rotating stars and giant planets

$$\omega^2 = 4\Omega^2(\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{\Omega}})^2 + N^2(\hat{\boldsymbol{k}} \times \hat{\boldsymbol{g}})^2$$

- Convective regions: inertial waves

  (Savonije et al.; Ogilvie & Lin; Wu; Ivanov & Papaloizou; Goodman & Lackner; Rieutord & Valdettaro)

- Radiative regions: inertia-gravity waves

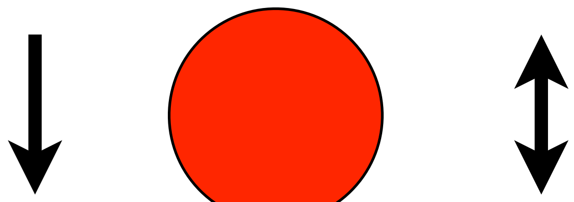
  (Zahn; Savonije & Papaloizou; Goldreich & Nicholson; Savonije & Witte)

Excitation, propagation, reflection, dissipation

Eigenvalue problem for normal modes is generally non-separable and ill-posed, so modes may not exist without diffusion

Internal gravity waves

Buoyancy (Brunt-Väisälä) frequency



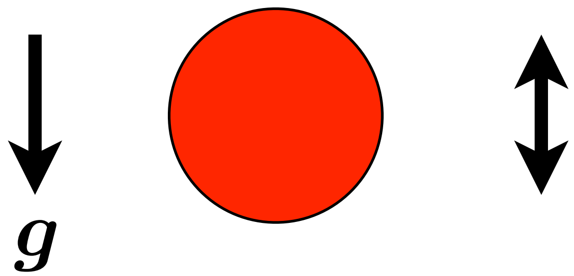
$\boldsymbol{g}$

$$N^2 = \frac{g}{\rho}\left(\left.\frac{\mathrm{d}\rho}{\mathrm{d}z}\right|_{\mathrm{ad}} - \frac{\mathrm{d}\rho}{\mathrm{d}z}\right)$$

$$= g\left(\frac{1}{\Gamma_1}\frac{\mathrm{d}\ln p}{\mathrm{d}z} - \frac{\mathrm{d}\ln\rho}{\mathrm{d}z}\right)$$

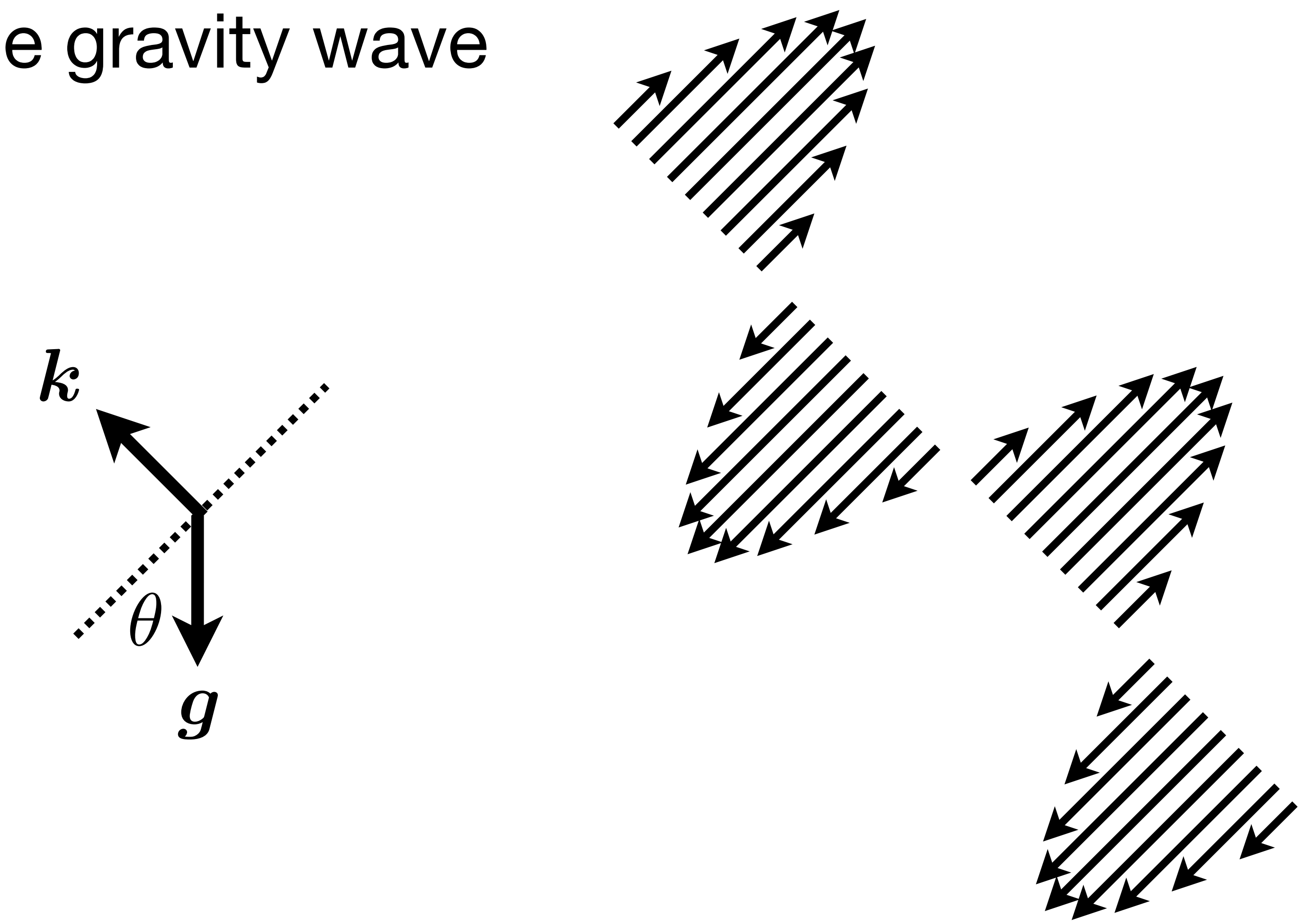
Internal gravity waves

Buoyancy (Brunt-Väisälä) frequency



$$
\begin{aligned}
N^2 &= \frac{g}{\rho}\left(\left.\frac{\mathrm{d}\rho}{\mathrm{d}z}\right|_{\mathrm{ad}} - \frac{\mathrm{d}\rho}{\mathrm{d}z}\right) \\
&= g\left(\frac{1}{\Gamma_1}\frac{\mathrm{d}\ln p}{\mathrm{d}z} - \frac{\mathrm{d}\ln\rho}{\mathrm{d}z}\right)
\end{aligned}
$$

Plane gravity wave



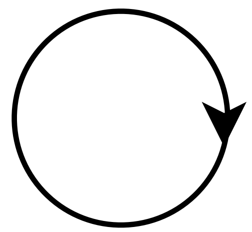
$$\omega = N\cos\theta$$

$$\omega^2 = N^2|\hat{\boldsymbol{k}} \times \hat{\boldsymbol{g}}|^2$$

Inertial waves

Horizontal oscillation

$\odot$ $\boldsymbol{\Omega}$

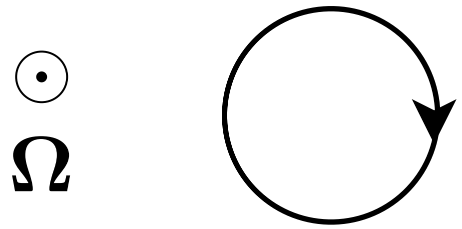


$$\left.\begin{aligned}\dot{u}_x &= 2\Omega\, u_y \\ \dot{u}_y &= -2\Omega\, u_x\end{aligned}\right\} \quad \omega = 2\Omega$$

(centrifugal force balanced by pressure)

Inertial waves

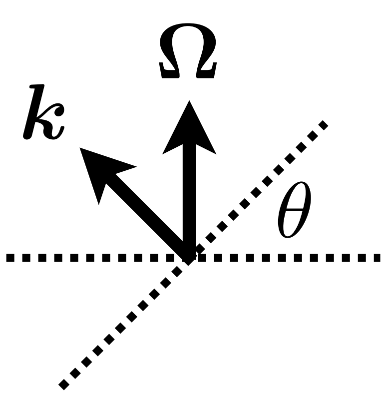
Horizontal oscillation

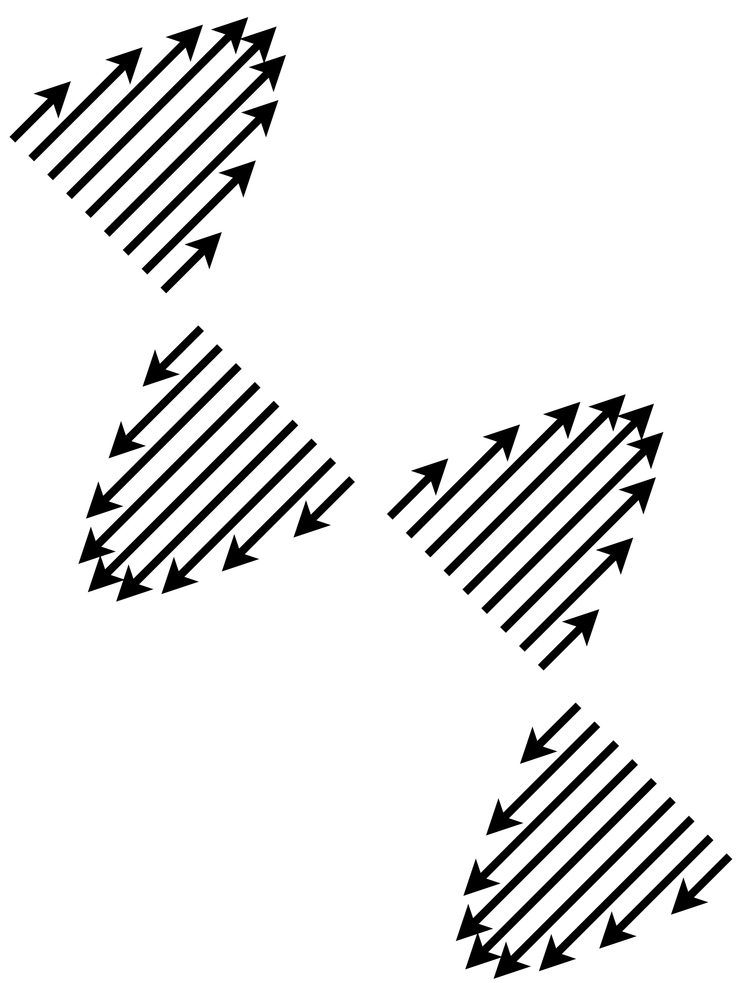


$$\left.\begin{aligned} \dot{u}_x &= 2\Omega\, u_y \\ \dot{u}_y &= -2\Omega\, u_x \end{aligned}\right\} \quad \omega = 2\Omega$$

(centrifugal force balanced by pressure)

Plane inertial wave



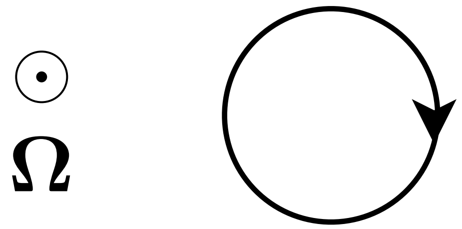


$$\omega = 2\Omega \cos\theta$$

$$\omega^2 = 4\Omega^2 (\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{\Omega}})^2$$

Inertial waves

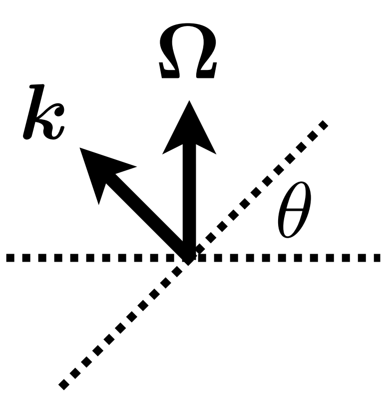
Horizontal oscillation

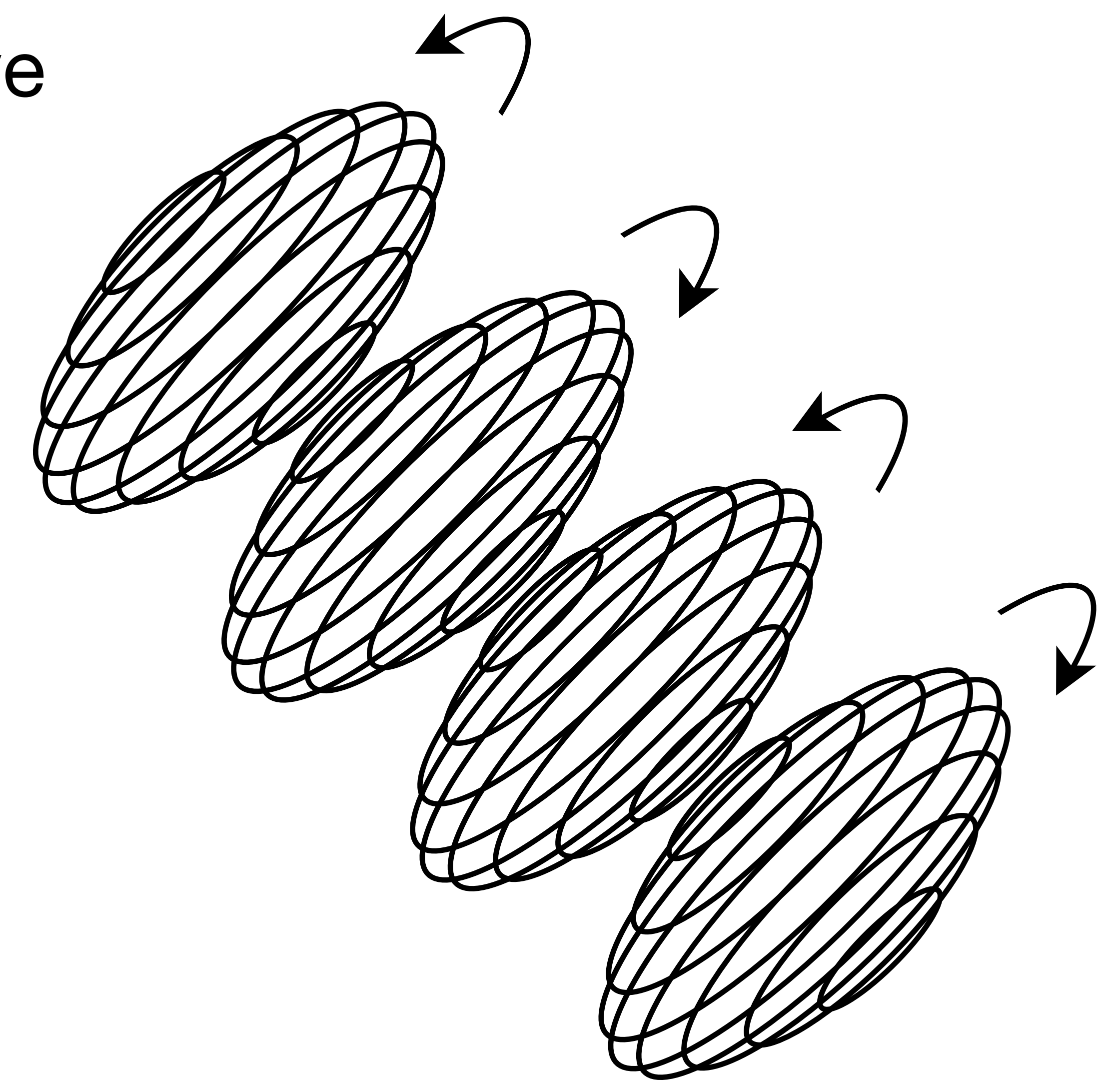


$$\left.\begin{aligned} \dot{u}_x &= 2\Omega\, u_y \\ \dot{u}_y &= -2\Omega\, u_x \end{aligned}\right\} \quad \omega = 2\Omega$$

(centrifugal force balanced by pressure)

Plane inertial wave



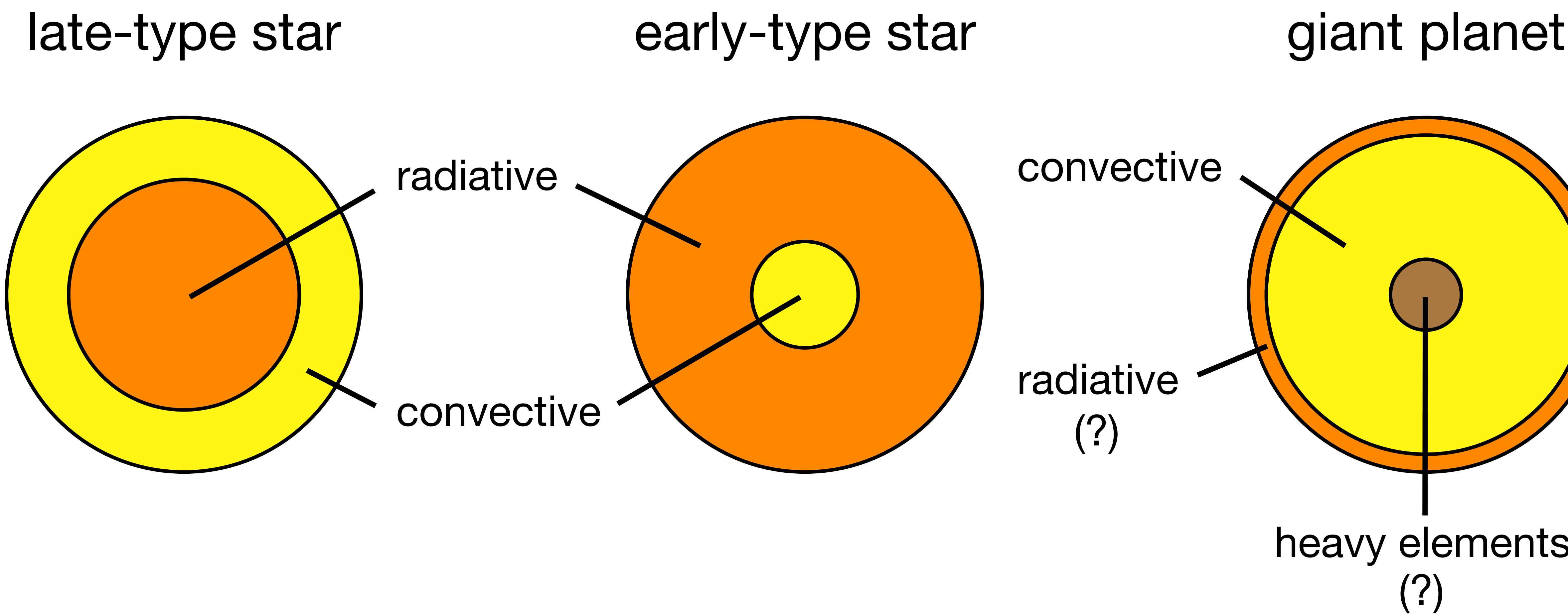


$$\omega = 2\Omega \cos\theta$$

$$\omega^2 = 4\Omega^2 (\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{\Omega}})^2$$

Linear tides in uniformly rotating unstratified fluids

Tides in convective regions of planets and stars



- Turbulent viscosity acting on tidal bulge?
- Excitation and dissipation of inertial waves?

Linear tides in barotropic fluid bodies

- Barotropic : no stable stratification or internal gravity waves
- Low-frequency tides in slowly rotating bodies :

$$\omega \sim \Omega \sim \epsilon \left(\frac{GM}{R^3} \right)^{1/2}, \quad \epsilon \ll 1$$

- Systematic theory based on expansion in powers of ϵ^2
- Displacement $\boldsymbol{\xi} = \boldsymbol{\xi}_{\mathrm{nw}} + \boldsymbol{\xi}_{\mathrm{w}}$
- Non-wavelike part :

  response of spherical body to tidal potential neglecting Coriolis

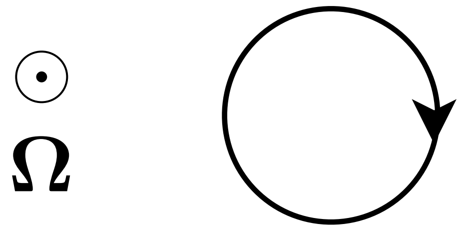
  (easily computed but different from classical equilibrium tide)
- Wavelike part :

  residual response (inertial waves)

  known body force from Coriolis force on non-wavelike part

Inertial waves

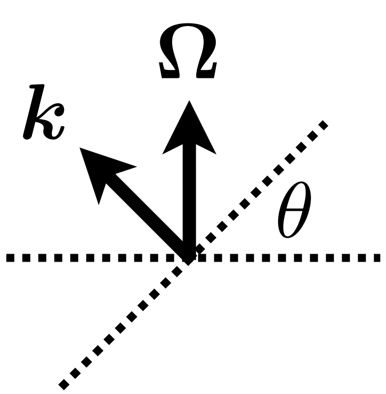
Horizontal oscillation

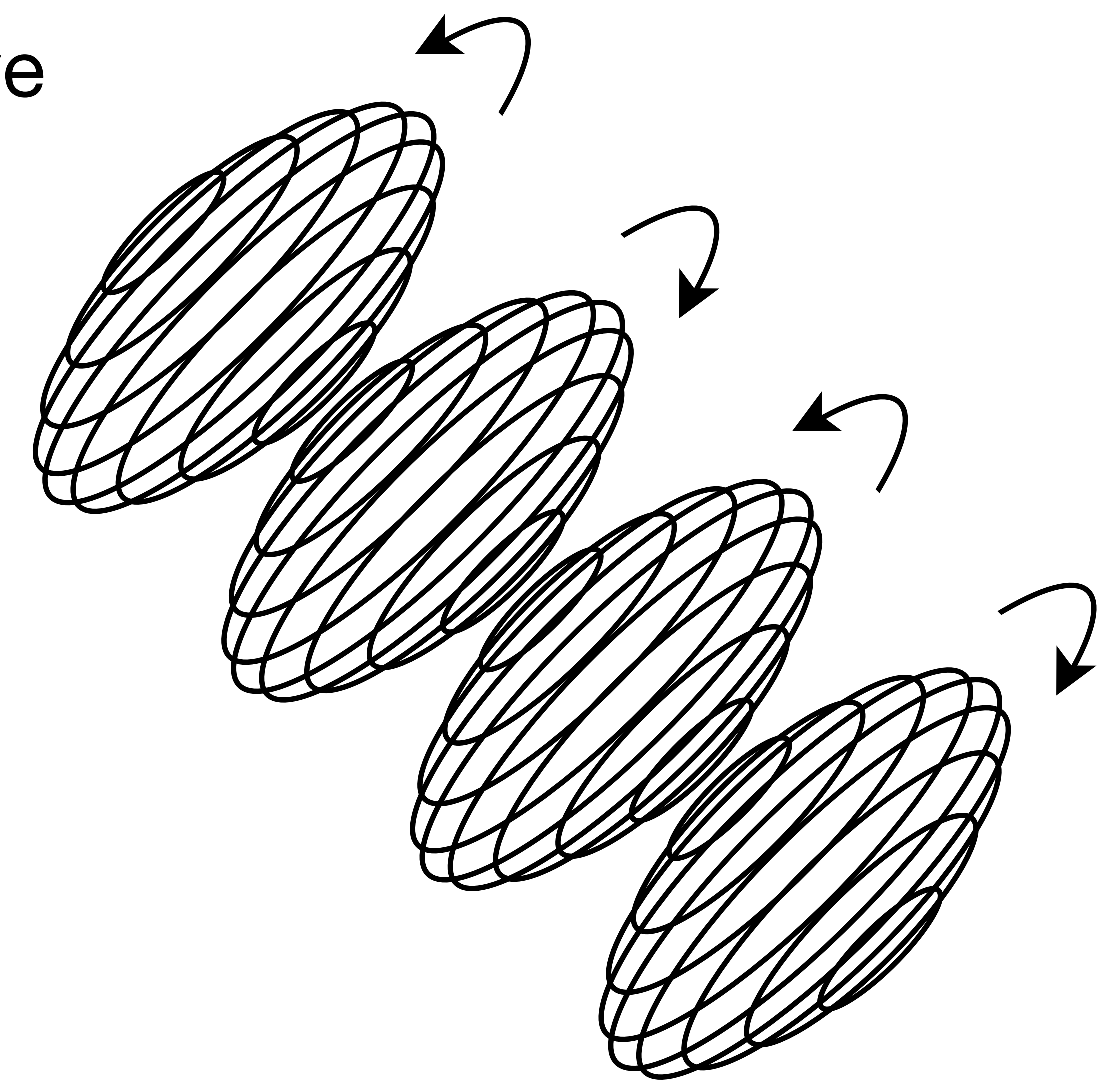


$$\left.\begin{aligned} \dot{u}_x &= 2\Omega\, u_y \\ \dot{u}_y &= -2\Omega\, u_x \end{aligned}\right\} \quad \omega = 2\Omega$$

(centrifugal force balanced by pressure)

Plane inertial wave



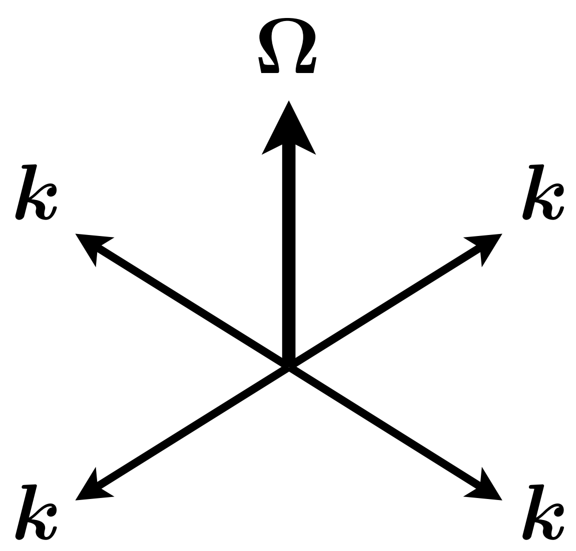


$$\omega = 2\Omega\cos\theta$$

$$\omega^2 = 4\Omega^2(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{\Omega}})^2$$

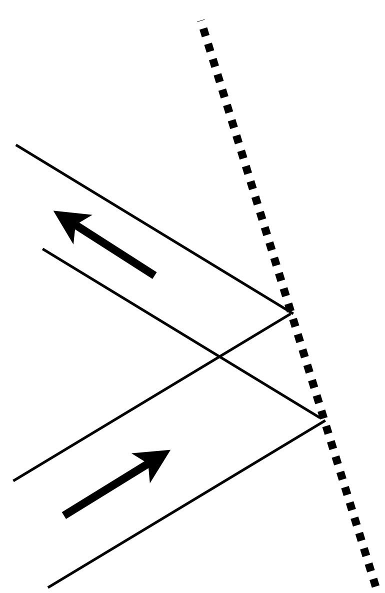
Peculiarities of low-frequency waves

Wave frequency depends only on *direction* of wavevector



Group velocity *perpendicular* to wavevector

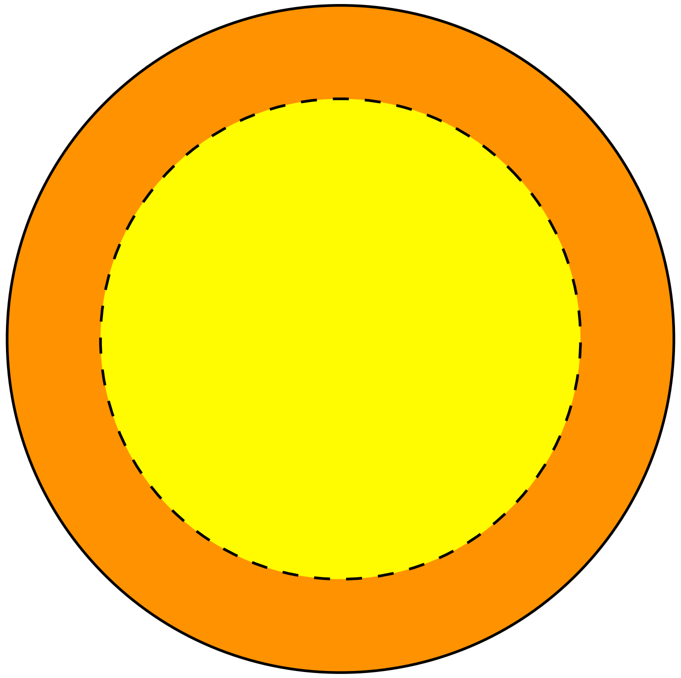
Focusing of beams at sloping boundaries



Reflection from interfaces...

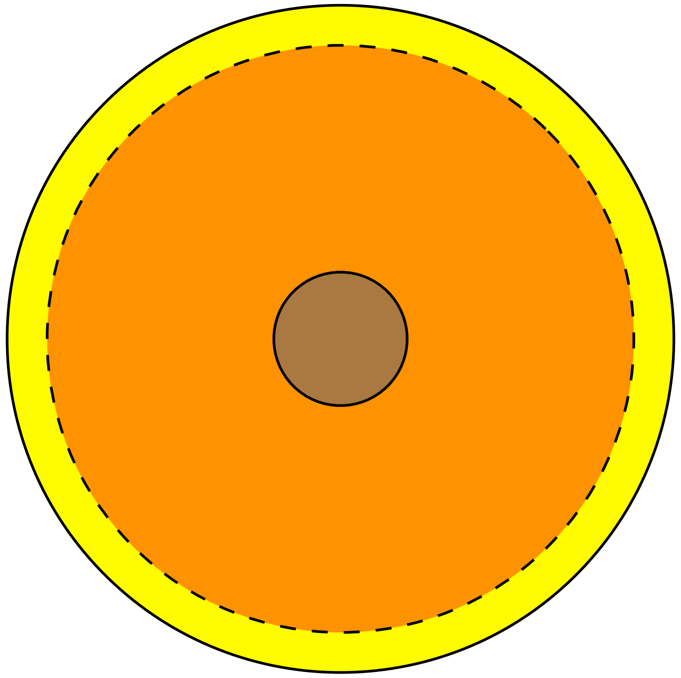
Inertial waves in convective regions

Solar-type star



[Savonije & Witte 2002]
Ogilvie & Lin 2007

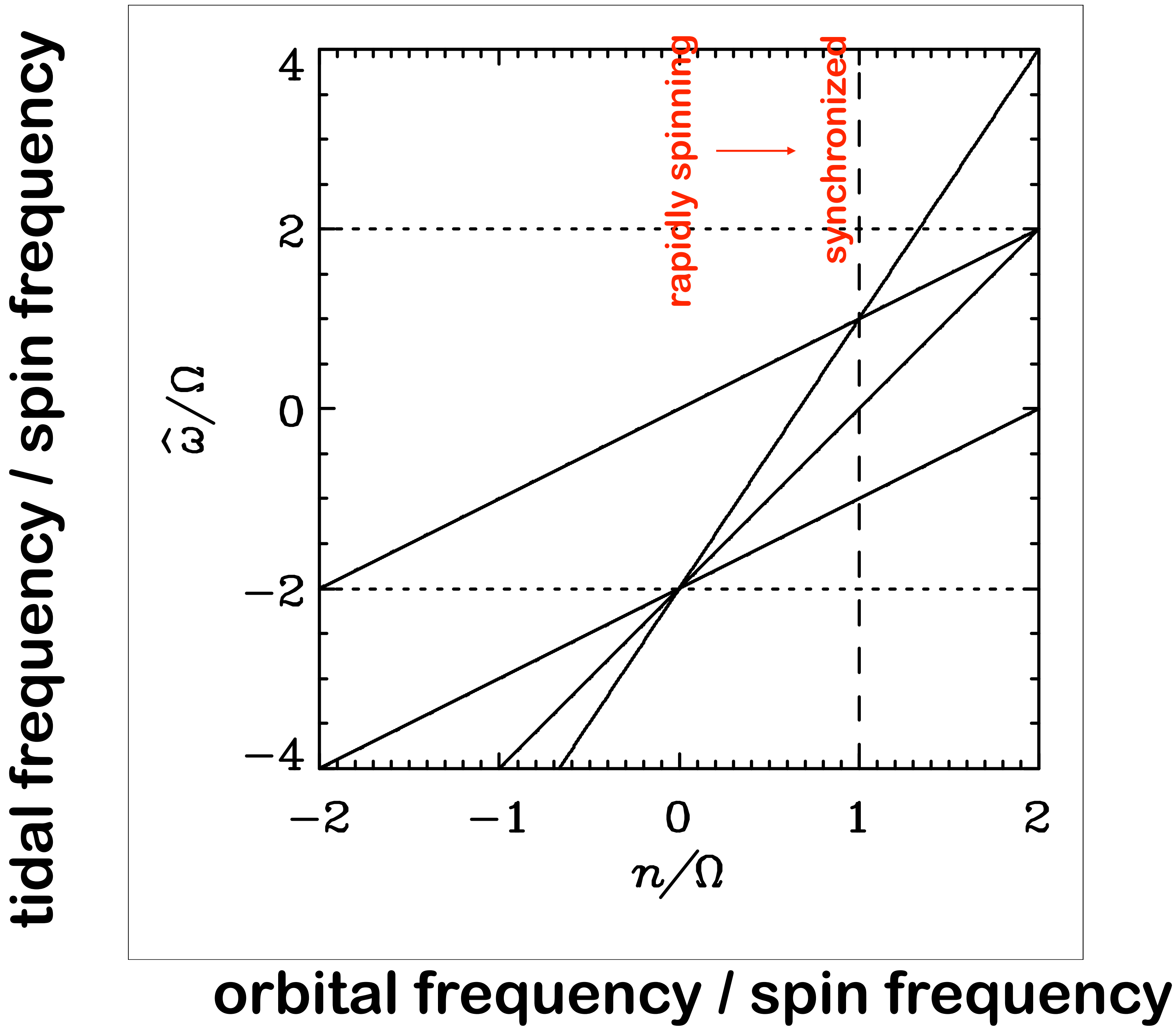
[Irradiated] giant planet



Ogilvie & Lin 2004
Wu 2005
Papaloizou & Ivanov 2005
Ivanov & Papaloizou 2007
Goodman & Lackner 2009
Ogilvie 2009
Rieutord & Valdettaro 2010

Inertial wave frequency range

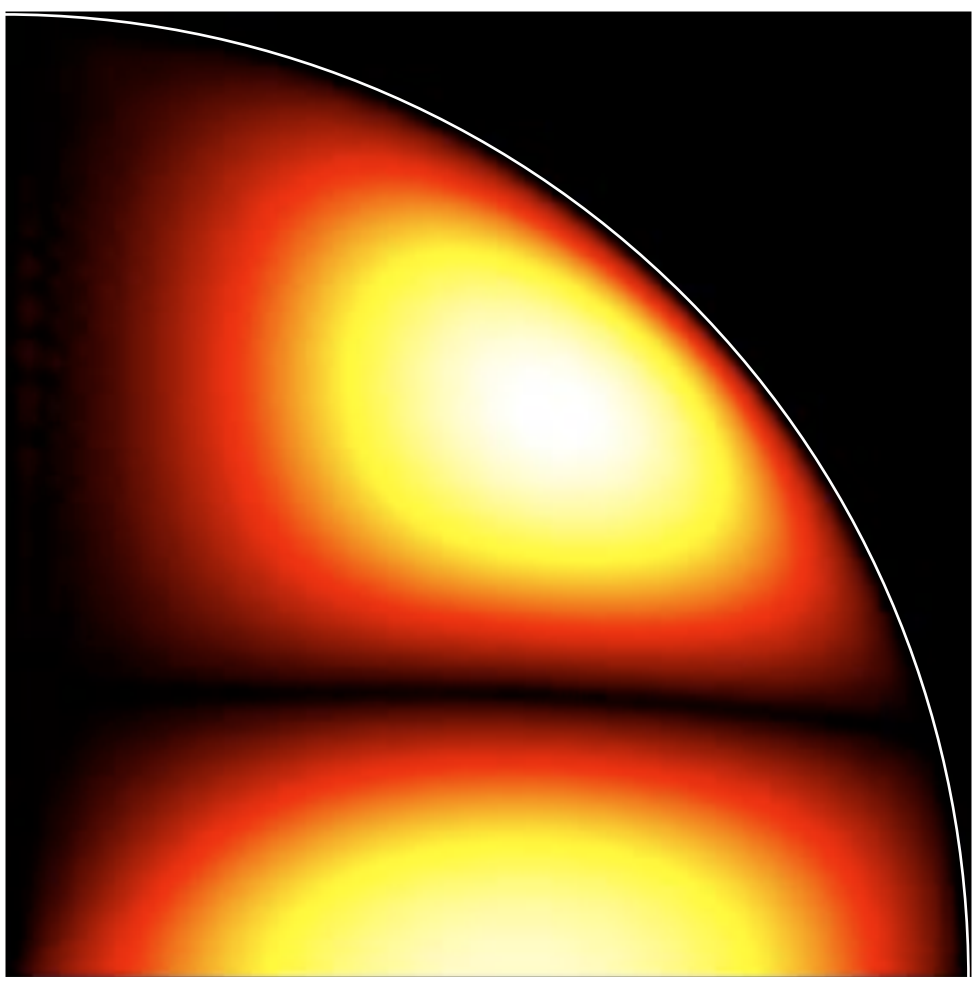
For a uniformly rotating body, $-2\Omega < \hat{\omega} < 2\Omega$

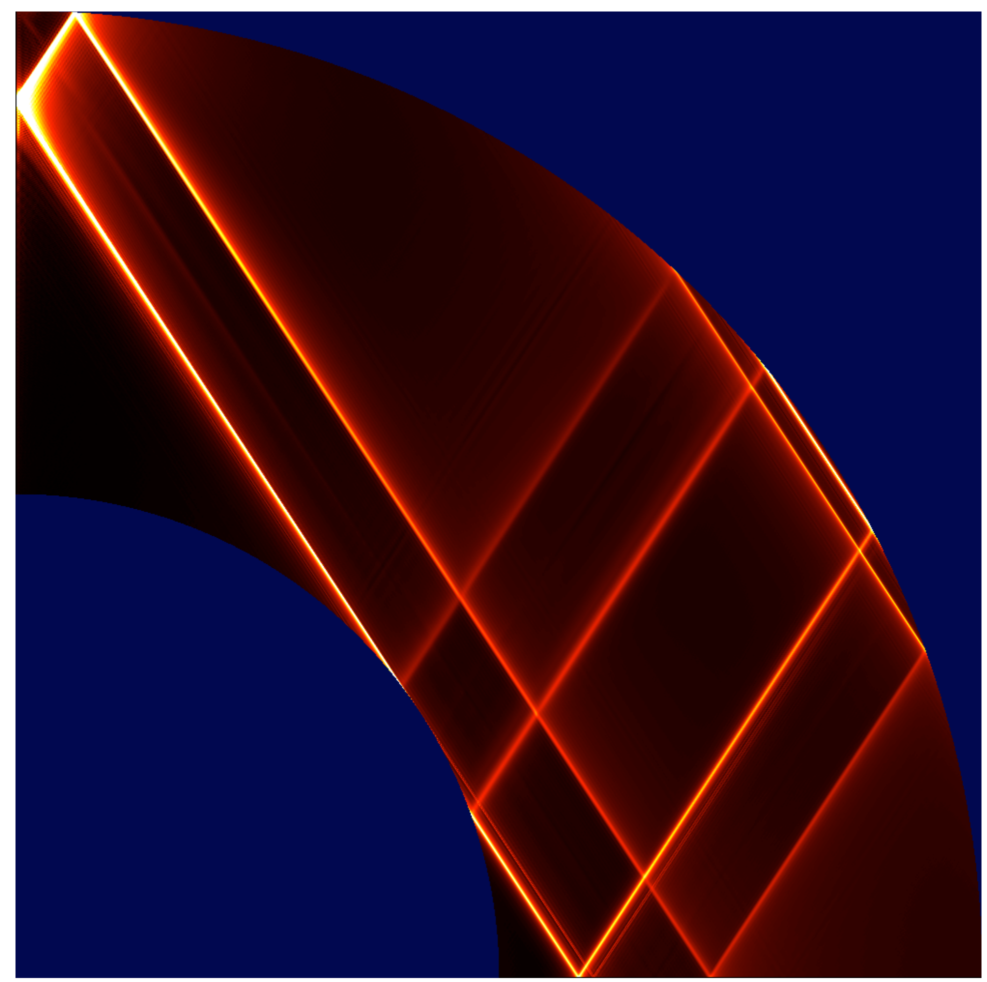


Inertial waves : modes or beams?

Dense or continuous spectrum, $-2\Omega < \hat{\omega} < 2\Omega$

- Tidal forcing excites normal modes (Wu; Papaloizou & Ivanov)
- Tidal forcing excites narrow beams (Ogilvie & Lin; Goodman & Lackner; Rieutord & Valdettaro)

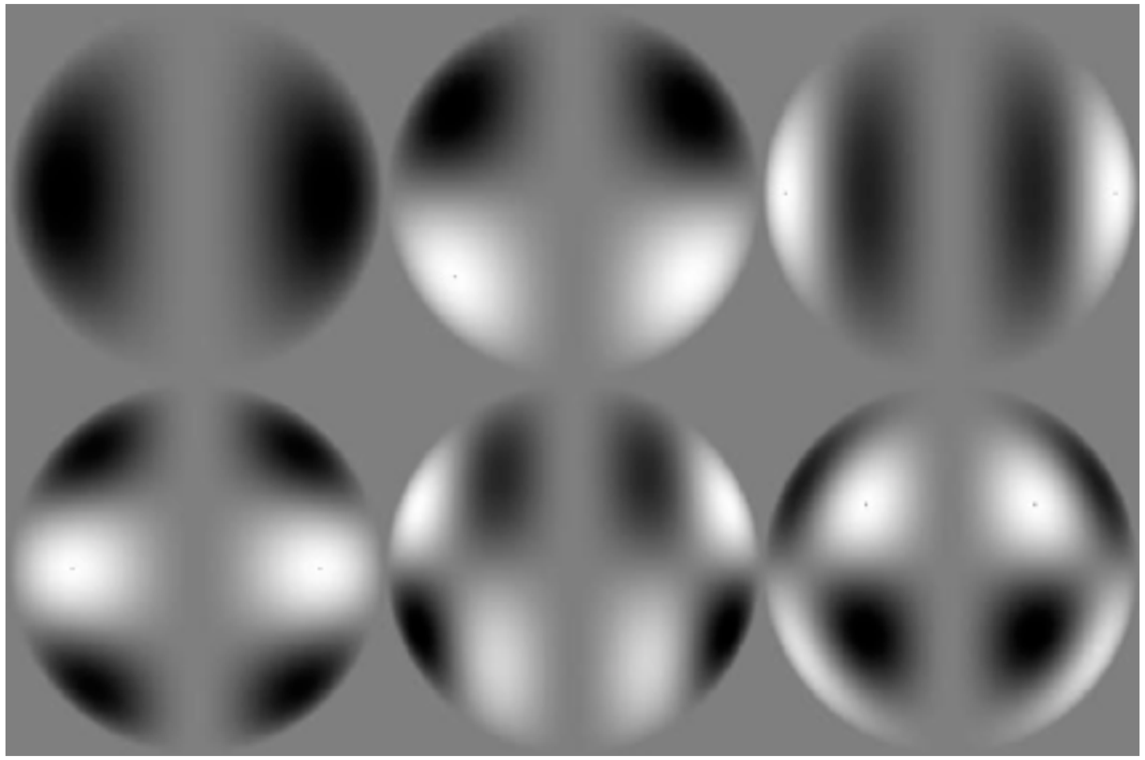




MODES / RESONANCE ?

Full sphere (Bryan 1889) :

- two-index set of smooth modes for each m
- discrete spectrum, dense in $(-2\Omega, 2\Omega)$

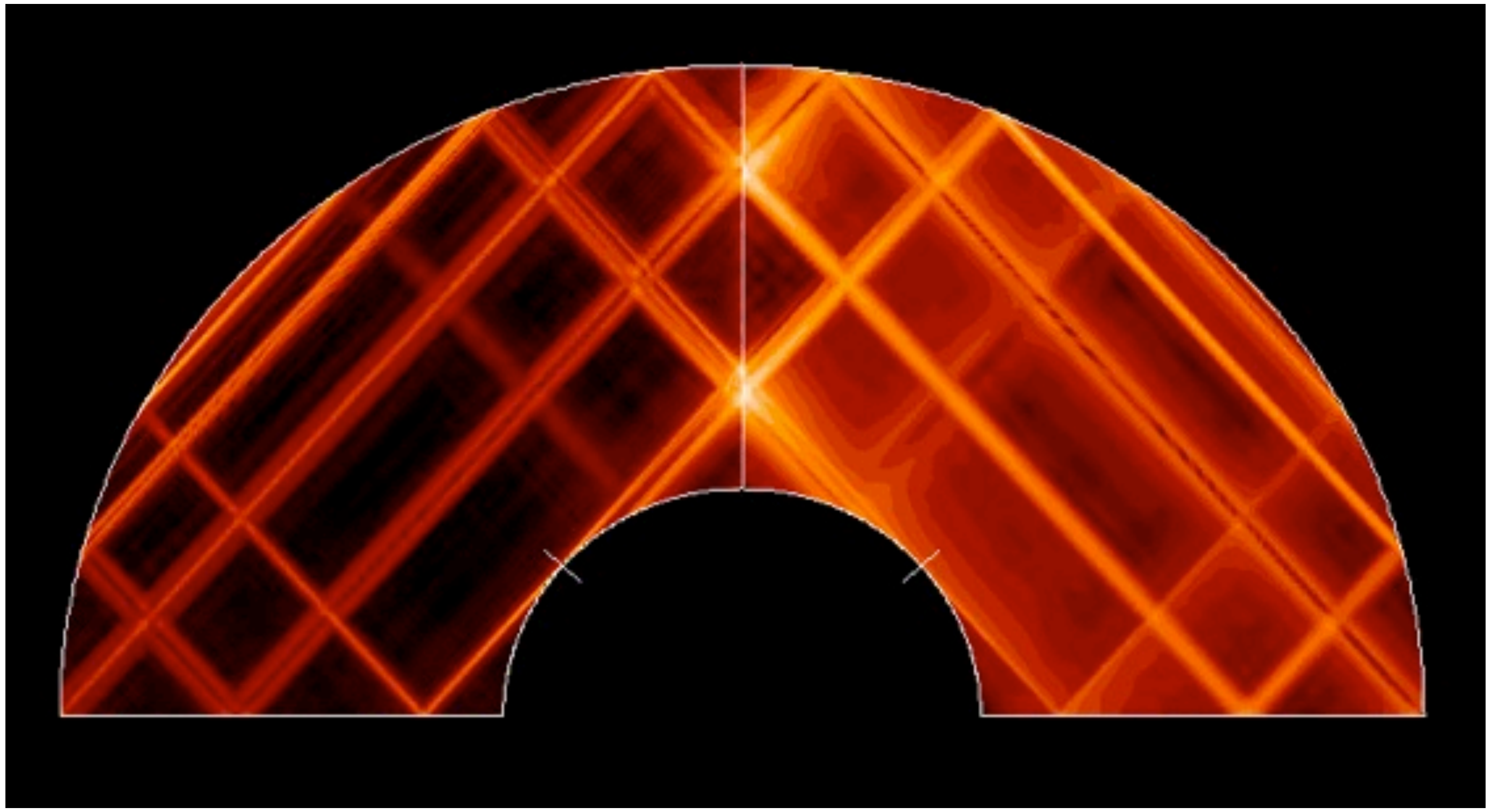


- no resonant excitation by Y_2^2 (homogeneous body)

INERTIAL WAVES IN A SHELL

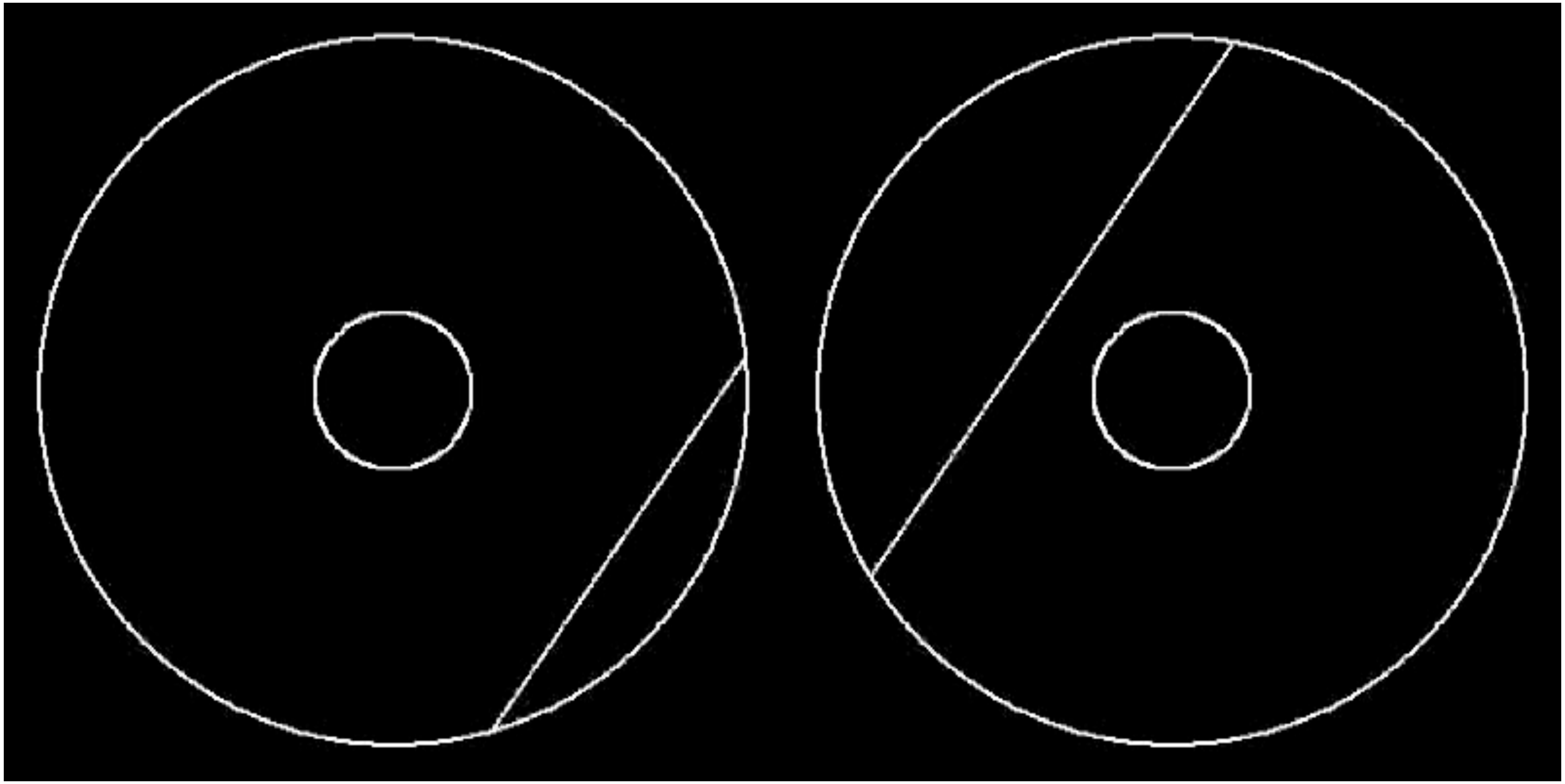
Spherical shell (Bretherton 1964, Stewartson 1972)

- no separation of variables, no smooth modes
- numerical calculations including viscosity
  (Rieutord et al. 2001)

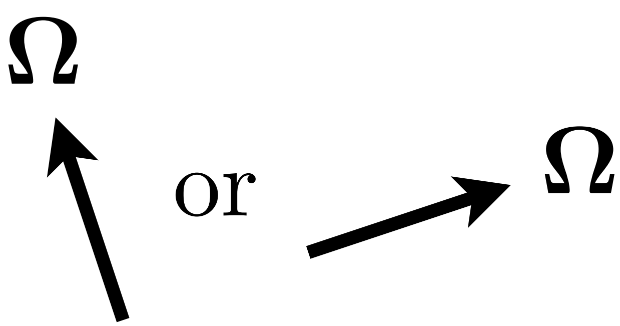


RAY PROPAGATION

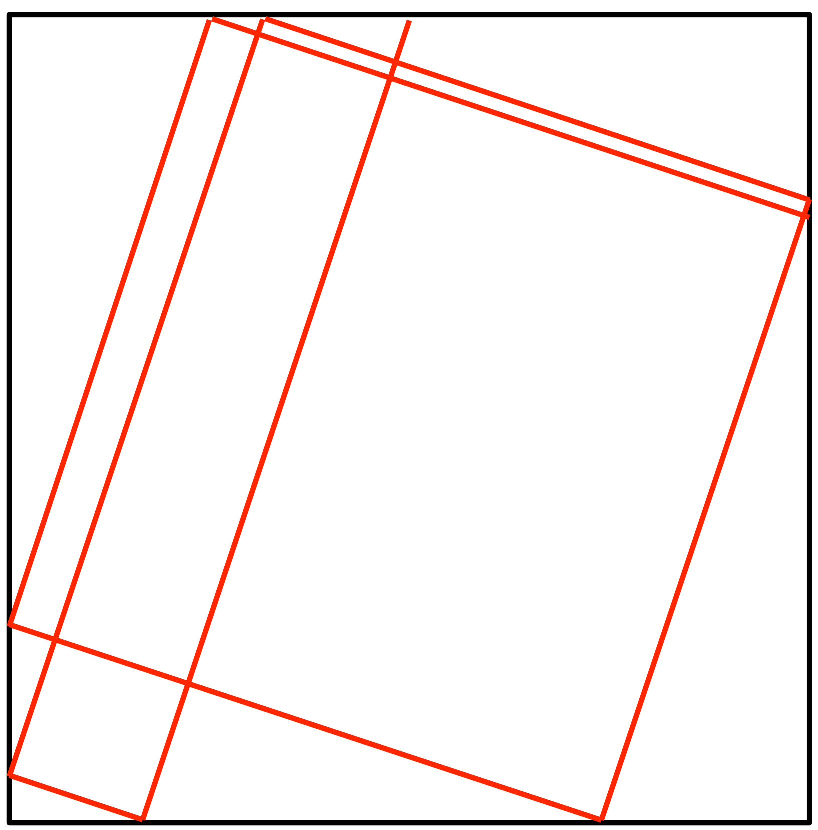
convergence to a wave attractor

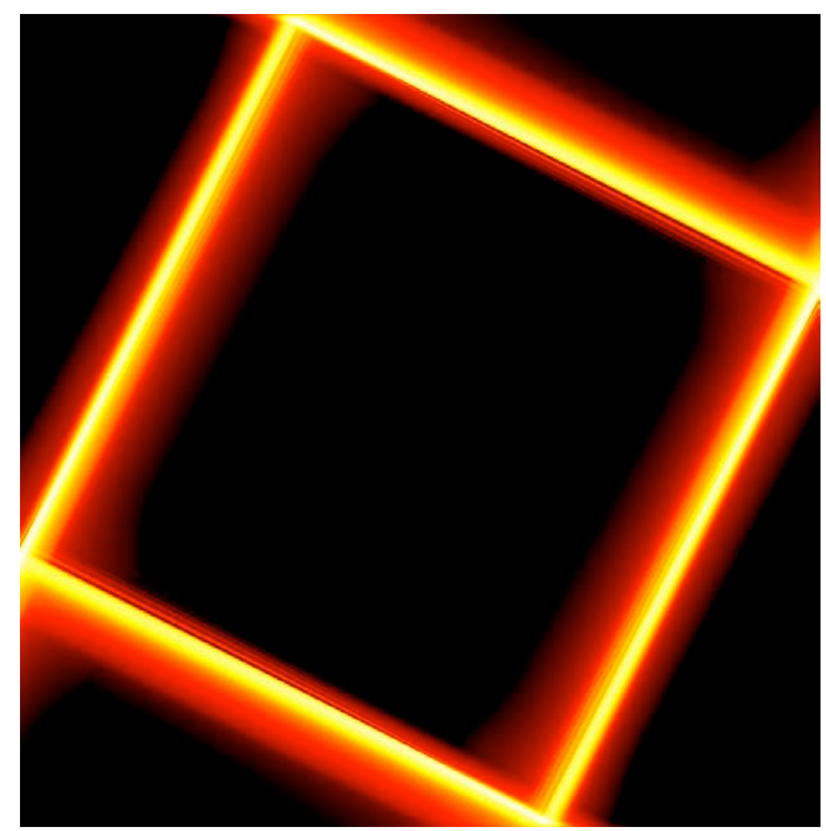


Attractive simplicity: inertial waves in a box

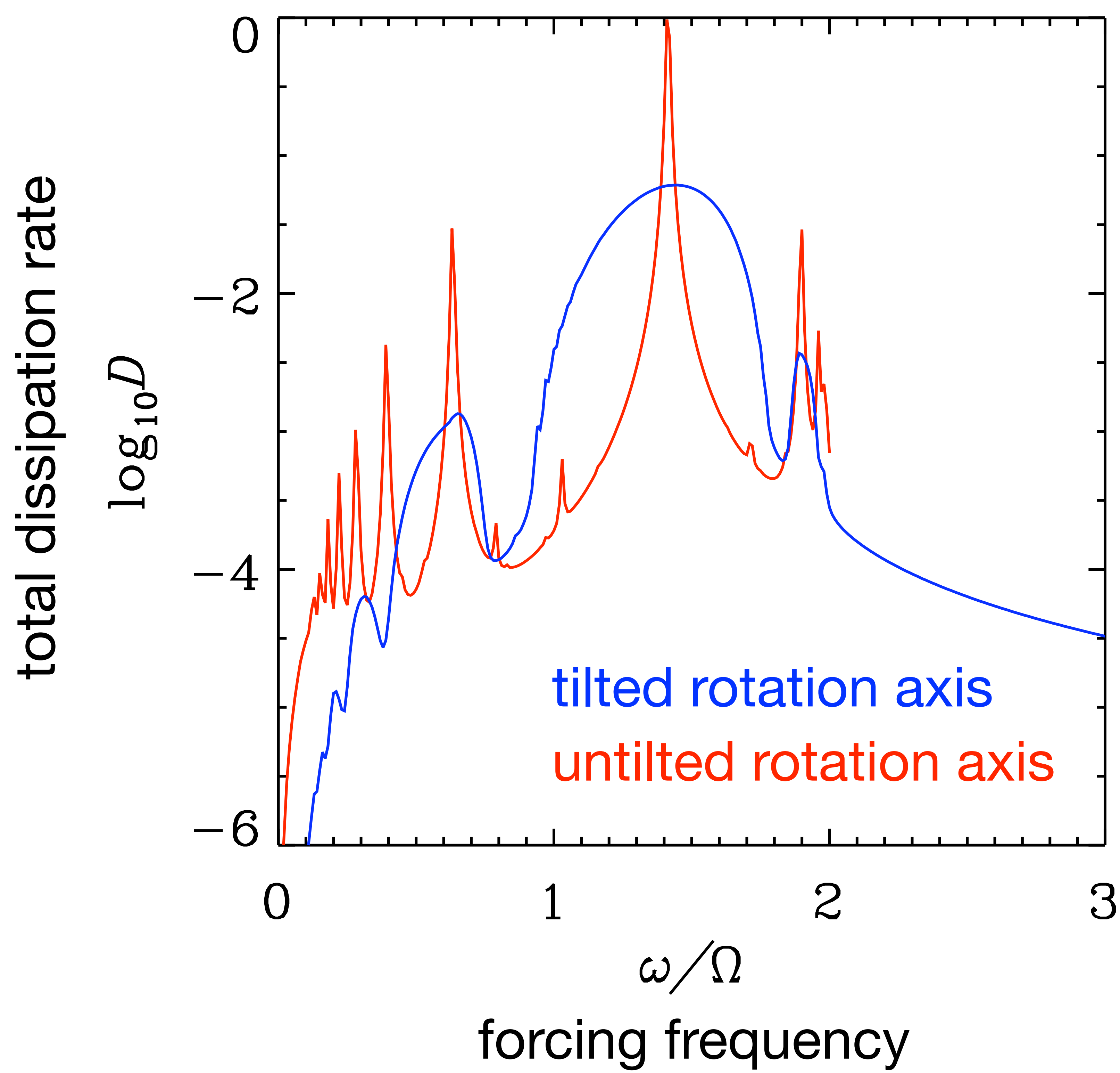


$$\omega = \Omega\sqrt{2}$$





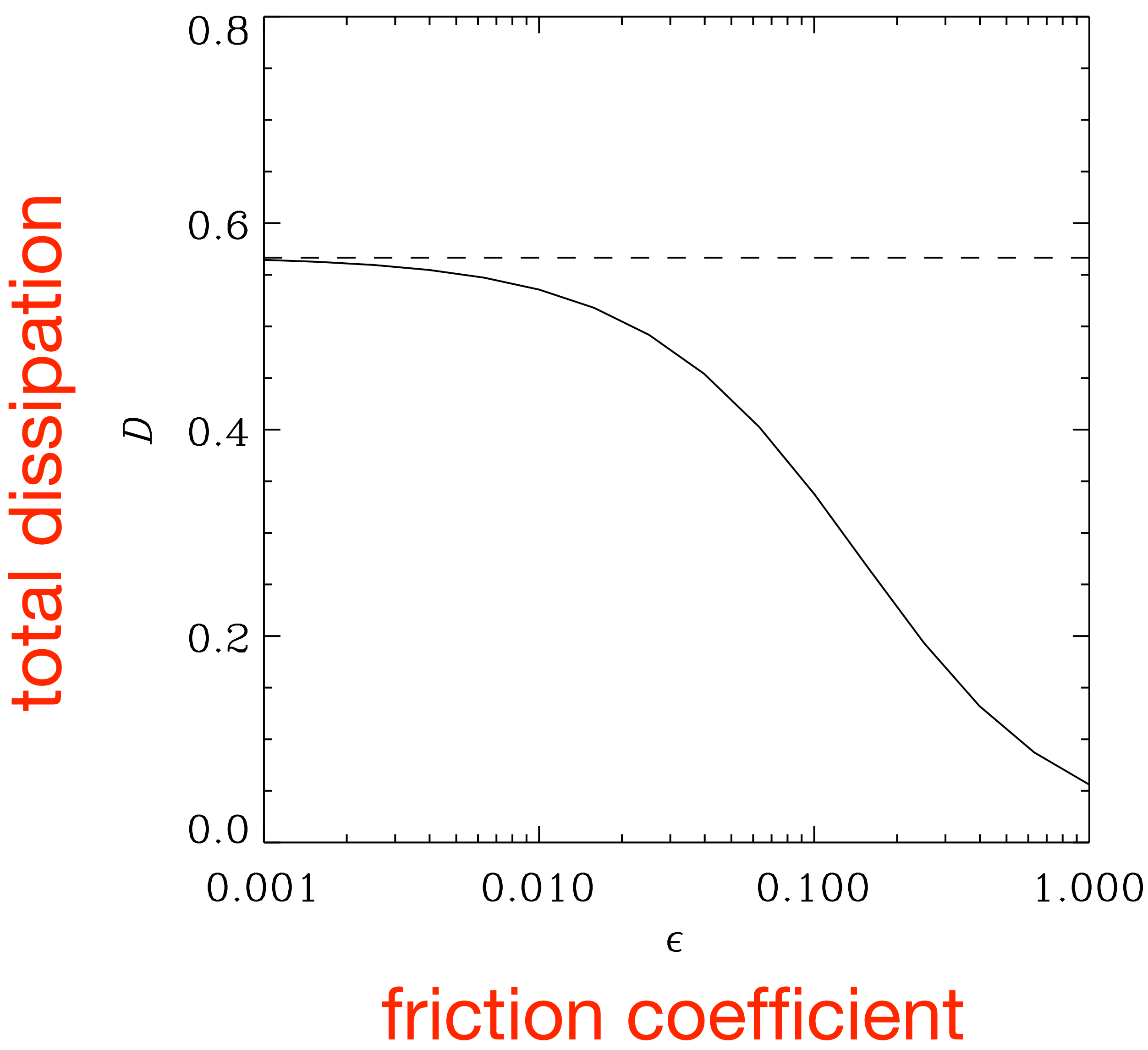
Wave attractors versus normal modes

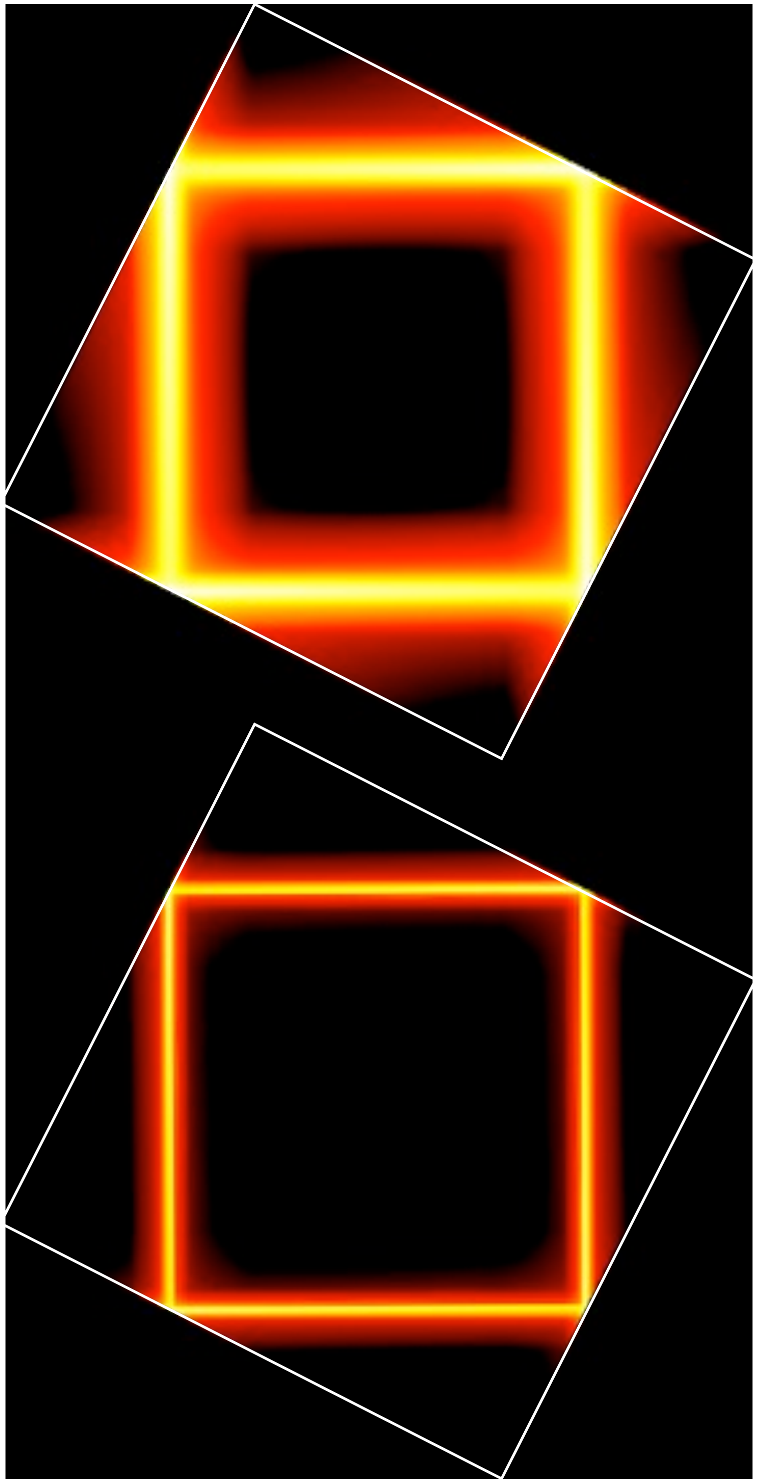


Attractor asymptotics

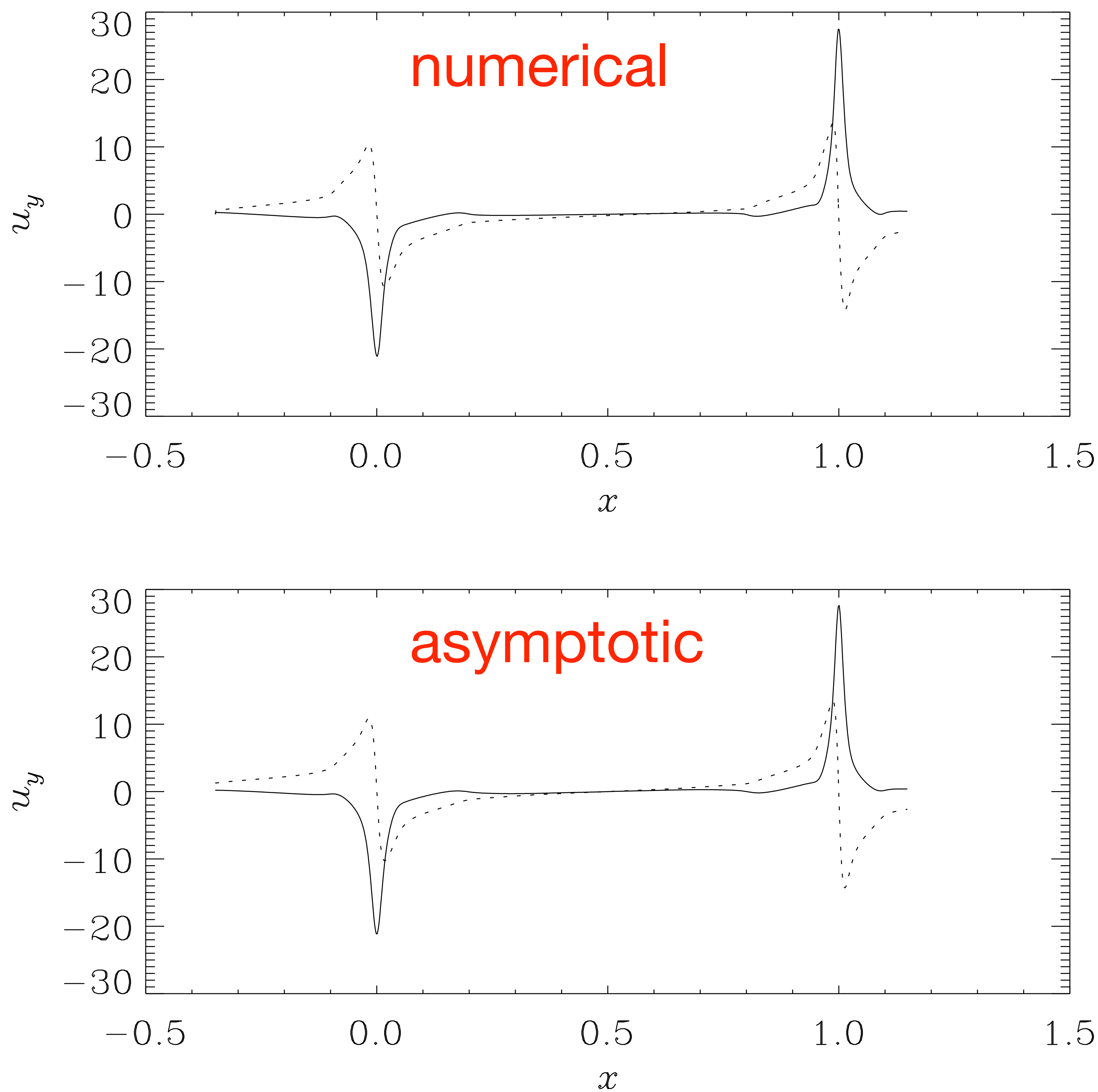
Ogilvie (2005)

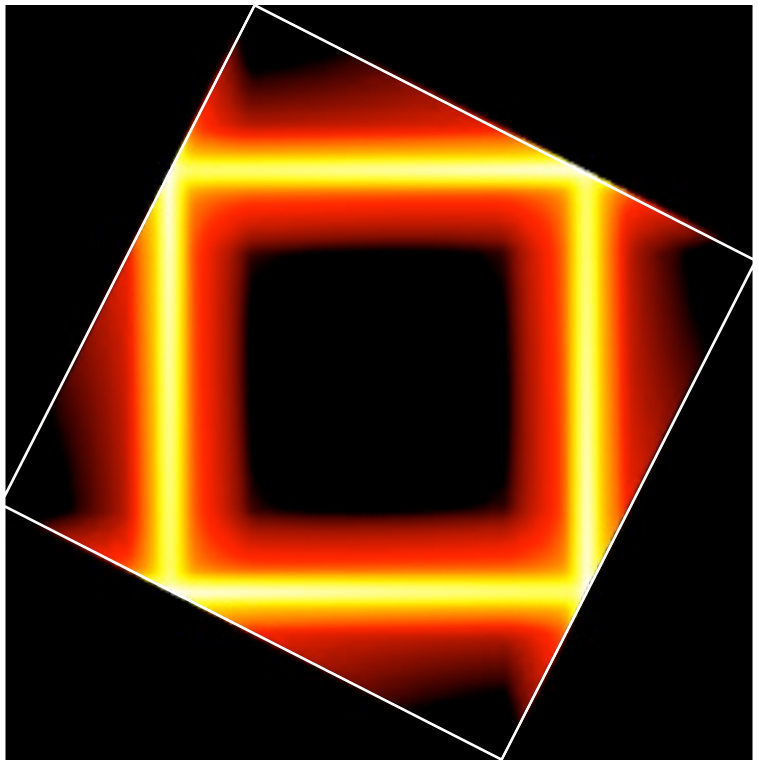
$$\mathrm{i}\frac{\partial^2\psi}{\partial x\partial y} - \epsilon\nabla^2\psi = f$$





Attractor asymptotics

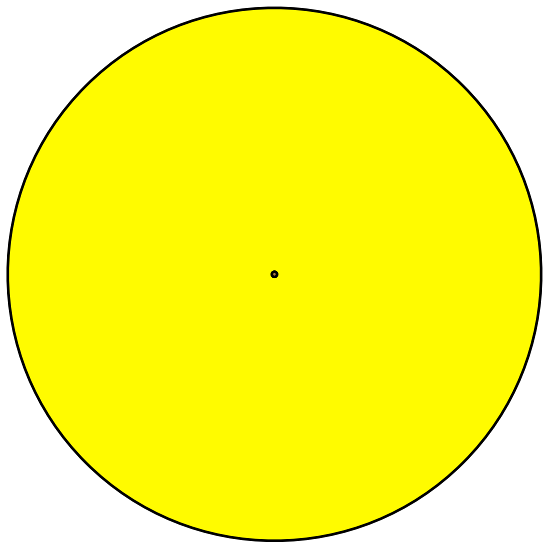


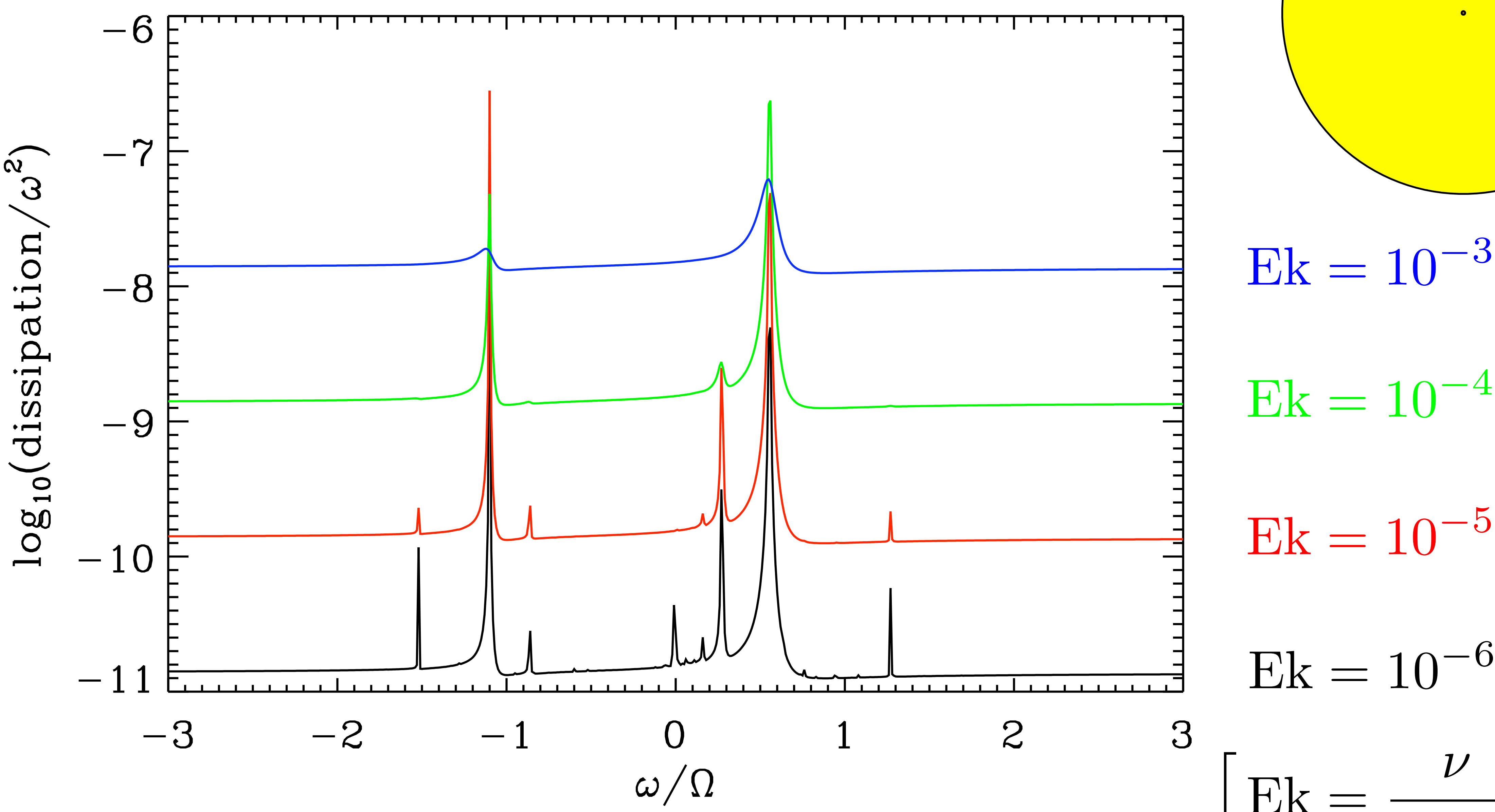


Responses of spheres and shells

Idealized problem : isentropic rotating fluid in spherical geometry

- Rigid core, fractional radius 0.01



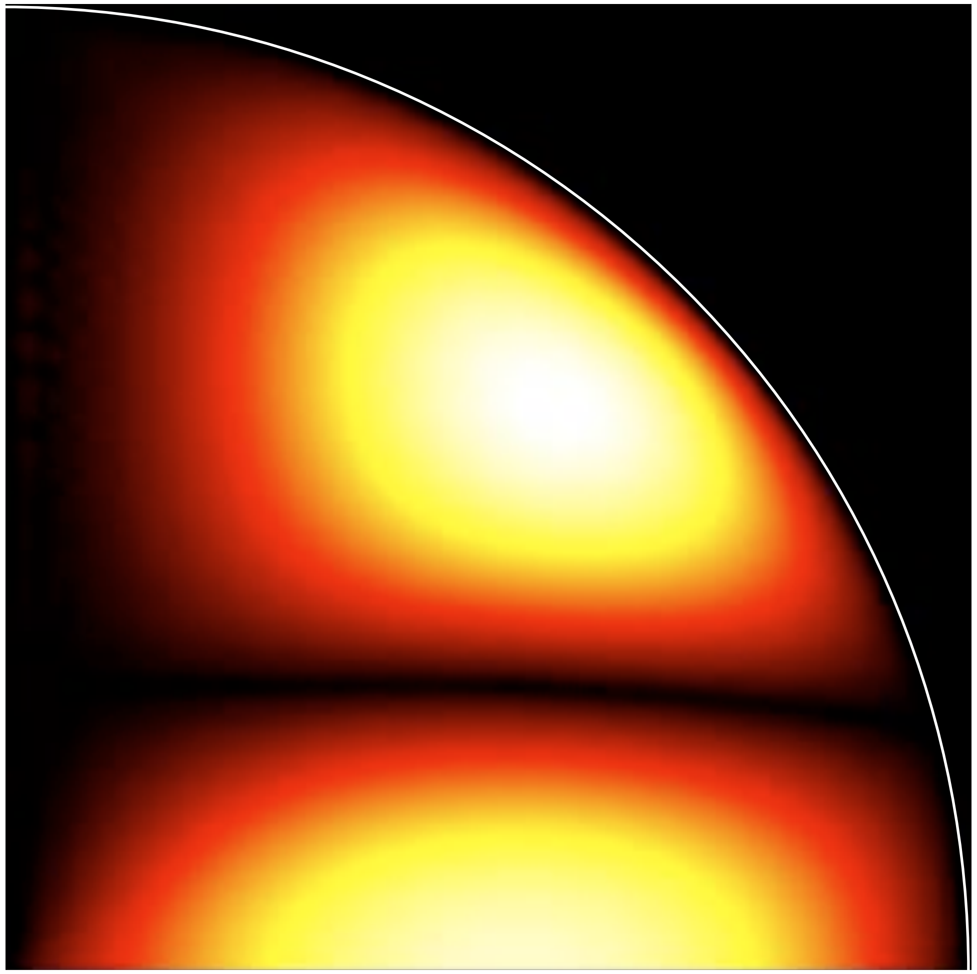


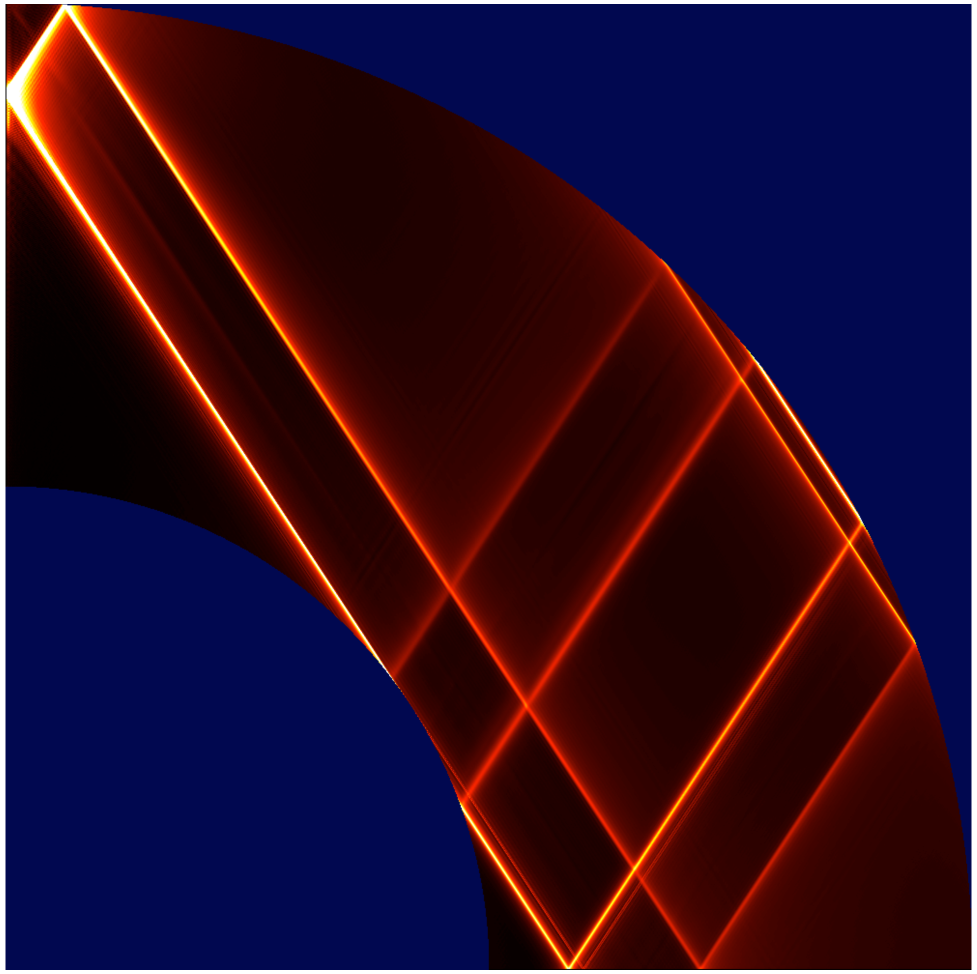
$$\left[\ \mathrm{Ek} = \frac{\nu}{2\Omega R^2}\ \right]$$

Inertial waves : modes or beams?

Dense or continuous spectrum, $-2\Omega < \hat{\omega} < 2\Omega$

- Tidal forcing excites normal modes (Wu; Papaloizou & Ivanov)
- Tidal forcing excites narrow beams (Ogilvie & Lin; Goodman & Lackner; Rieutord & Valdettaro)

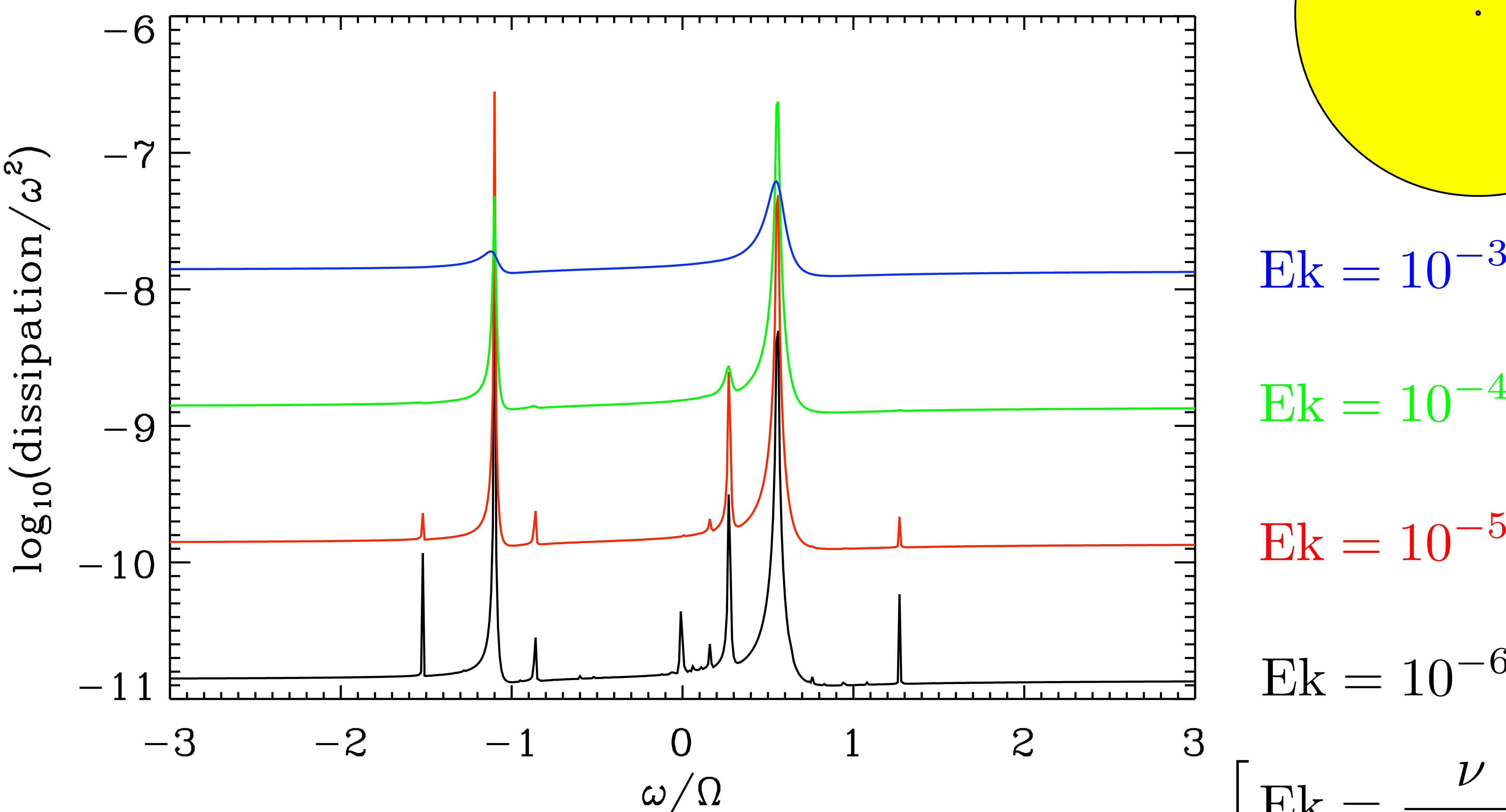


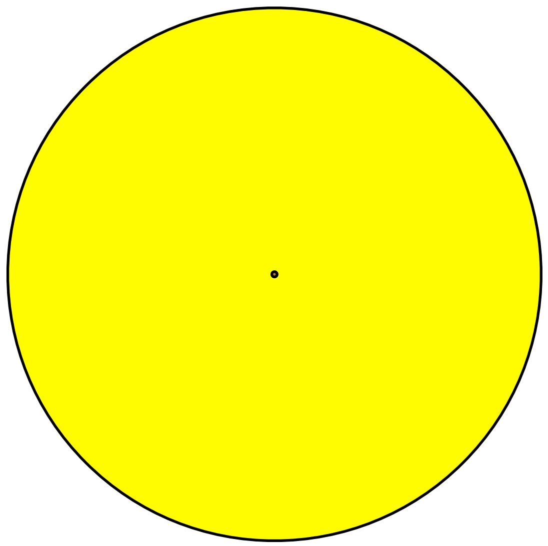


Responses of spheres and shells

Idealized problem : isentropic rotating fluid in spherical geometry

- Rigid core, fractional radius 0.01



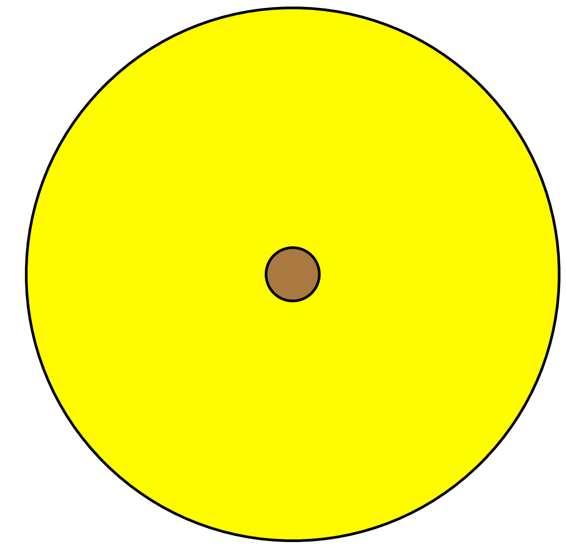


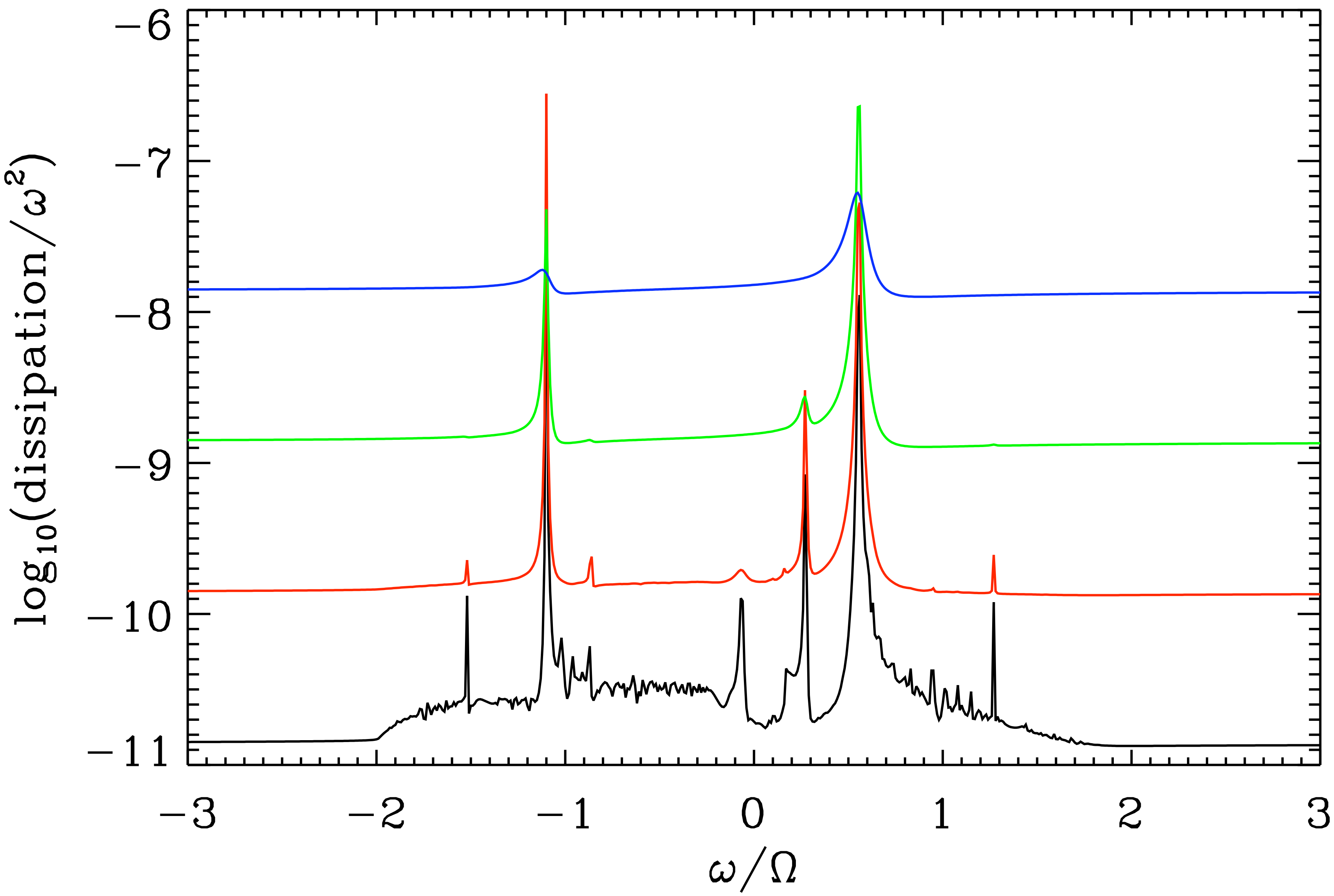
$$\left[\mathrm{Ek} = \frac{\nu}{2\Omega R^2} \right]$$

Responses of spheres and shells

Idealized problem : isentropic rotating fluid in spherical geometry

- Rigid core, fractional radius 0.1

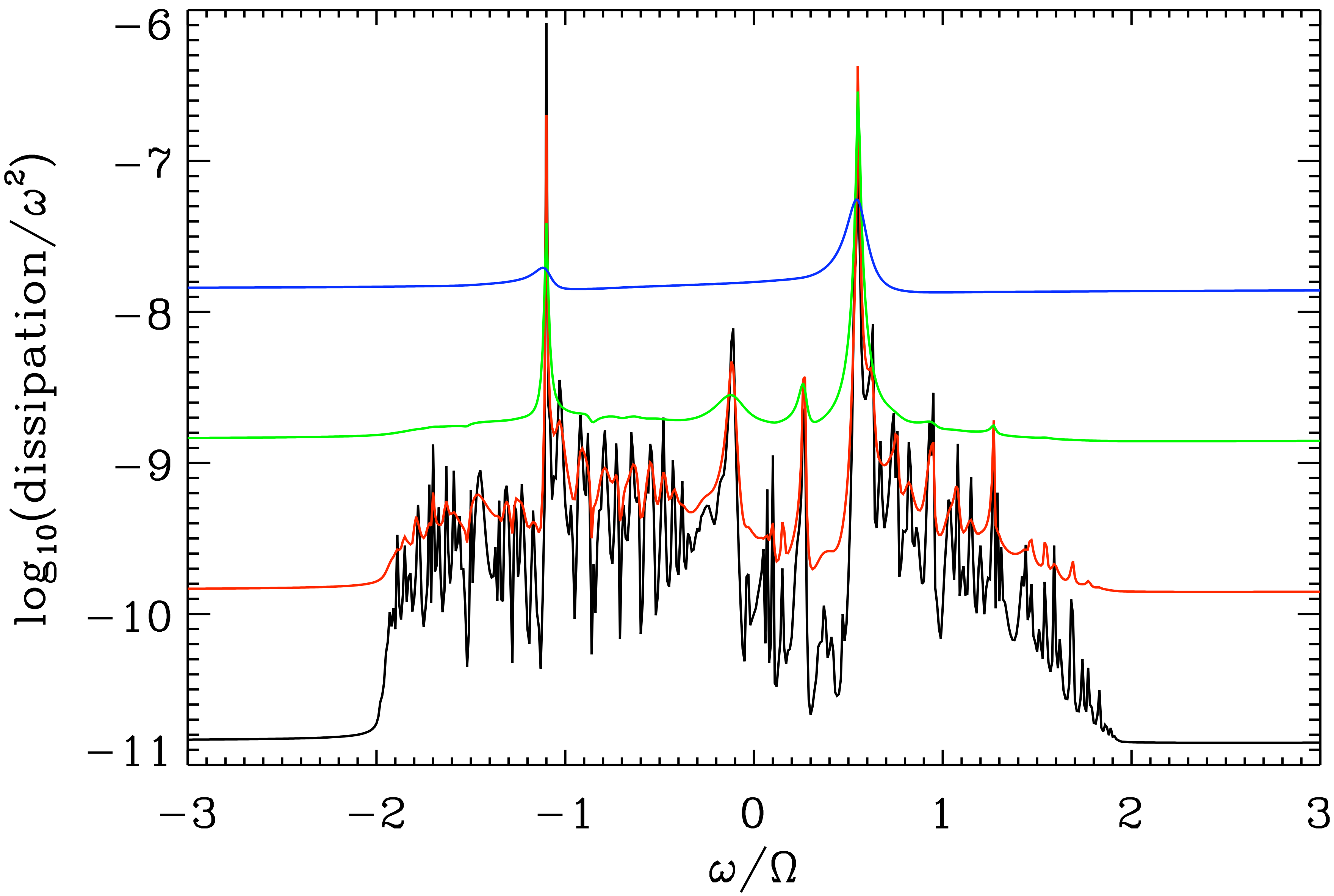


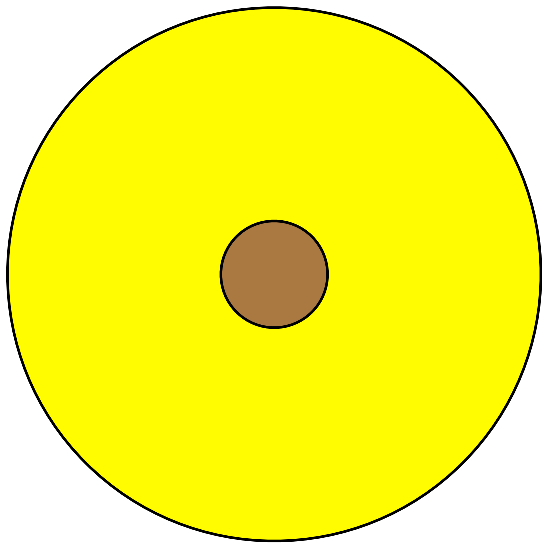


Responses of spheres and shells

Idealized problem : isentropic rotating fluid in spherical geometry

- Rigid core, fractional radius 0.2

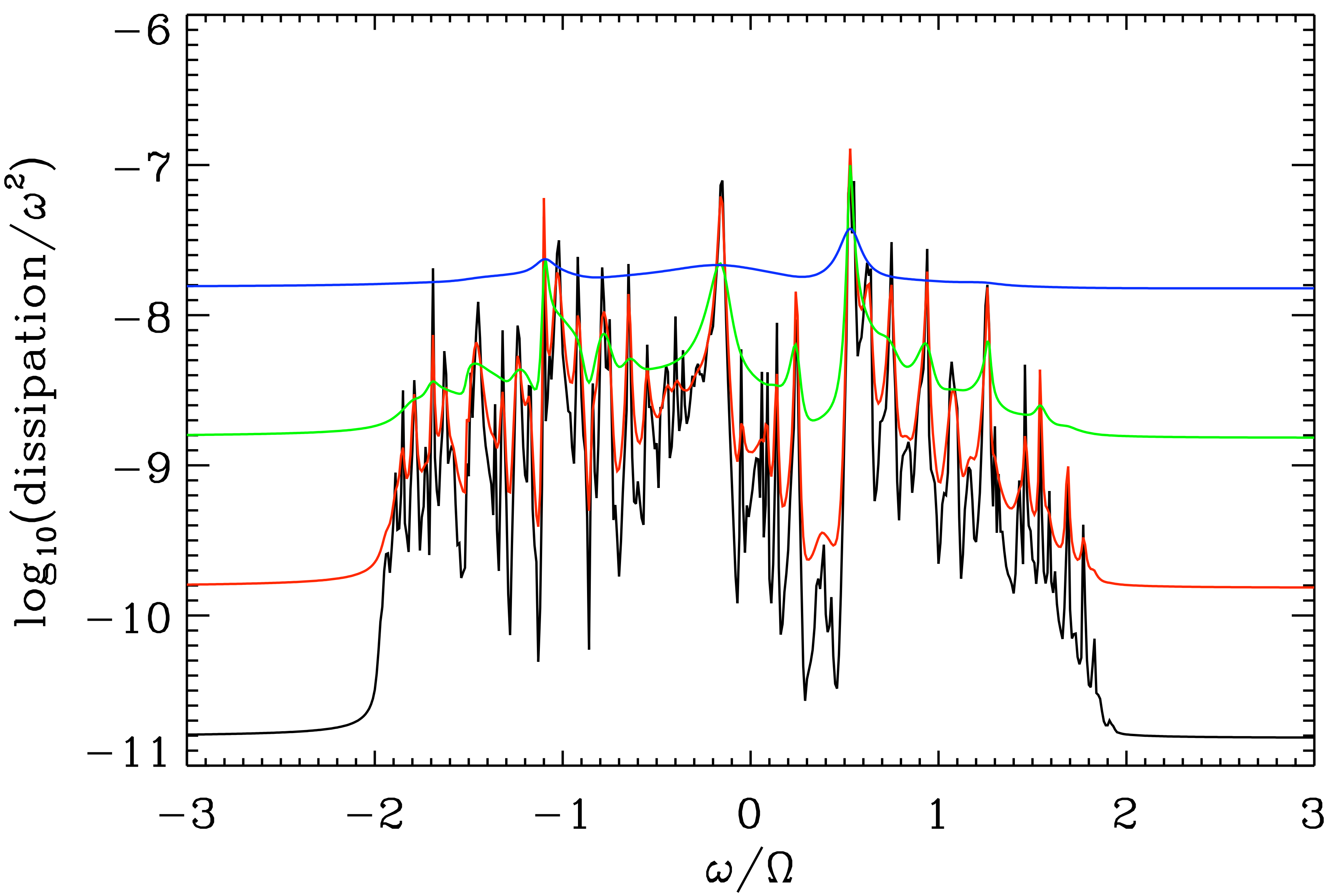


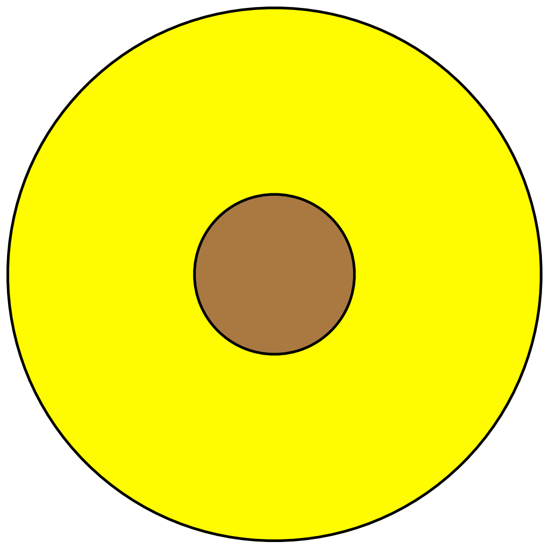


Responses of spheres and shells

Idealized problem : isentropic rotating fluid in spherical geometry

- Rigid core, fractional radius 0.3



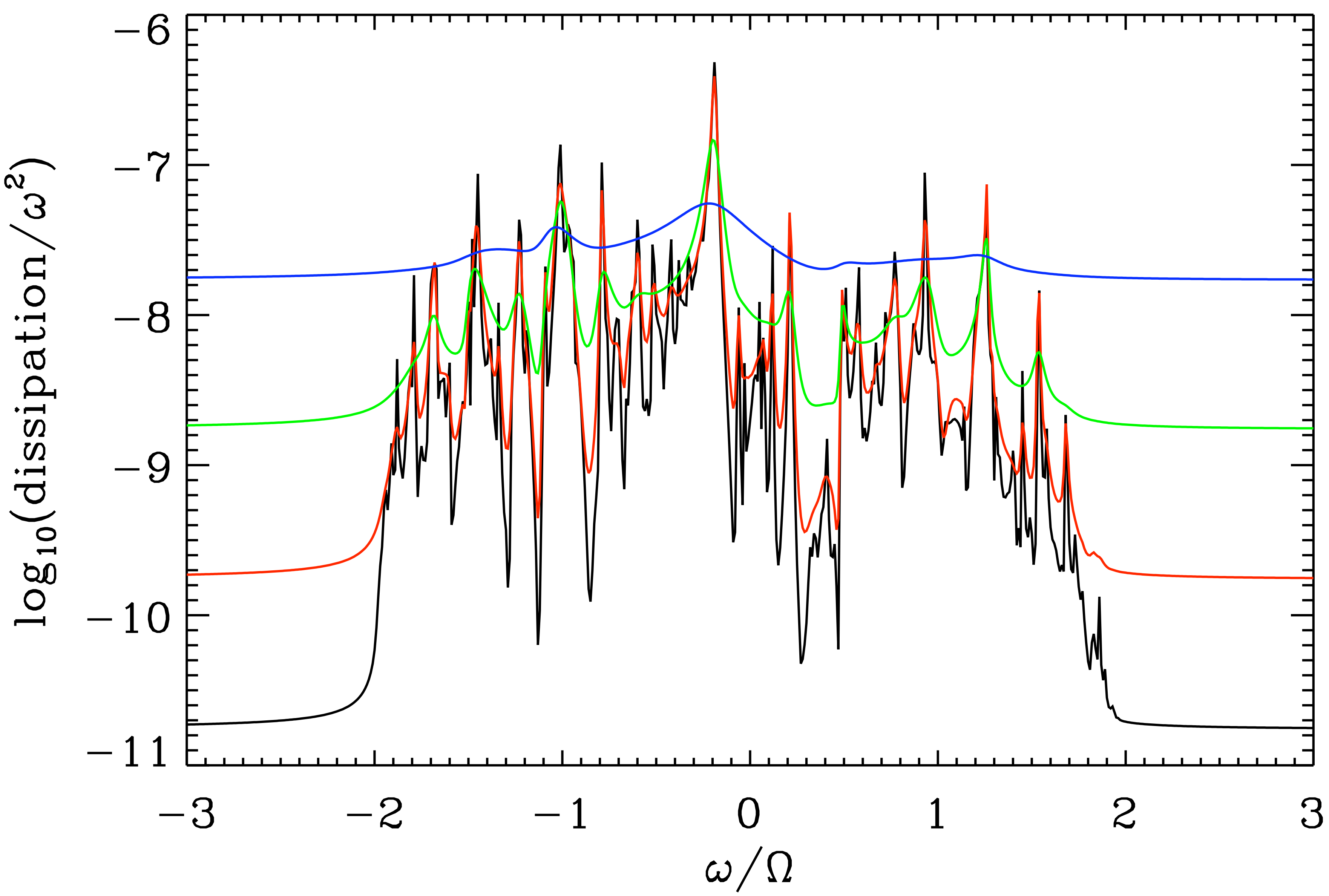


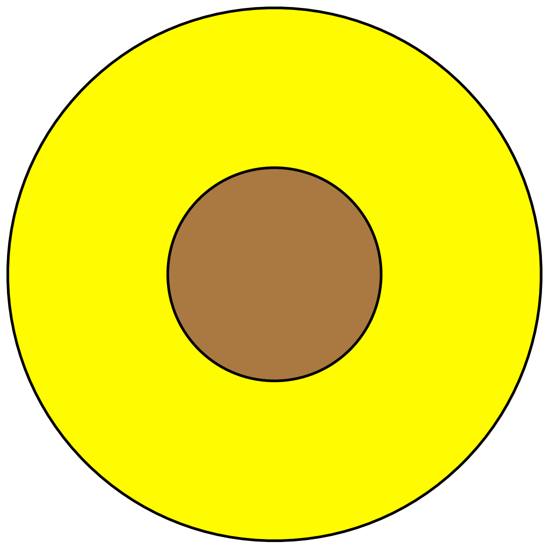
2011年12月2日金曜日

Responses of spheres and shells

Idealized problem : isentropic rotating fluid in spherical geometry

- Rigid core, fractional radius 0.4

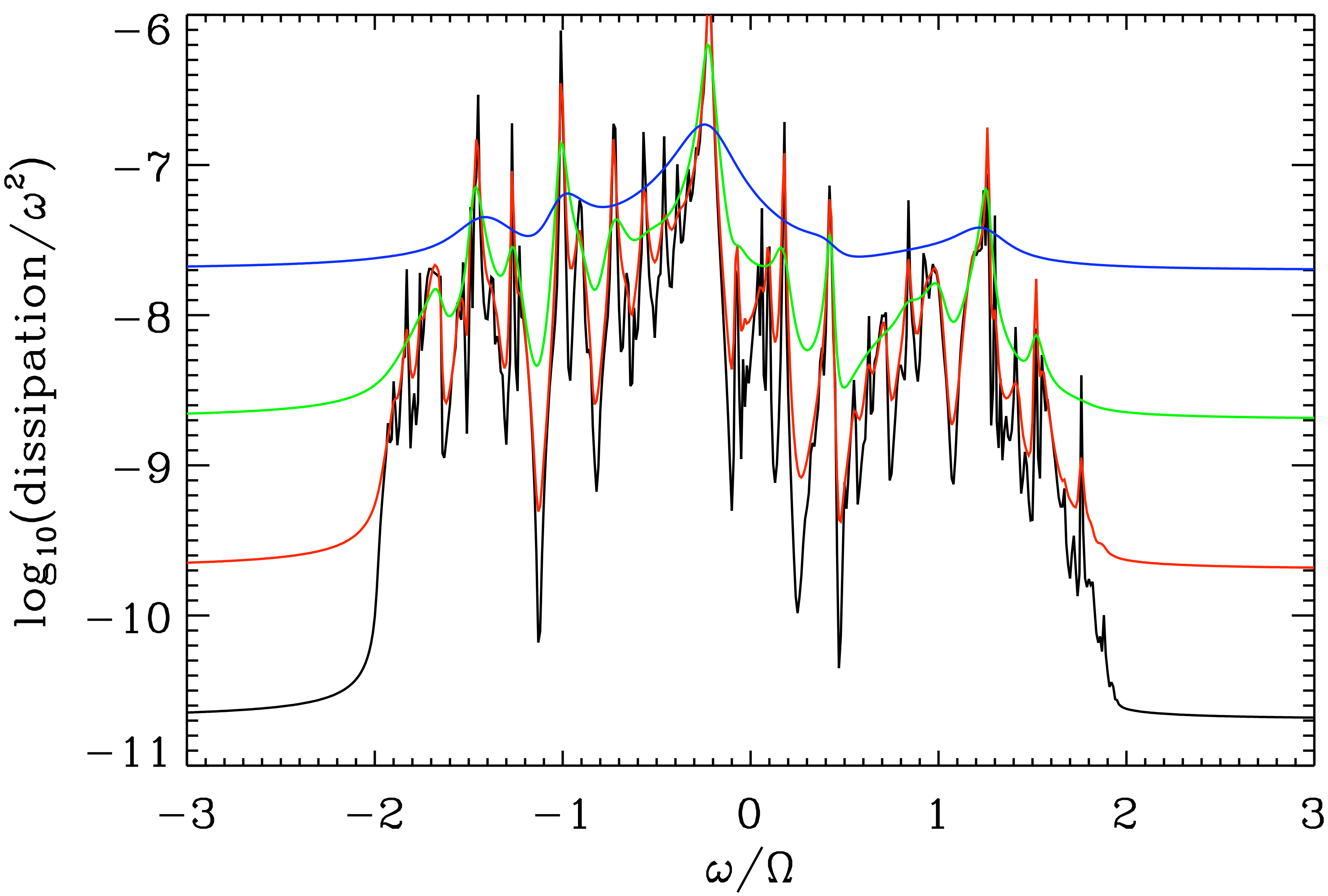


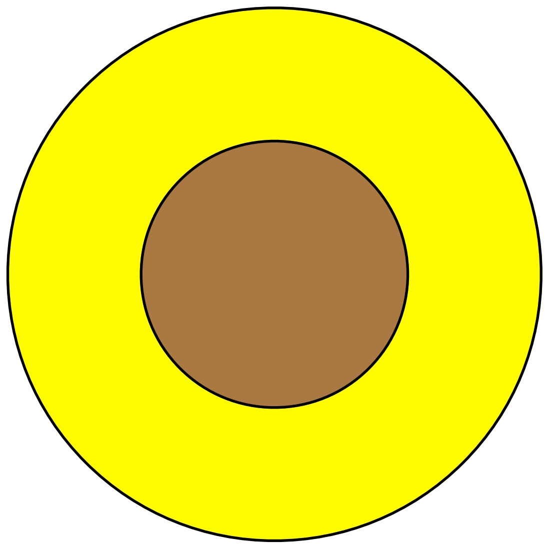


Responses of spheres and shells

Idealized problem : isentropic rotating fluid in spherical geometry

- Rigid core, fractional radius 0.5

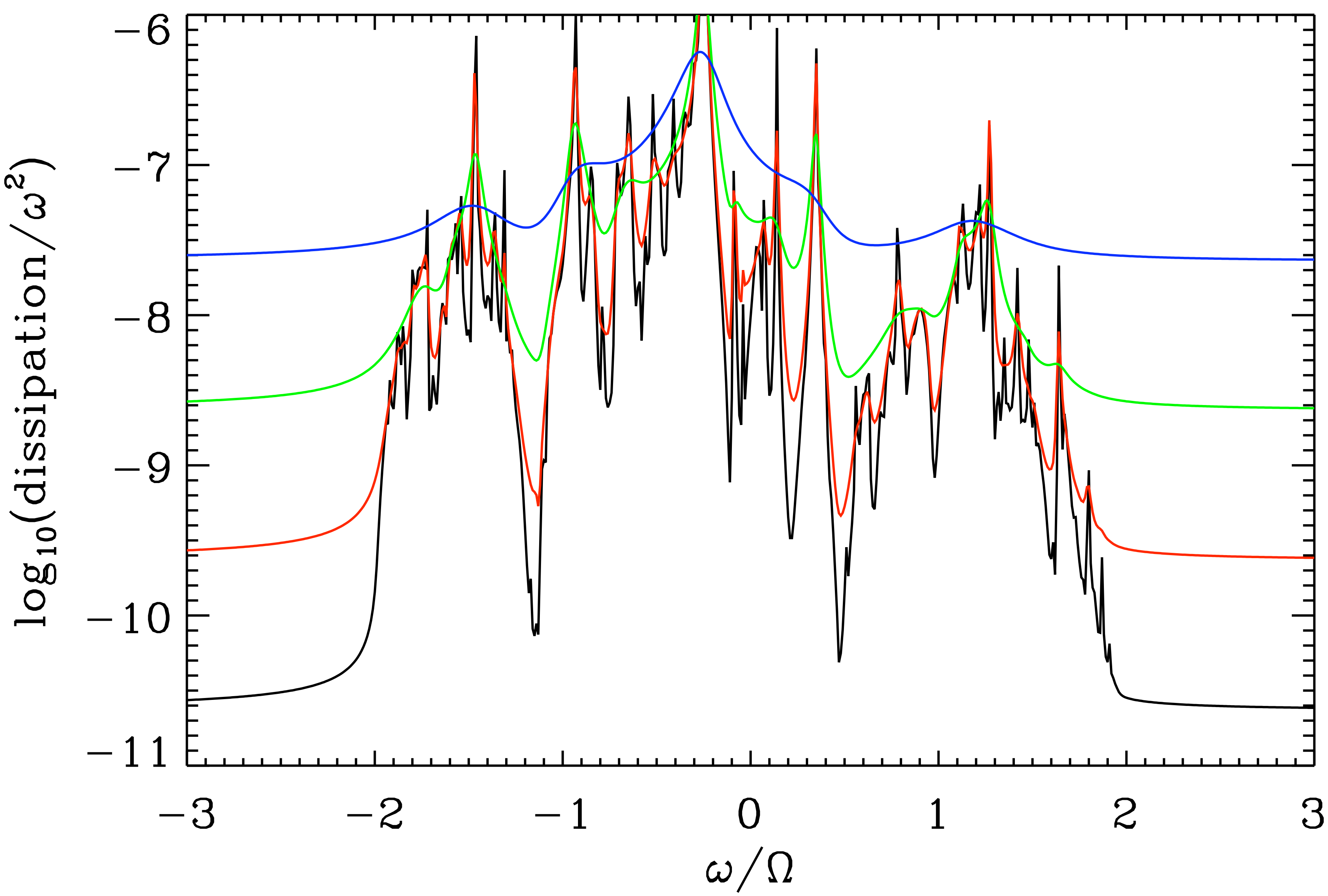


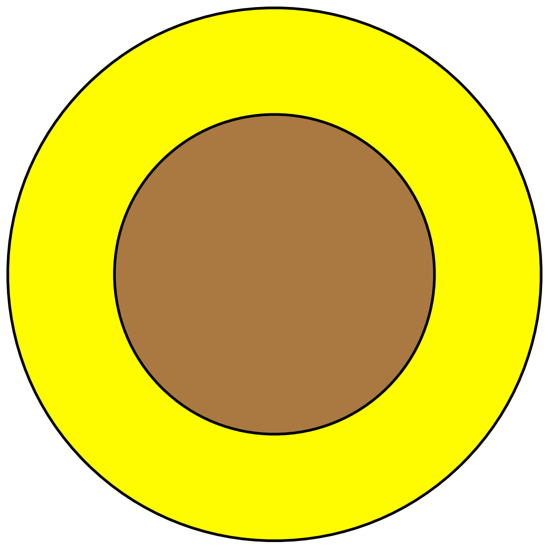


Responses of spheres and shells

Idealized problem : isentropic rotating fluid in spherical geometry

- Rigid core, fractional radius 0.6

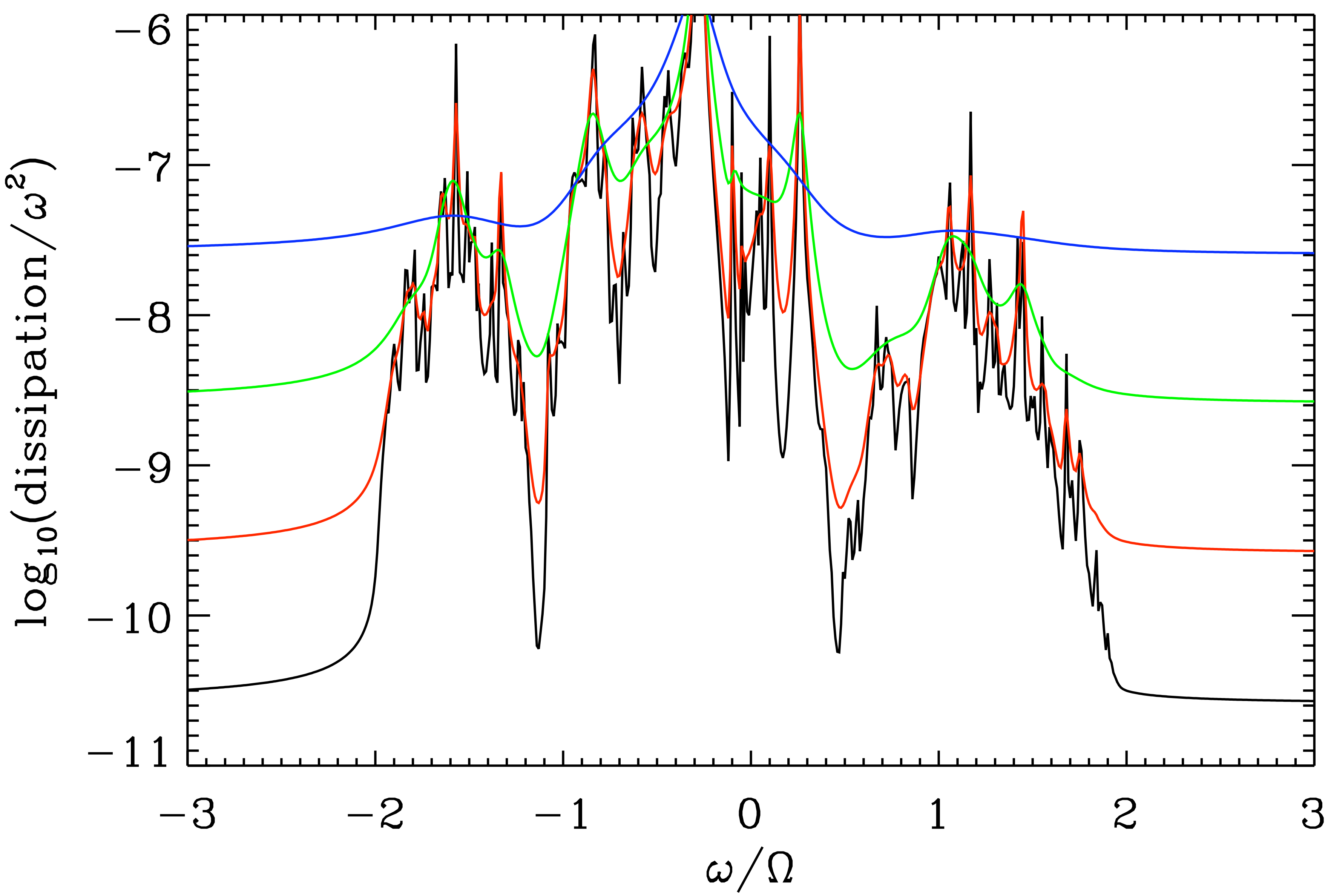


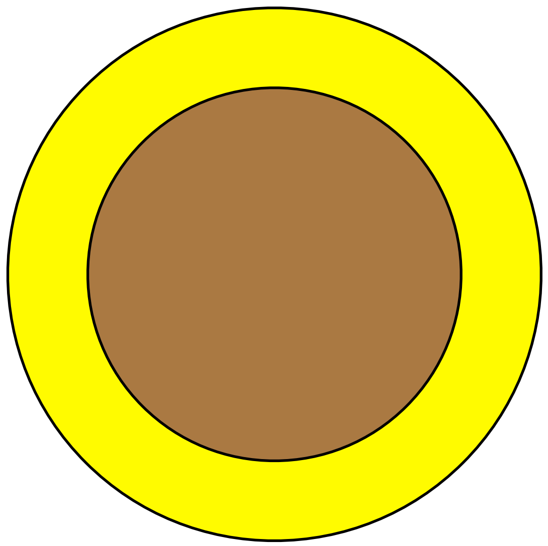


Responses of spheres and shells

Idealized problem : isentropic rotating fluid in spherical geometry

- Rigid core, fractional radius 0.7

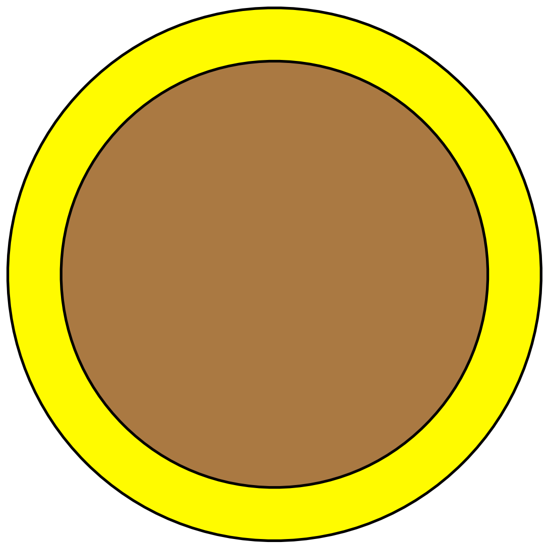


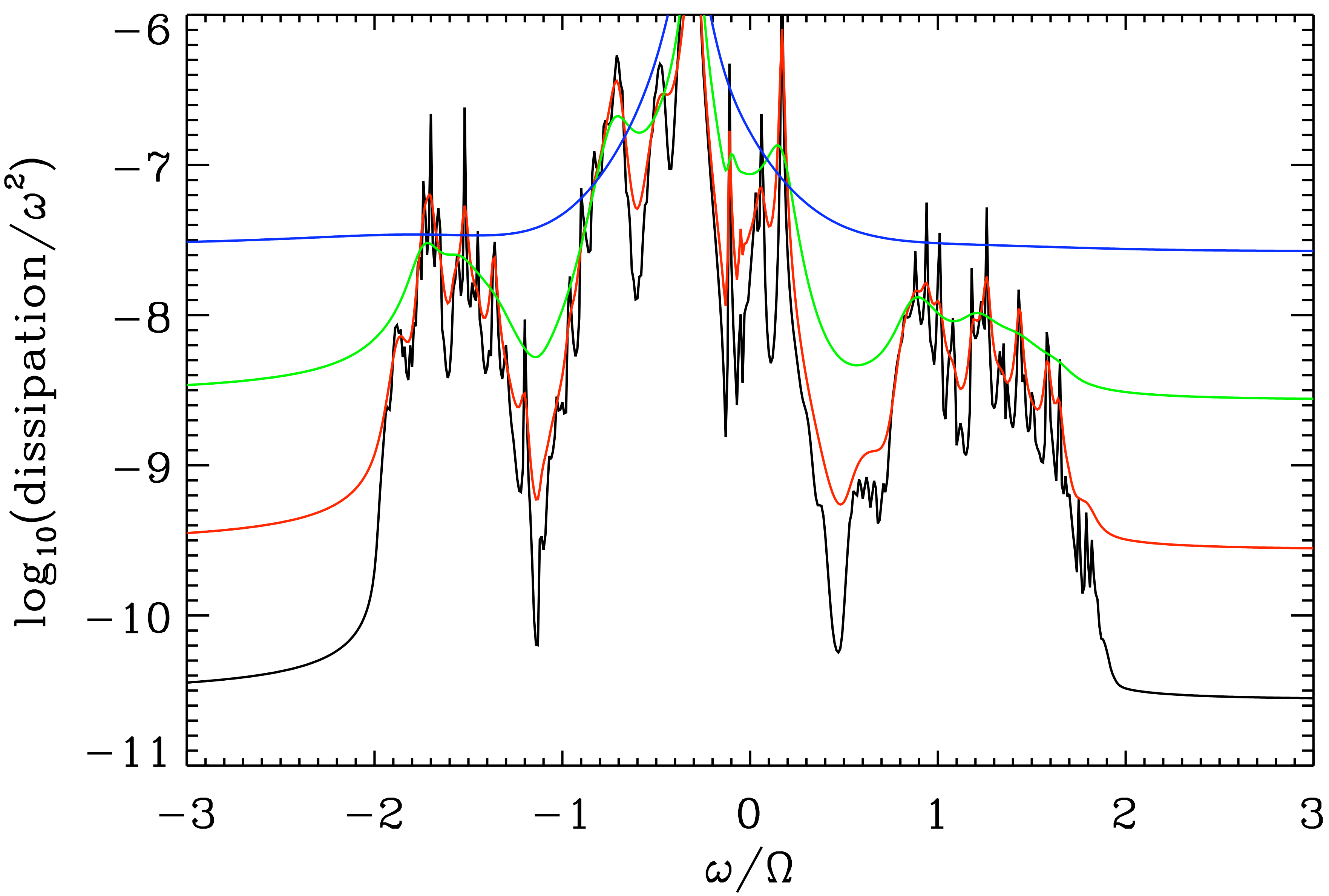


Responses of spheres and shells

Idealized problem : isentropic rotating fluid in spherical geometry

- Rigid core, fractional radius 0.8

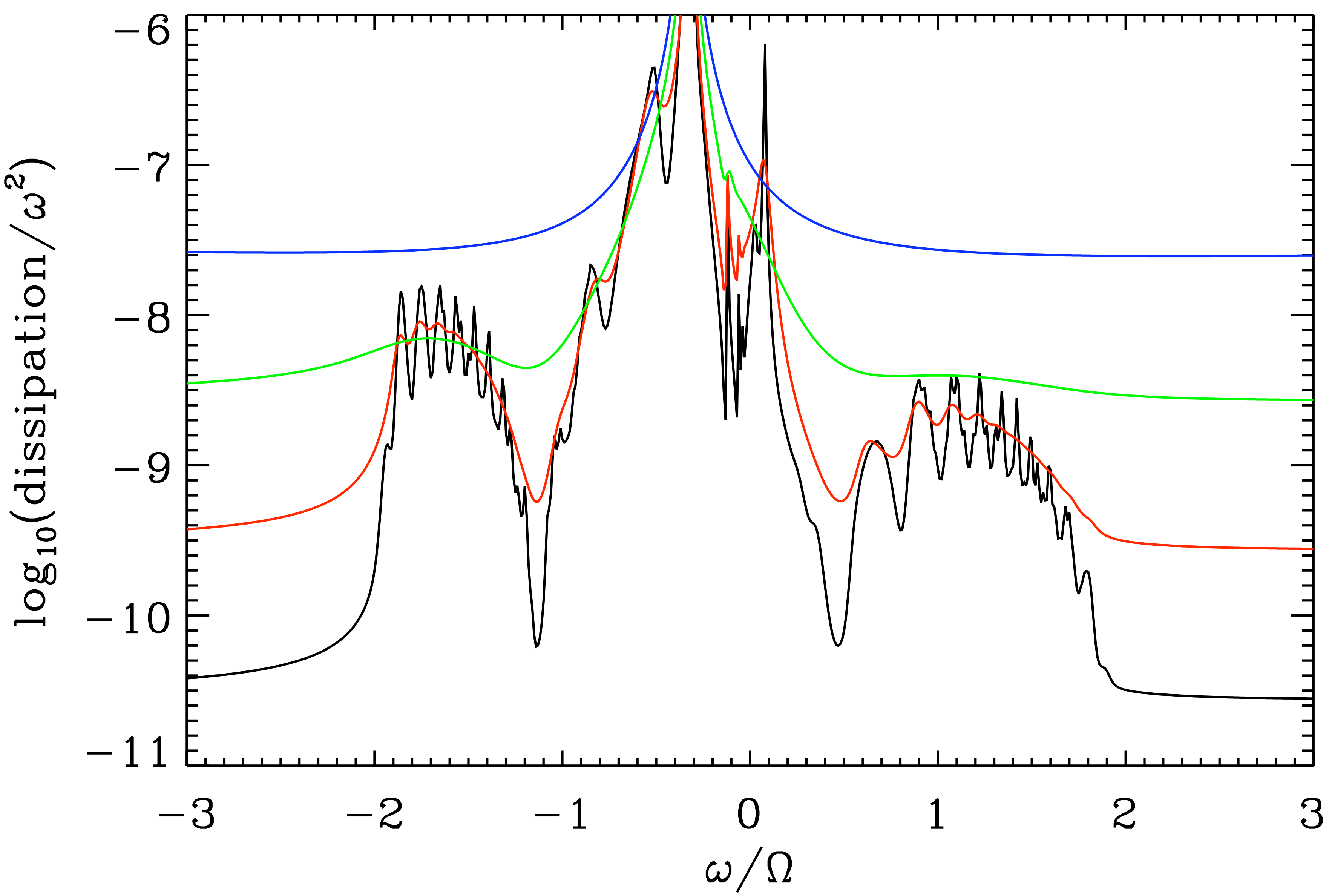


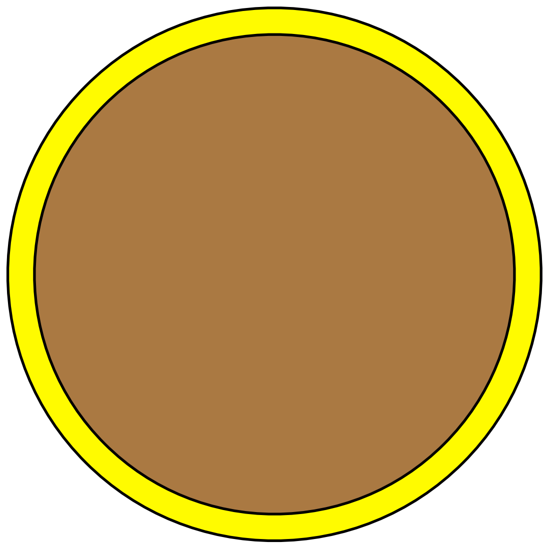


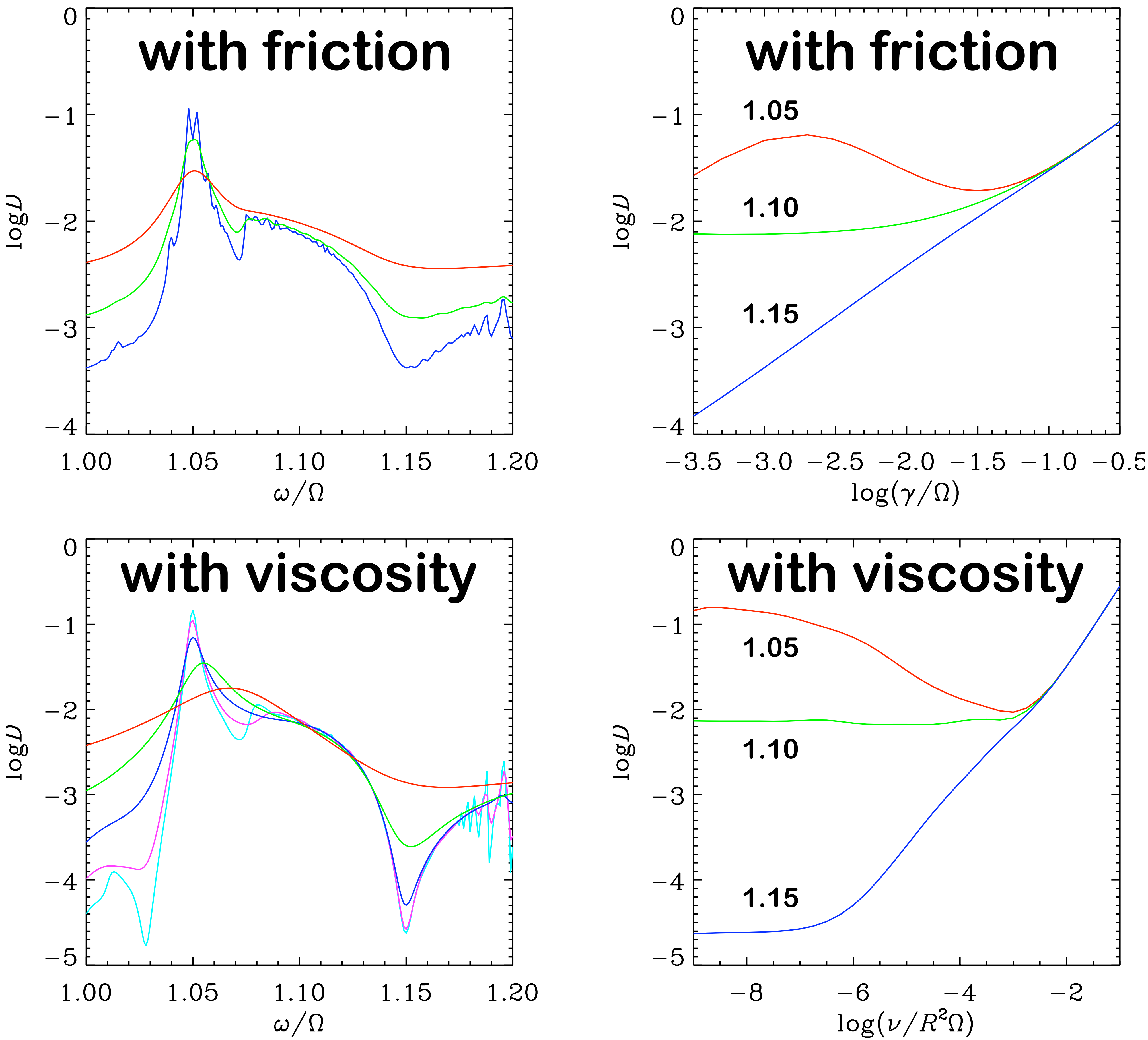
Responses of spheres and shells

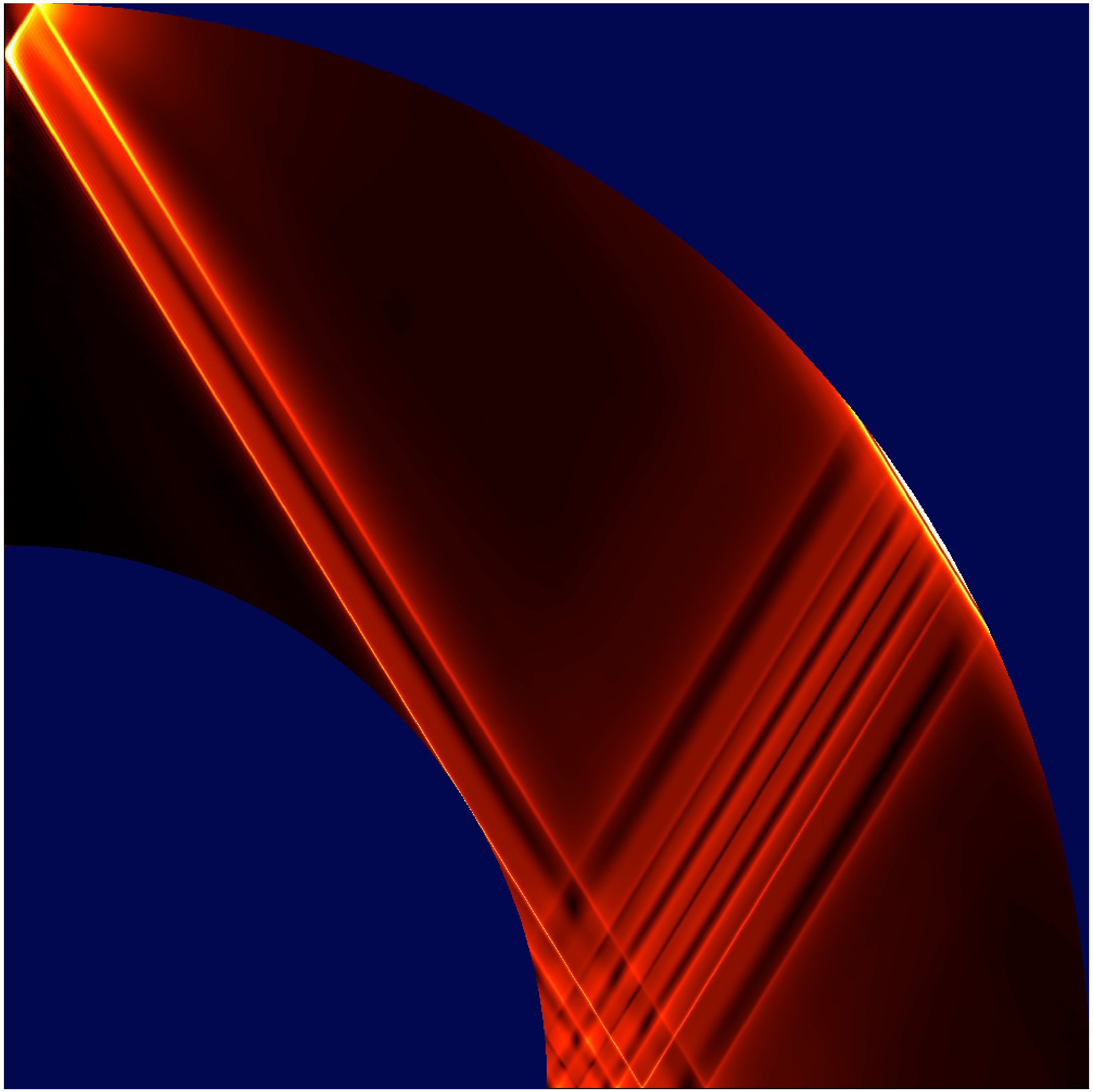
Idealized problem : isentropic rotating fluid in spherical geometry

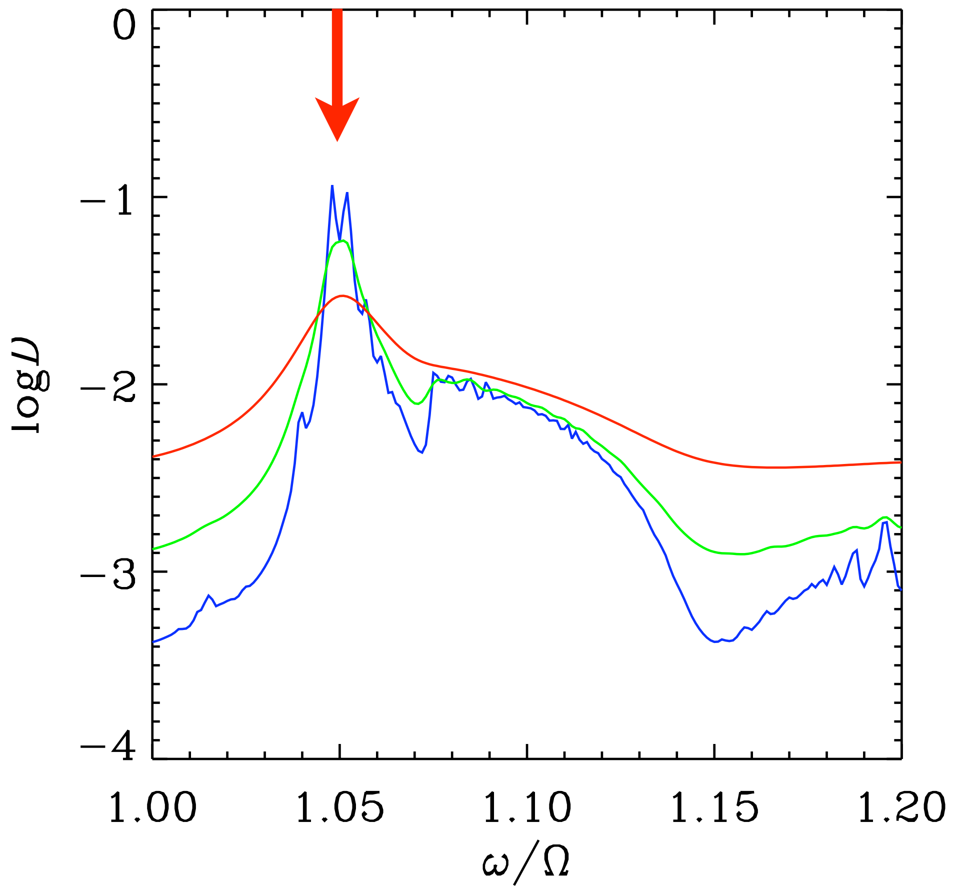
- Rigid core, fractional radius 0.9

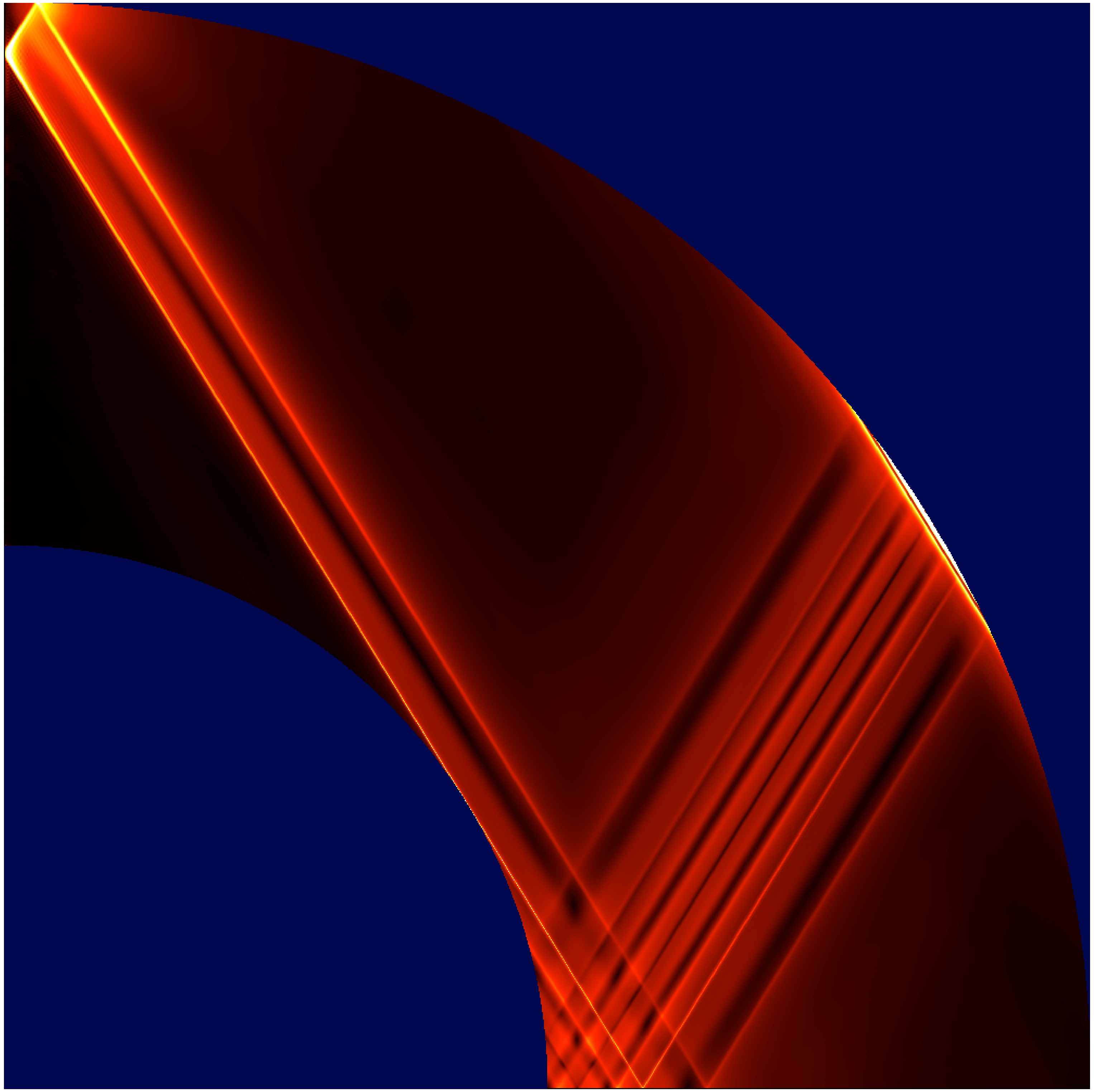


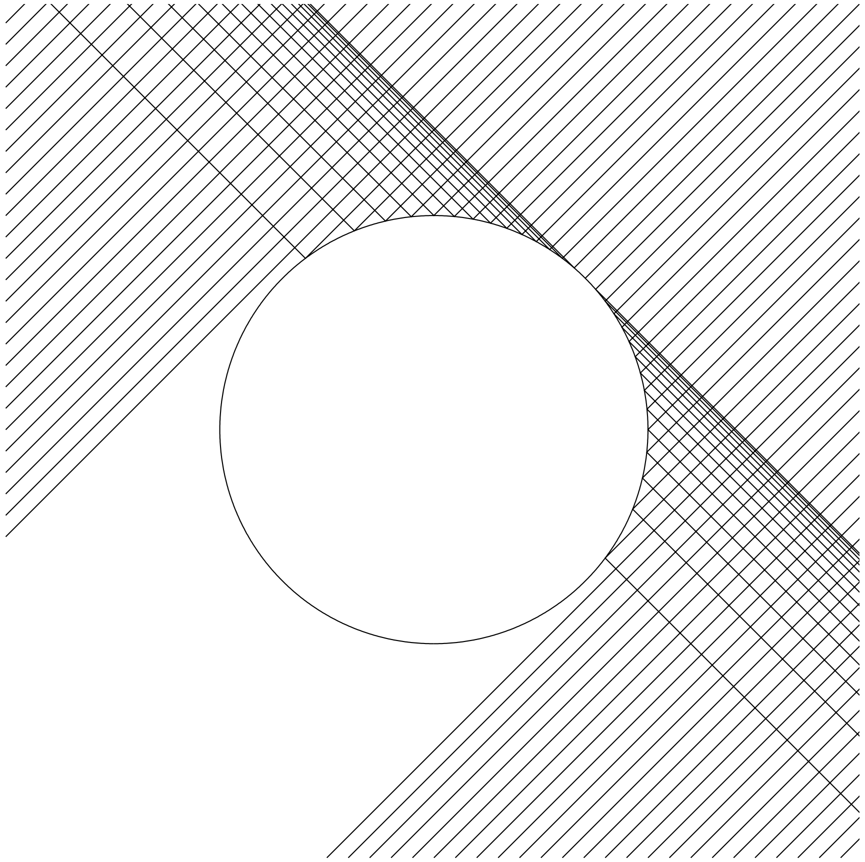



with friction
with friction
1.05
1.10
1.15
with viscosity
with viscosity
1.05
1.10
1.15
log D
ω/Ω
log(γ/Ω)
log(ν/R²Ω)
0
−1
−2
−3
−4
−5
1.00
1.05
1.10
1.15
1.20
−3.5
−3.0
−2.5
−2.0
−1.5
−1.0
−0.5
−8
−6
−4
−2

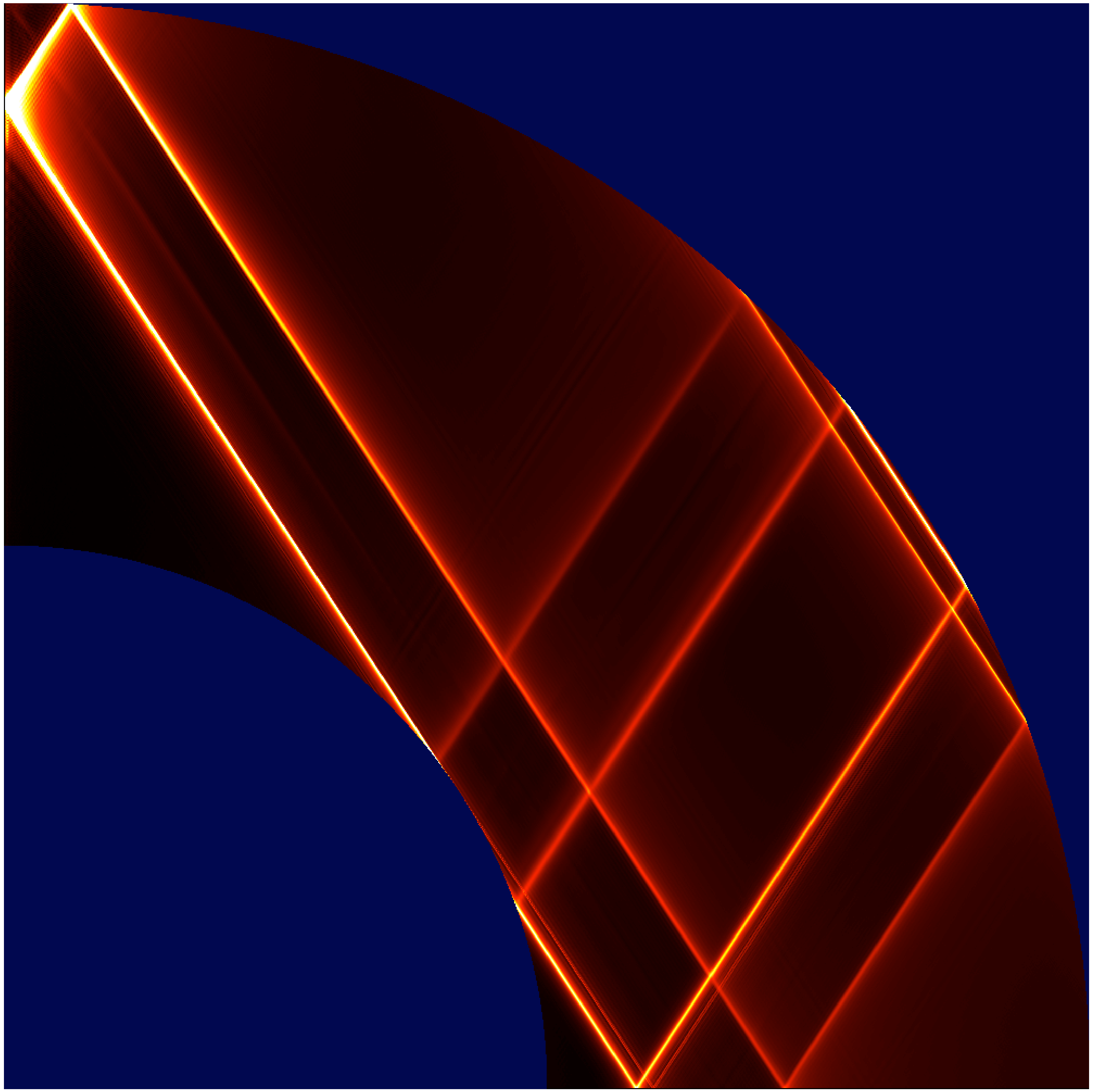


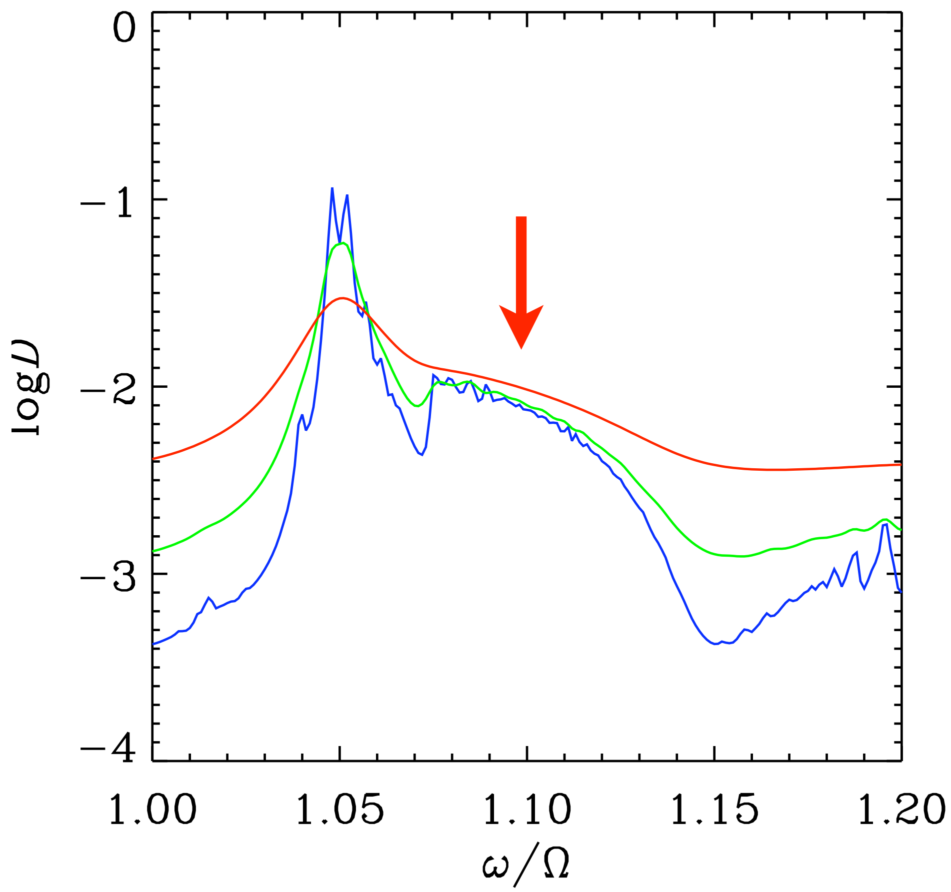

0
−1
−2
−3
−4
log D
1.00
1.05
1.10
1.15
1.20
ω/Ω

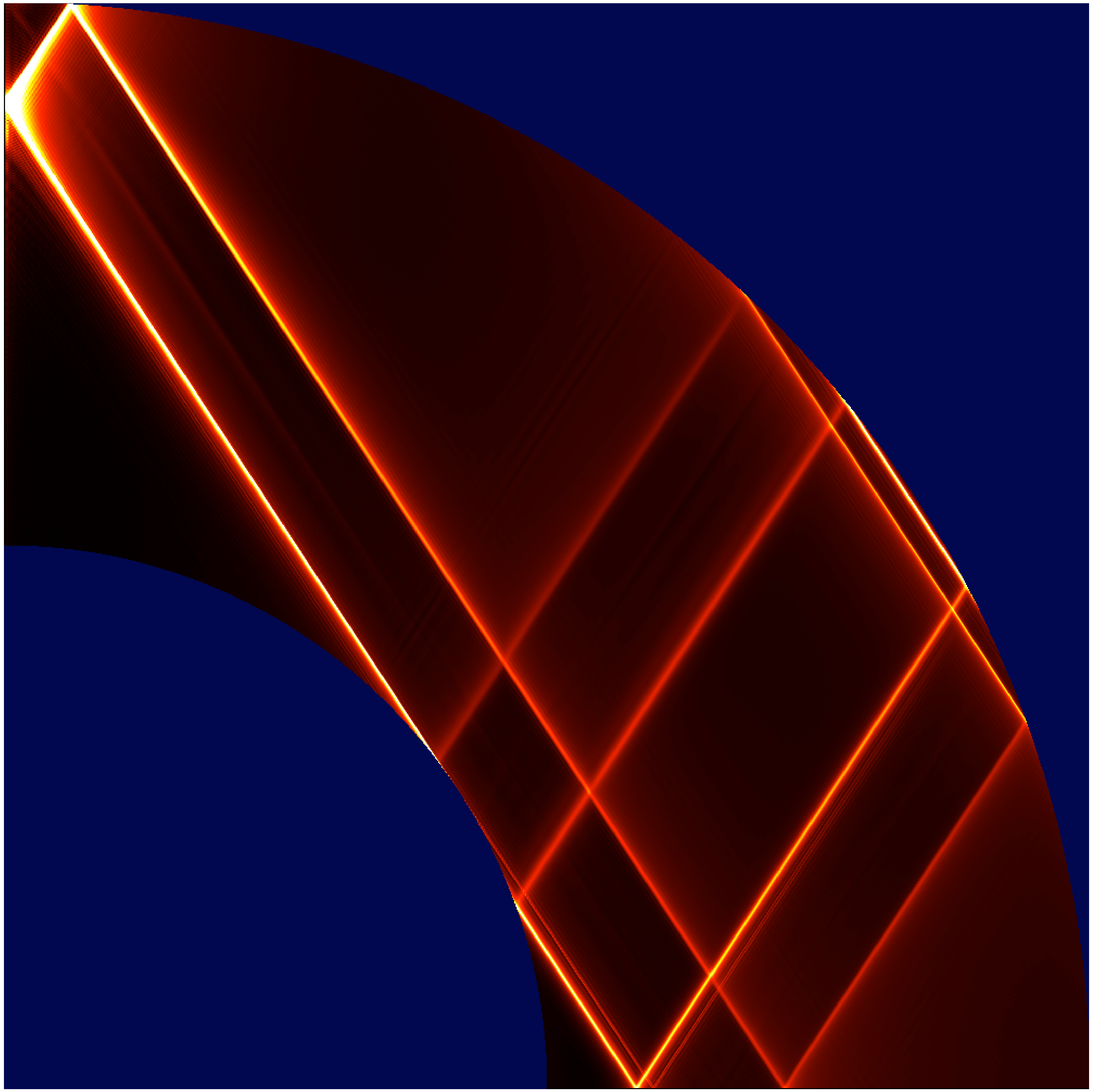


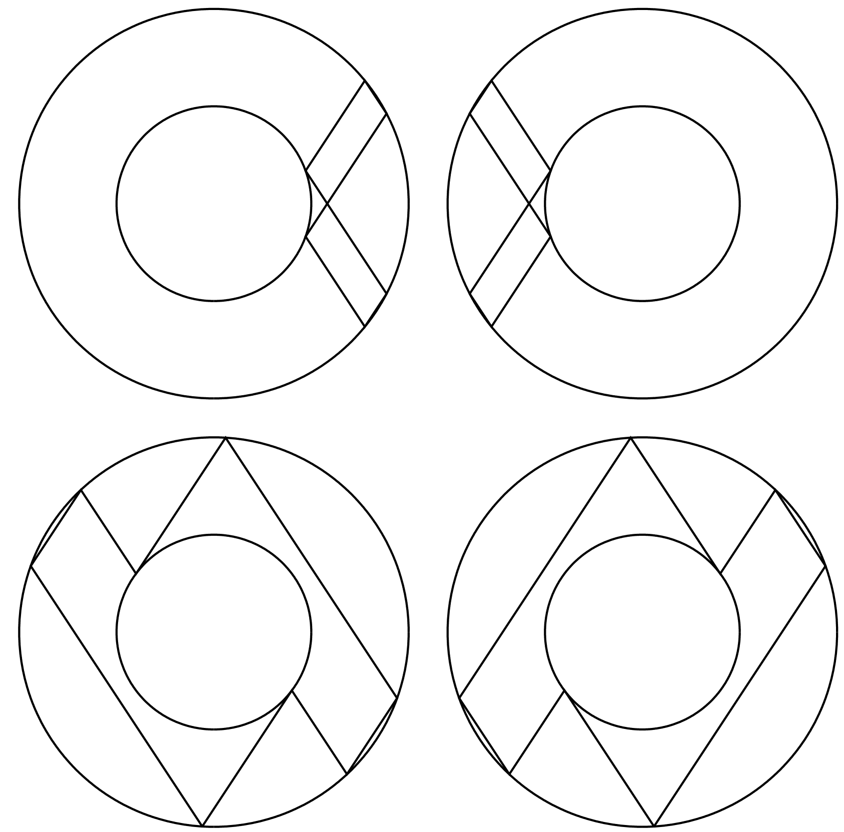


critical latitude
singularity

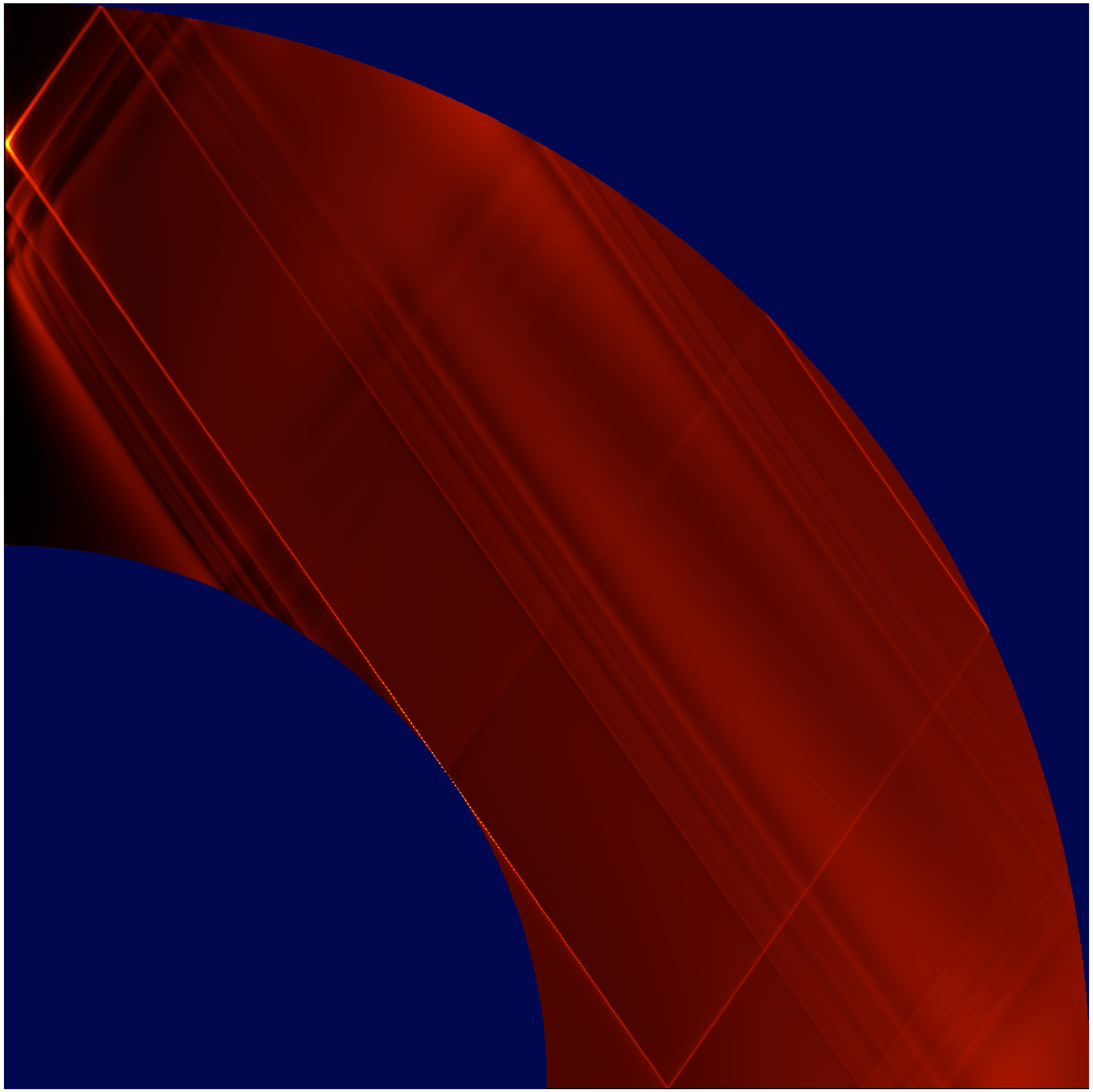


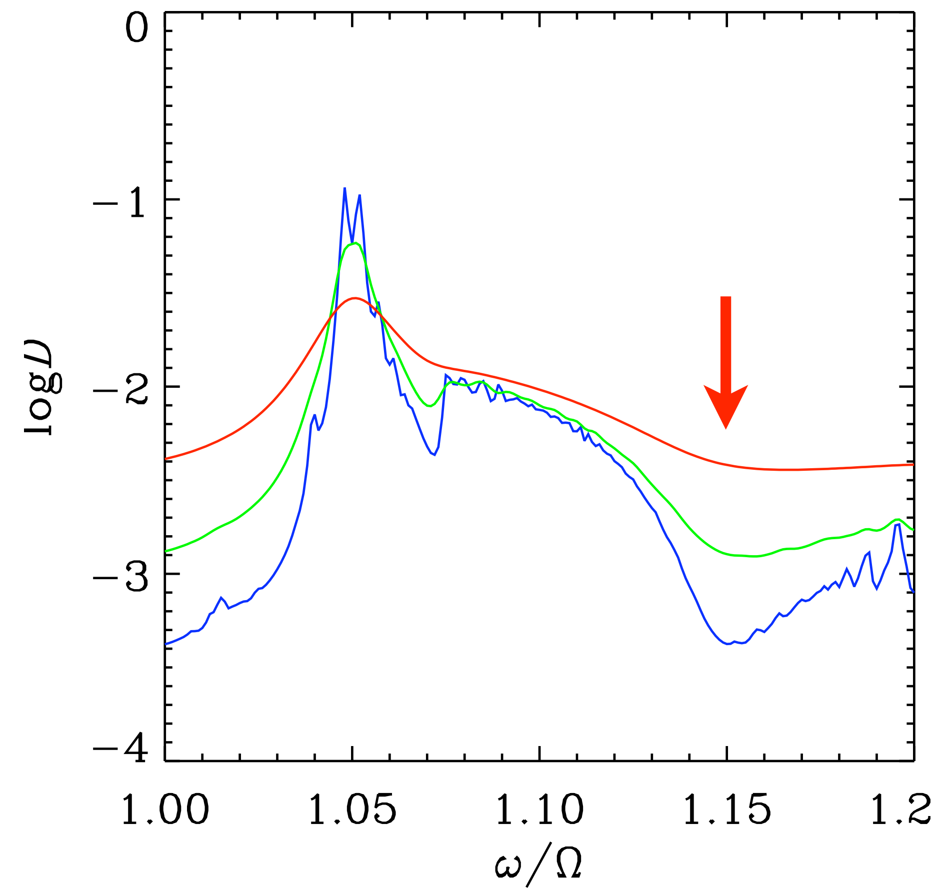

0
−1
−2
−3
−4
log D
1.00
1.05
1.10
1.15
1.20
ω/Ω





wave attractors





0
−1
−2
−3
−4
log D
1.00
1.05
1.10
1.15
1.20
ω/Ω

Mixed metaphors

Mixed metaphors

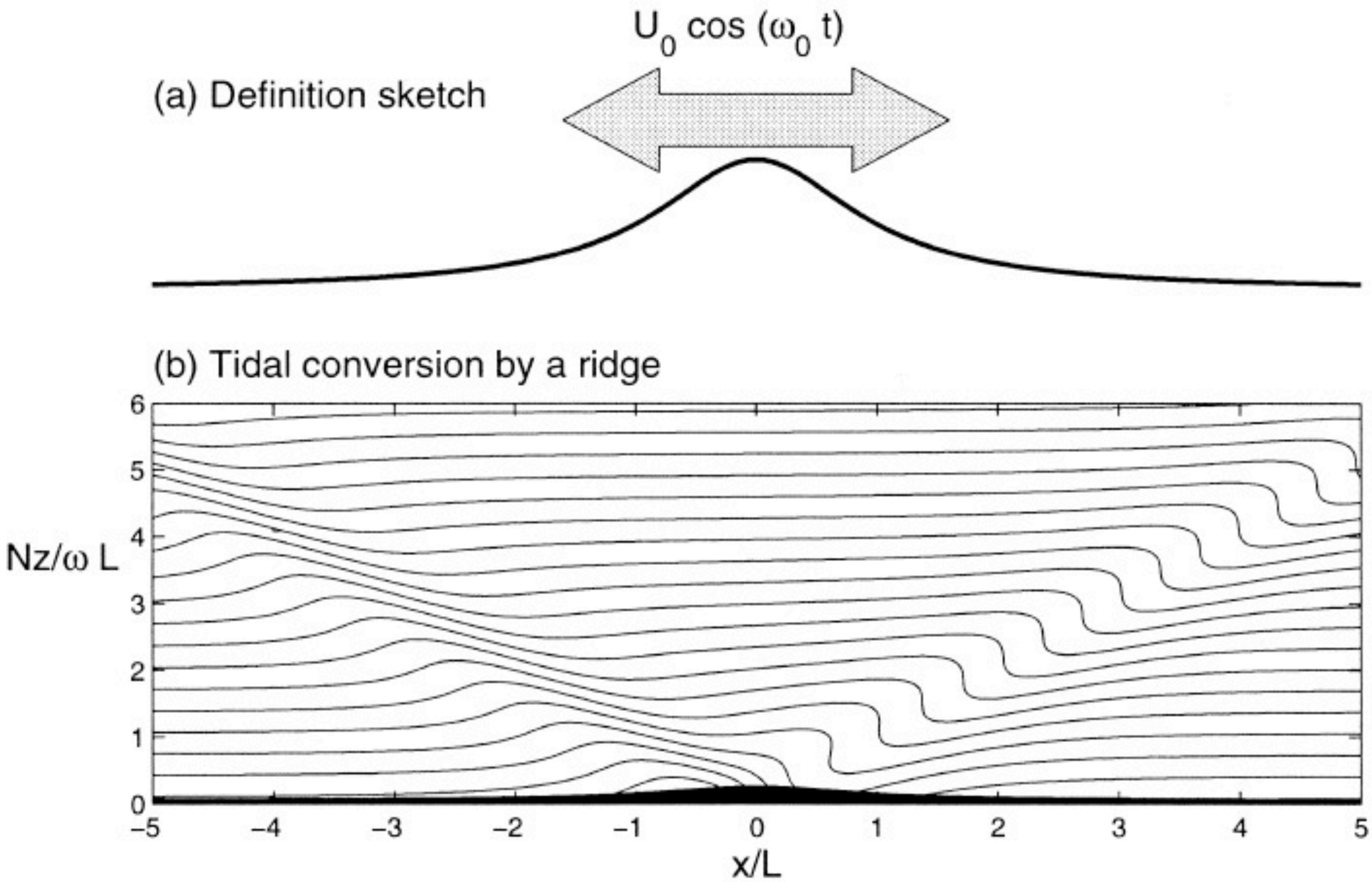
wave attractor ⇔ black hole [M. Rieutord]

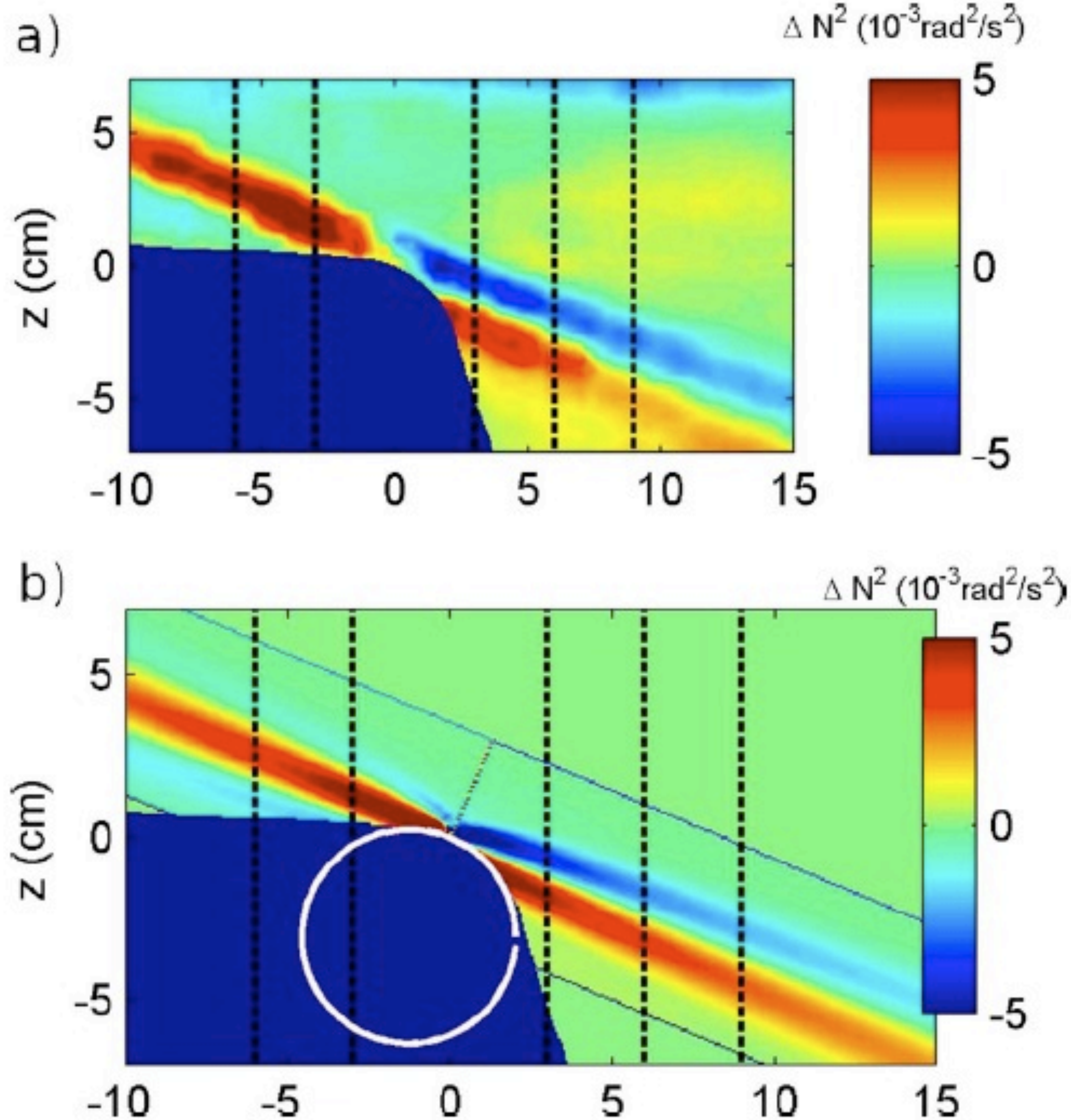
Mixed metaphors

wave attractor ⇔ black hole [M. Rieutord]

critical latitude ⇔ Hawking radiation [J. Goodman]

LLEWELLYN SMITH AND YOUNG



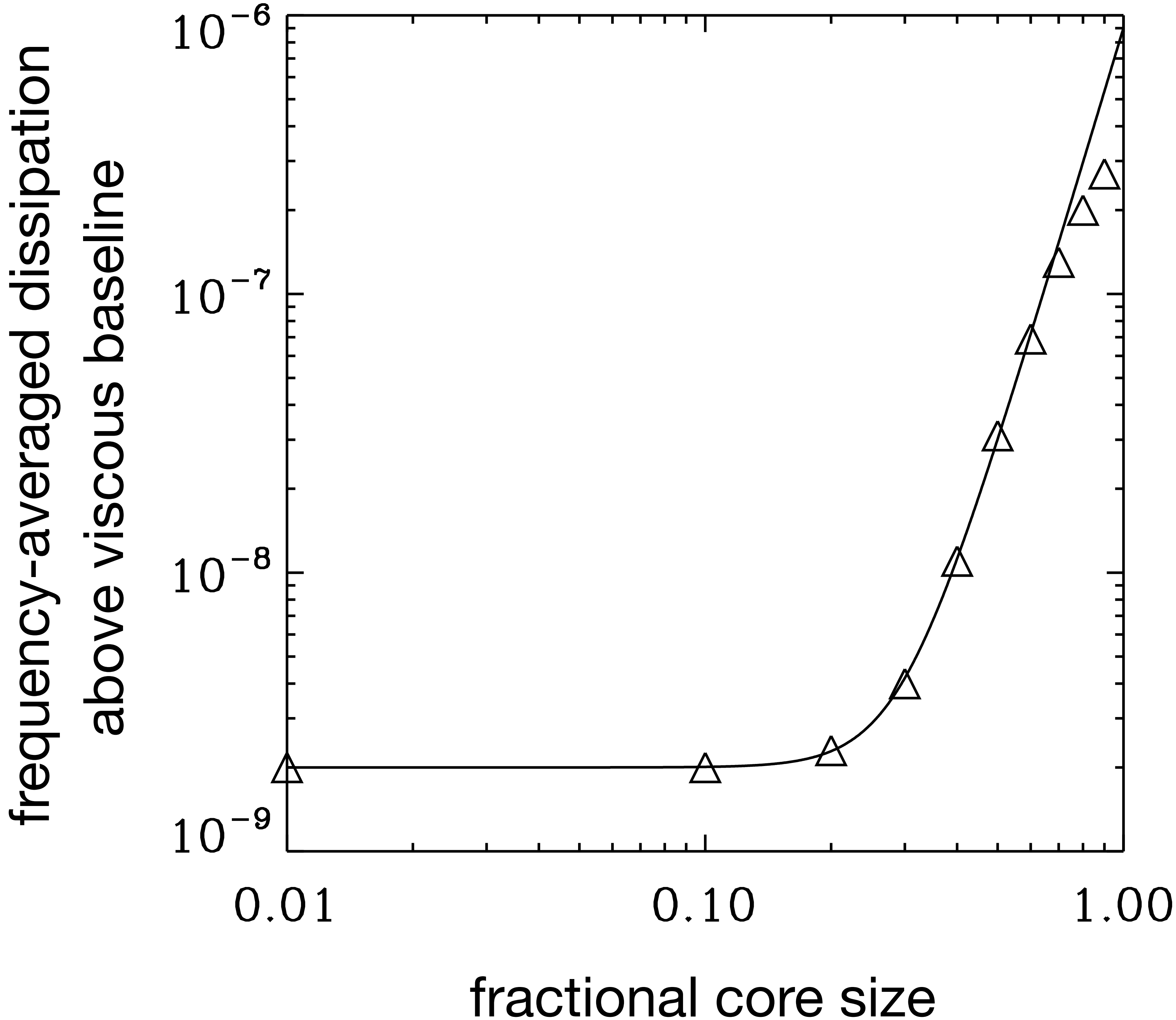


experiment

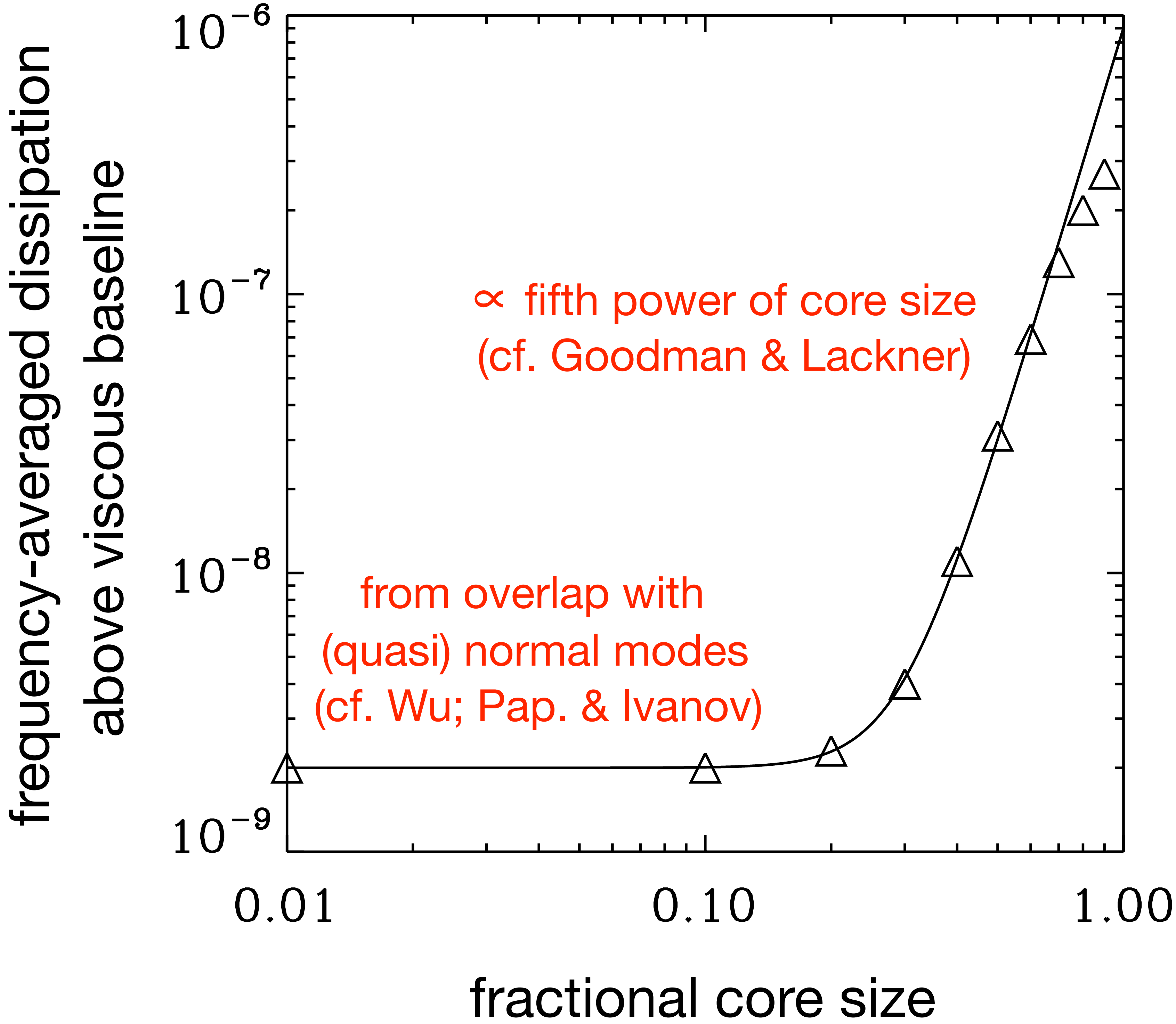
theory
(cf. Hurley)

Gostiaux & Dauxois (2007)

Dependence on core size



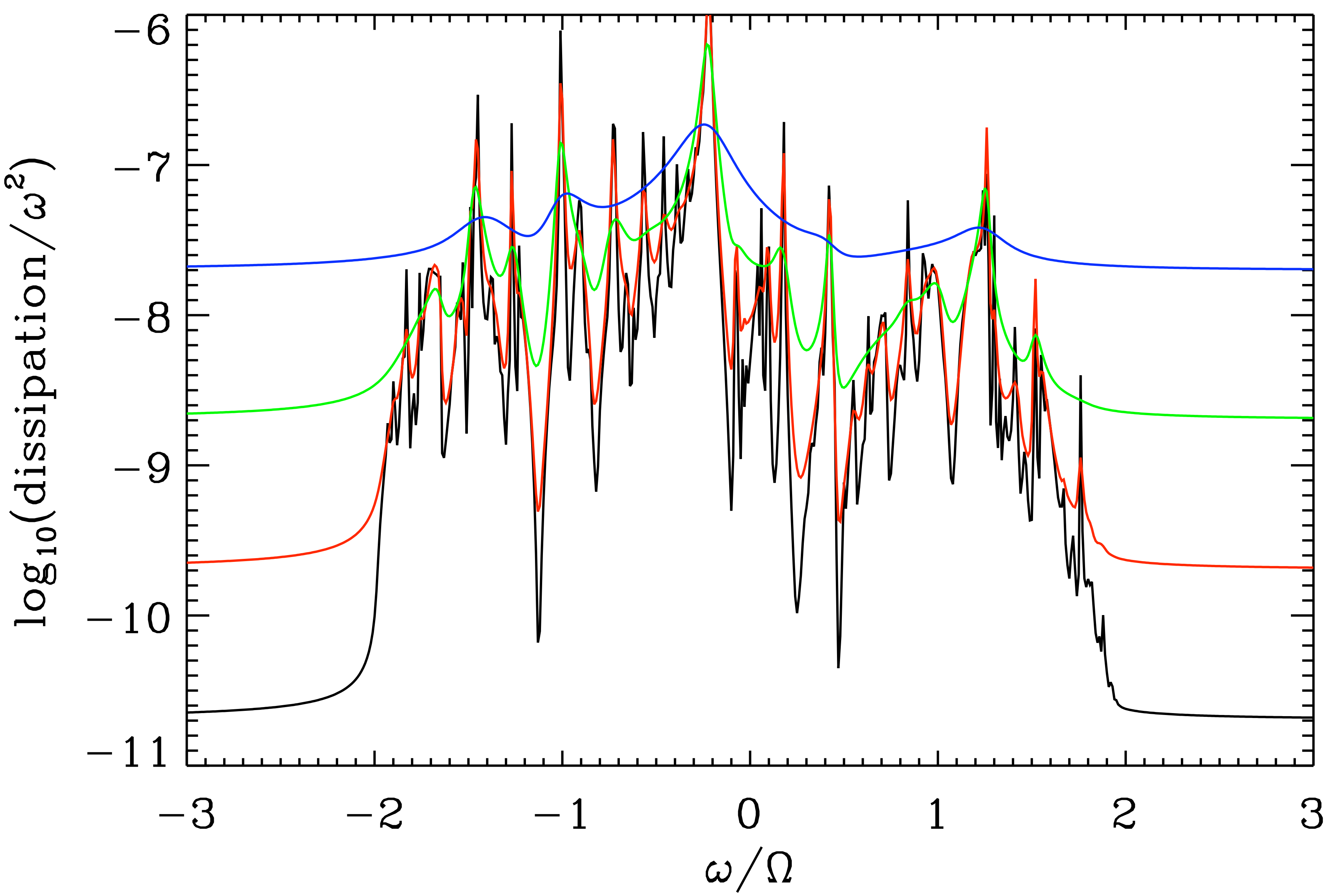
Dependence on core size

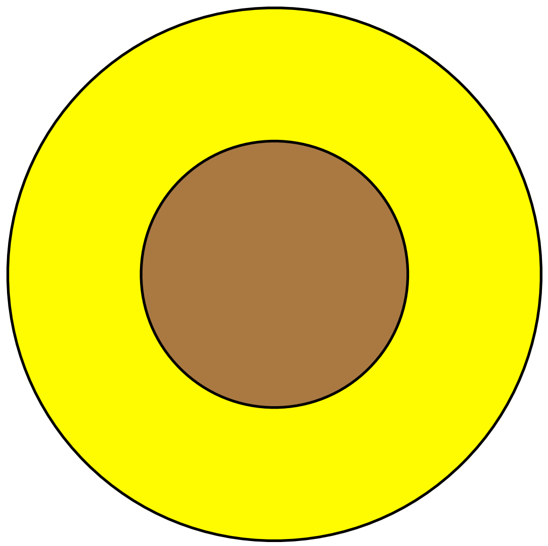


Rigid versus fluid core

Idealized problem : isentropic rotating fluid in spherical geometry

- Rigid core



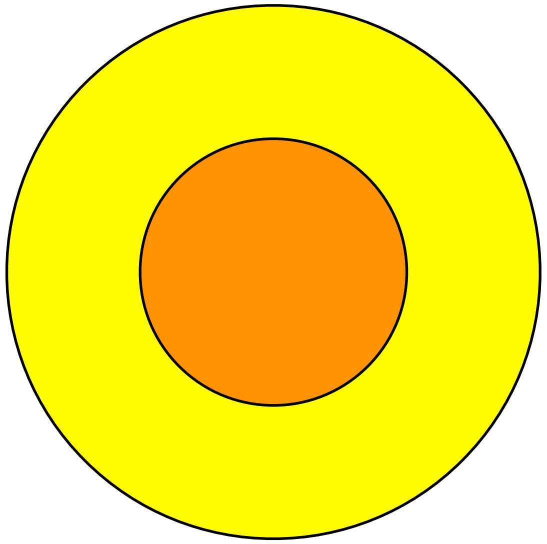


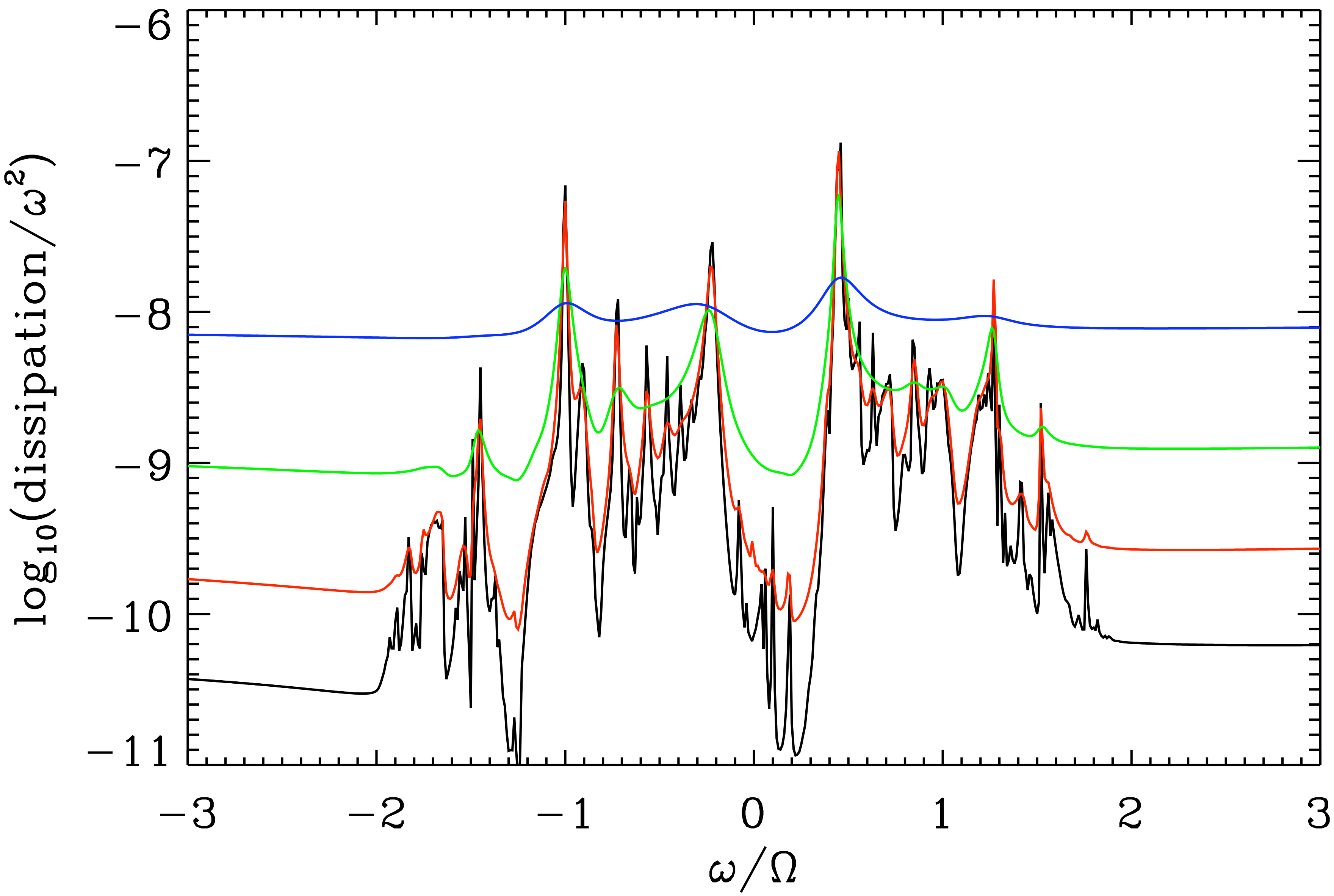
2011年12月2日金曜日

Rigid versus fluid core

Idealized problem : isentropic rotating fluid in spherical geometry

- Fluid core, density jump by factor 2





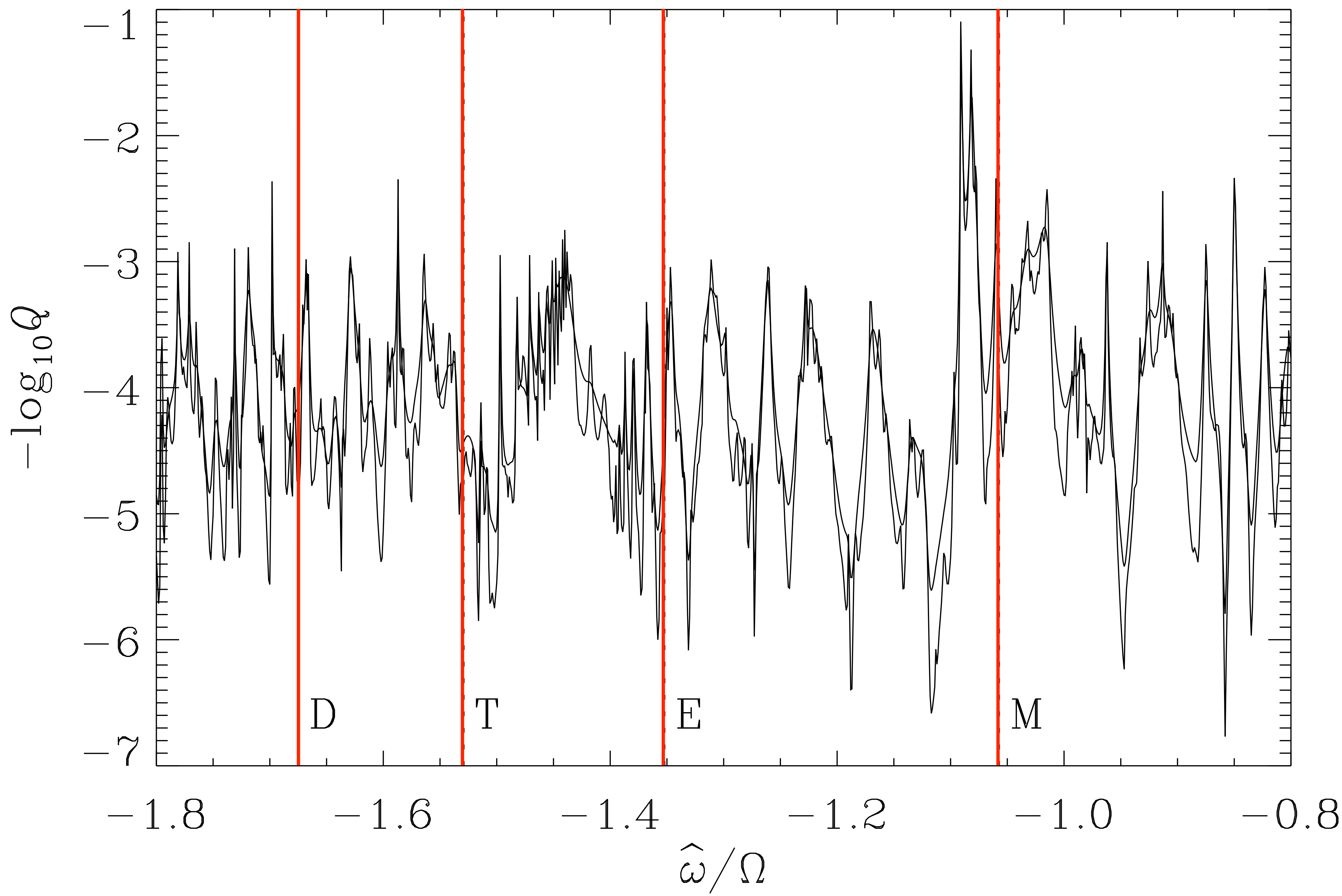
Responses of spheres and shells

- Full spheres with smooth density profiles support normal modes
- Some tidal overlap with normal modes occurs, leading to resonant peaks in the response, if the density is non-uniform
- Presence of a core and/or density jumps enhances tidal response but concepts of normal modes and resonance are less relevant
- Enhanced dissipation for tidal frequencies (in rotating frame) $-2\Omega < \omega < 2\Omega$ relevant for synchronization and circularization
- Frequency-averaged Q strongly dependent on internal stucture but not on viscosity; for intermediate core sizes,

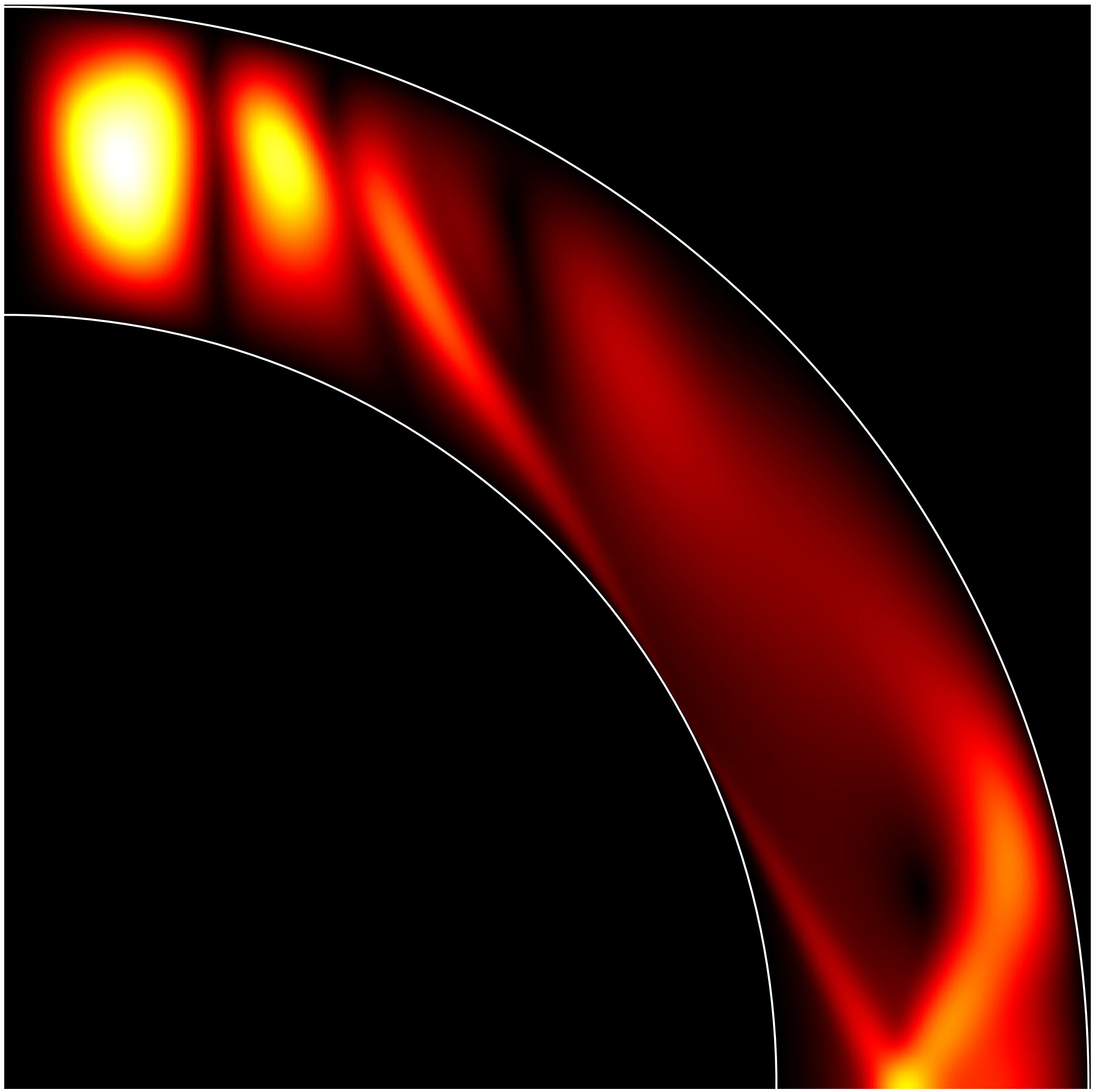
$$\left\langle \frac{1}{Q'} \right\rangle_\omega \approx \left(\frac{R_\mathrm{c}}{R_\mathrm{p}} \right)^5 \left(\frac{\Omega}{\Omega_\mathrm{dyn}} \right)^2$$

- Strong frequency dependence in cases of low viscosity

INERTIAL WAVES IN SATURN



STELLAR APPLICATION



inertial-wave response of convective zone

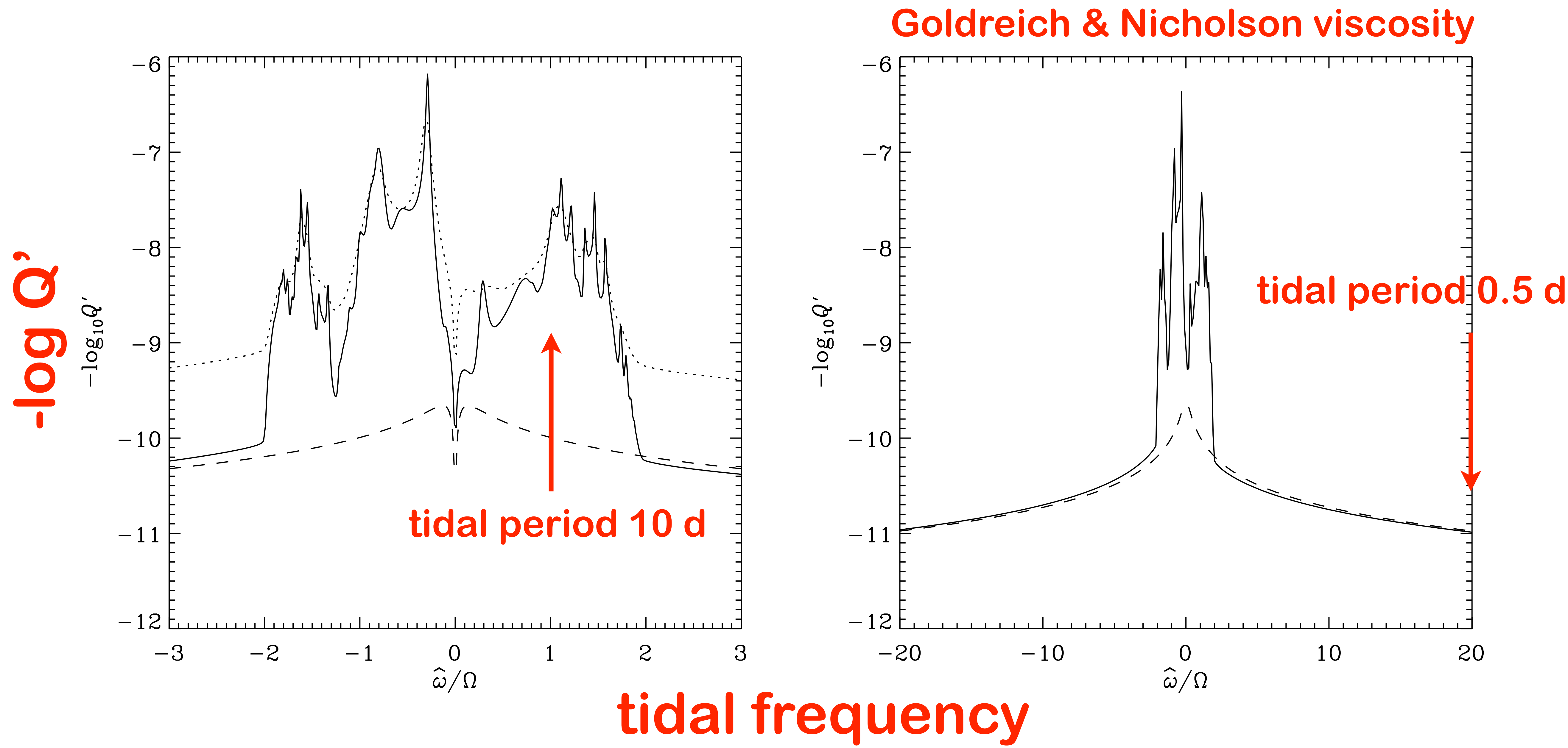
tidal frequency equal to spin frequency

relevant to binary circularization but not planet hosts

Inertial waves in a solar-type star

Ogilvie & Lin (2007)

- solar model, but spin period 10 days
- dissipation in convective zone only



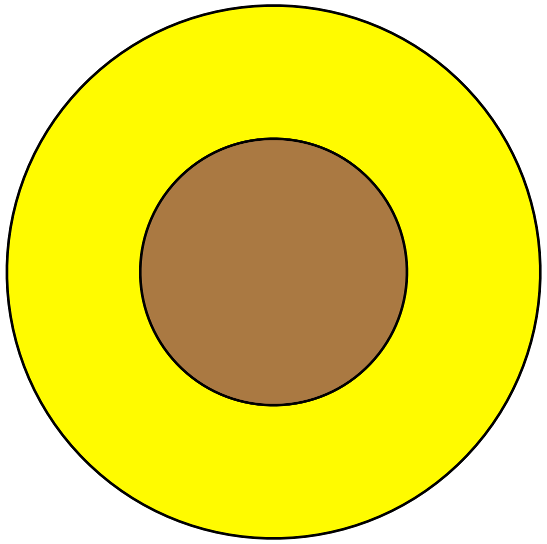
Impulsive forcing of inertial waves

- Consider inertial waves driven by body force $\propto \delta(t)$
- Impulsive response is smooth and readily computable (ODEs)
- Will subsequently resolve into complicated pattern of inertial waves which may dissipate through linear or nonlinear processes
- (Kinetic) energy of impulsive response gives a broad frequency average of the response function
- Equivalent to frequency-average of tidal time lag, or $\frac{1}{\omega Q}$
- Robust result, dependent only on gross structure
- Also more directly applicable to highly eccentric orbits

Impulsive energy transfer / frequency-averaged dissipation

- Homogeneous fluid body with rigid core
  - Sectoral harmonics $m = l$

$$\hat{E} \propto \frac{\alpha^{2l+1}}{1 - \alpha^{2l+1}} \qquad \alpha = \frac{r_{\rm in}}{R}$$



  - Tesseral harmonics $m < l$

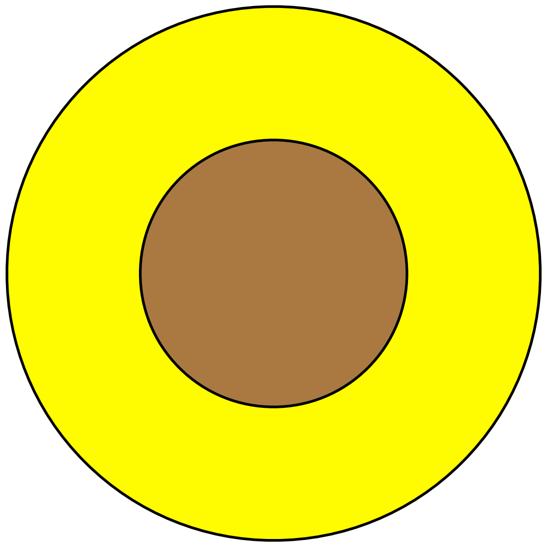
$$\hat{E} \propto \frac{1}{1 - \alpha^{2l+1}} \qquad (+\text{ term as above})$$

but beware trivial inertial modes with $l = 2$

Impulsive energy transfer / frequency-averaged dissipation

- Homogeneous fluid body with rigid core
  - Sectoral harmonics $m = l$

$$\hat{E} \propto \frac{\alpha^{2l+1}}{1-\alpha^{2l+1}} \qquad \alpha = \frac{r_{\rm in}}{R}$$

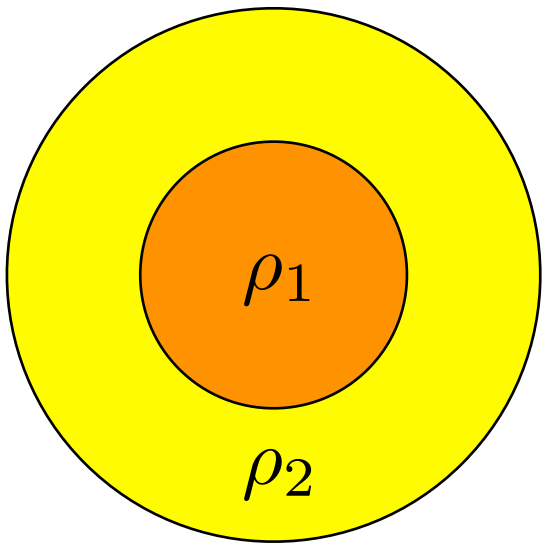


  - Tesseral harmonics $m < l$

$$\hat{E} \propto \frac{1}{1-\alpha^{2l+1}} \qquad (+ \text{ term as above})$$

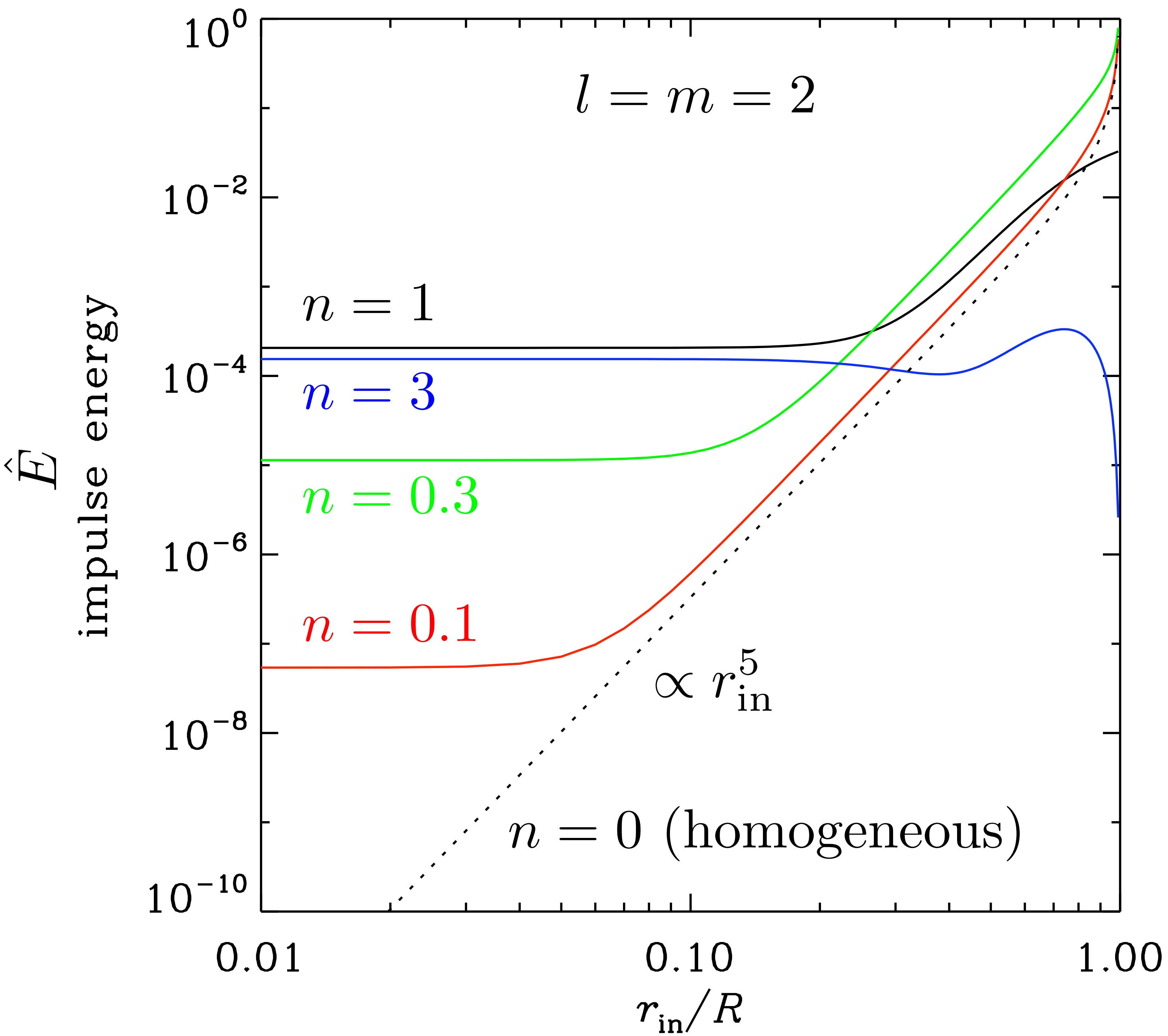
but beware trivial inertial modes with $l = 2$

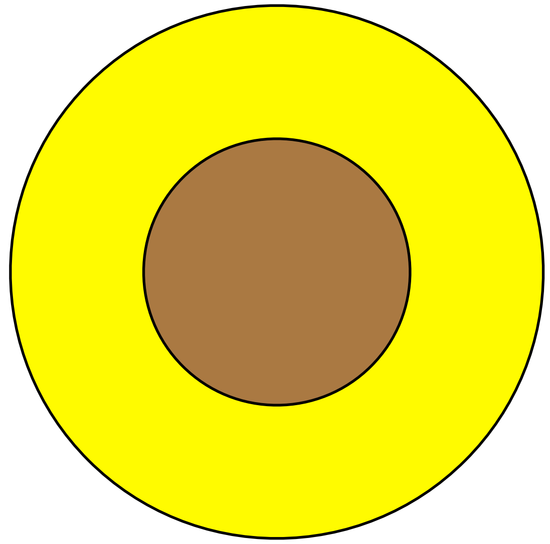
- Two homogeneous fluids
  - Similar but weaker result
  - Strengthened if densities differ greatly



Impulsive energy transfer / frequency-averaged dissipation

- Polytrope with rigid core $p \propto \rho^{1+1/n}$





Processes affecting inertial waves at small scales

Viscous dissipation

Effective viscous dissipation by turbulent convection

Deflection by meridional circulation

Deflection (scattering) by turbulent convection (random medium)

Interaction with magnetic fields, and Ohmic dissipation

Wave breaking / parametric instability

Imperfect reflection from boundaries / interfaces

Viscous and Ohmic dissipation

Local dispersion relation in incompressible MHD

$$(\omega_\nu \omega_\eta - \omega_\mathrm{a}^2)^2 = 4(\hat{\boldsymbol{k}} \cdot \boldsymbol{\Omega})^2 \omega_\eta^2$$

$$\omega_\nu = \omega - \boldsymbol{k} \cdot \boldsymbol{u} + \mathrm{i}\nu k^2$$

$$\omega_\eta = \omega - \boldsymbol{k} \cdot \boldsymbol{u} + \mathrm{i}\eta k^2 \qquad \eta = \frac{1}{\mu_0 \sigma}$$

$$\omega_\mathrm{a} = \boldsymbol{k} \cdot \boldsymbol{v}_\mathrm{a}$$

$$\boldsymbol{v}_\mathrm{a} = (\mu_0 \rho)^{-1/2} \boldsymbol{B}$$

Viscous and Ohmic dissipation

$$(\omega_\nu \omega_\eta - \omega_{\rm a}^2)^2 = 4(\hat{\boldsymbol{k}} \cdot \boldsymbol{\Omega})^2 \omega_\eta^2$$

Magnetic Prandtl number $\quad \mathrm{Pm} = \frac{\nu}{\eta} \ll 1$

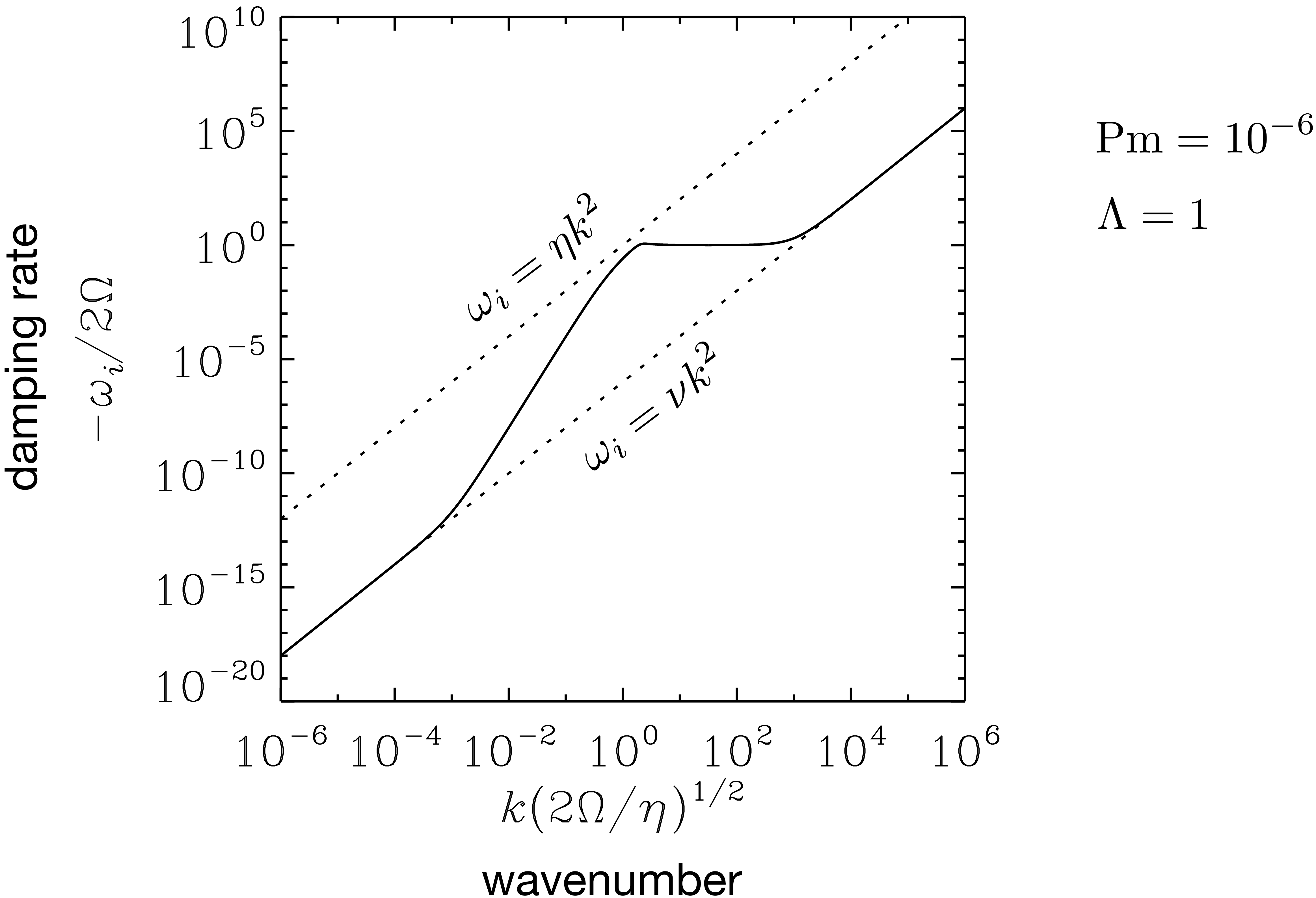
Elsasser number $\quad \Lambda = \frac{v_{\rm a}^2}{2\Omega\eta} = O(1)$

Magnetic coupling scale $\quad k_{\rm a} = \frac{2\Omega}{v_{\rm a}}$

Resistive scale $\quad k_\eta = \left(\frac{2\Omega}{\eta}\right)^{1/2} = \Lambda^{1/2} k_{\rm a}$

Viscous scale $\quad k_\nu = \left(\frac{2\Omega}{\nu}\right)^{1/2} = \mathrm{Pm}^{-1/2} k_\eta$

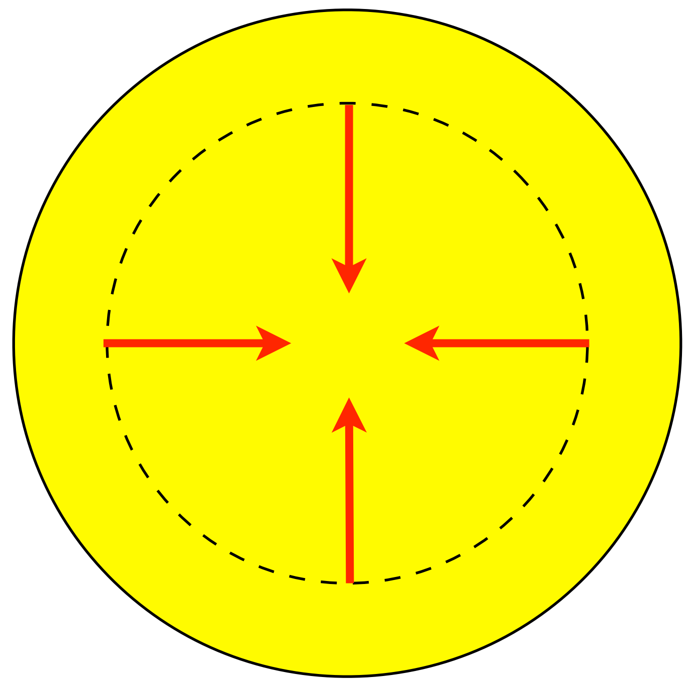
Viscous and Ohmic dissipation



Breaking internal gravity waves at the centre of a star

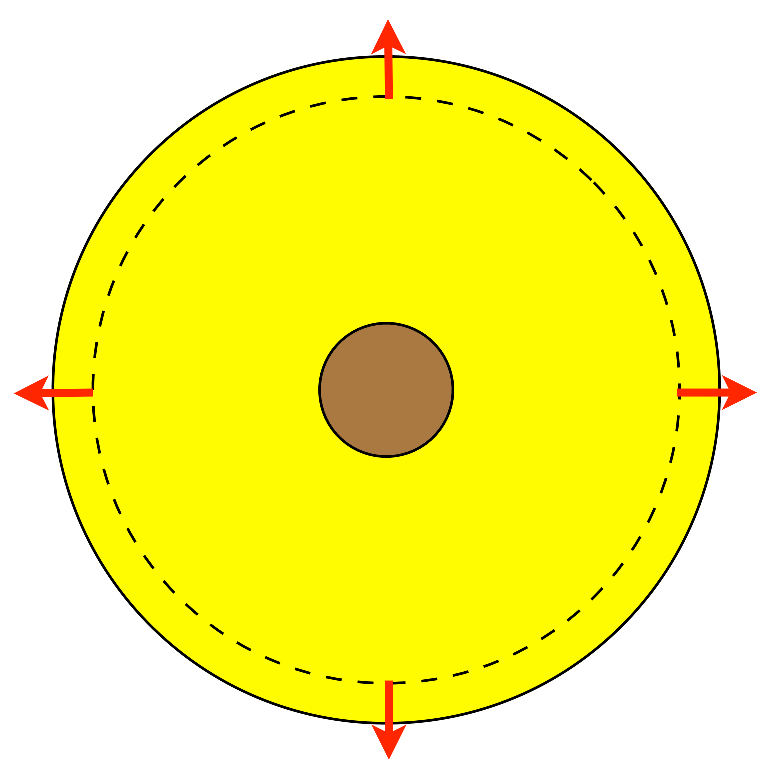
Inertia-gravity waves in radiative regions

Solar-type star



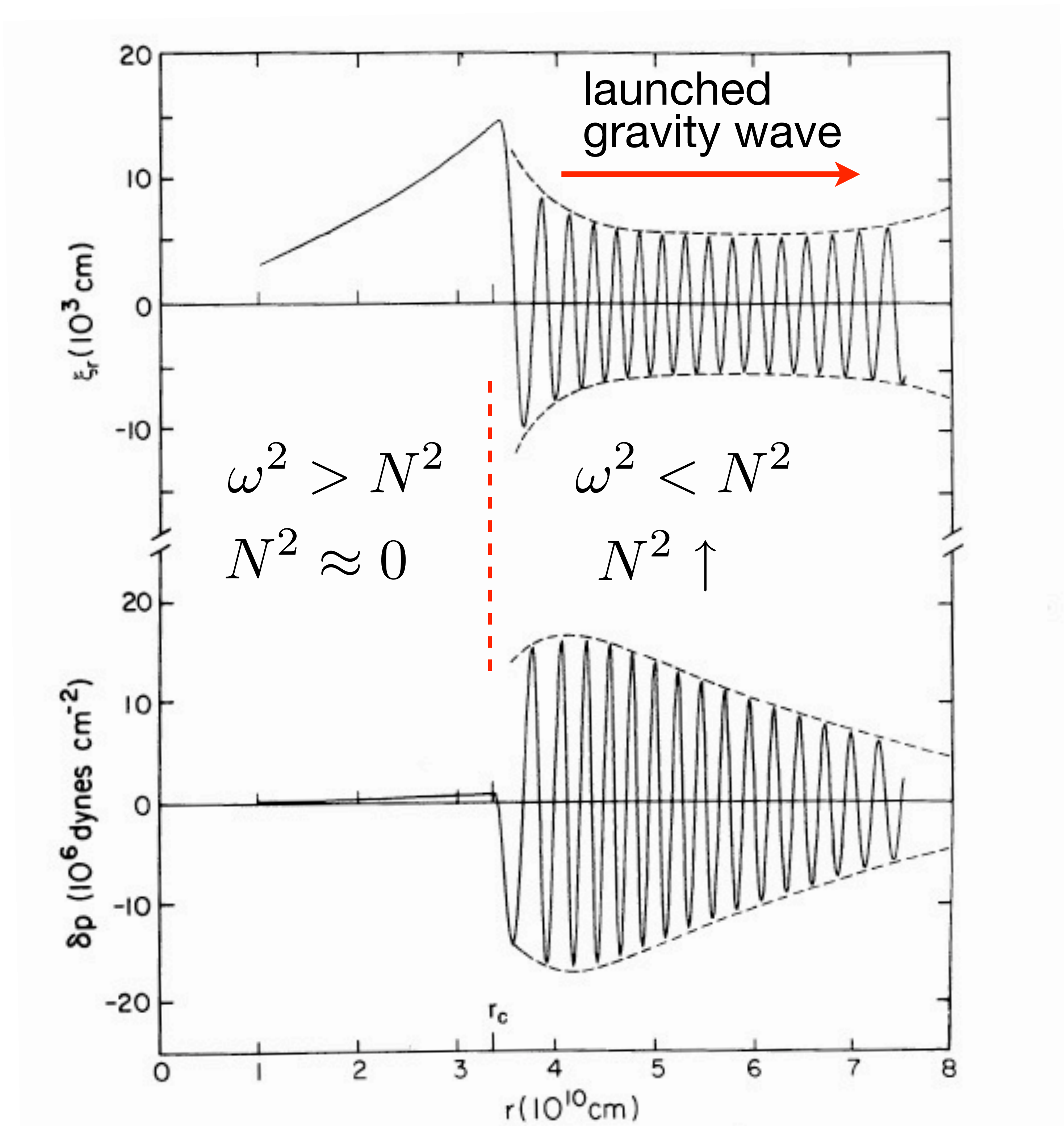
Goodman & Dickson 1998
Terquem et al. 1998
Savonije & Witte 2002
Ogilvie & Lin 2007
Barker & Ogilvie 2010

Irradiated giant planet



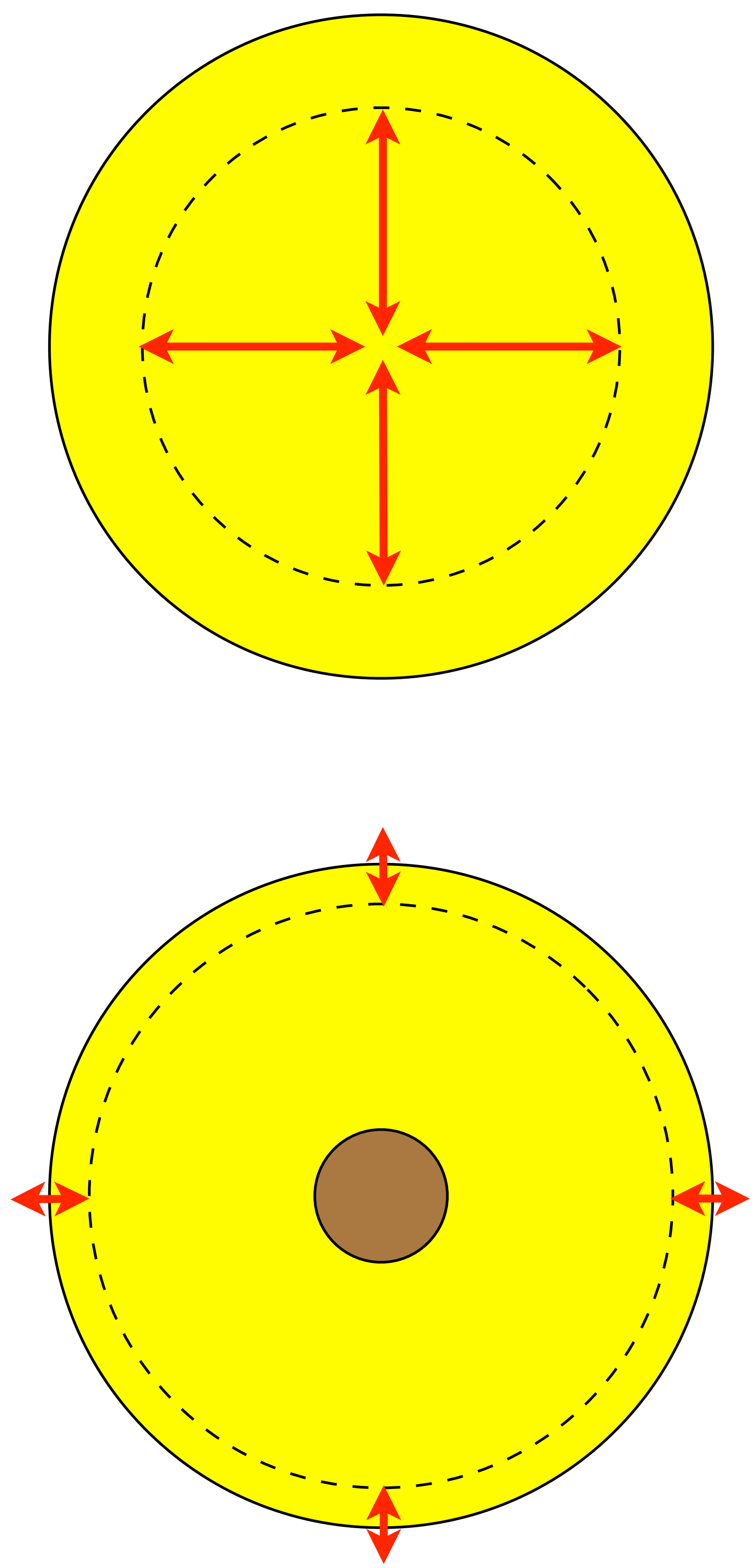
[Ioannou & Lindzen 1993]
Lubow et al. 1997
Ogilvie & Lin 2004
[Gu & Ogilvie 2009]
[Arras & Socrates 2010]

Inertia-gravity waves in radiative regions



Goldreich & Nicholson 1989

Inertia-gravity waves: resonant modes or breaking waves?



Inertia-gravity waves in radiative regions

Savonije & Witte 2002

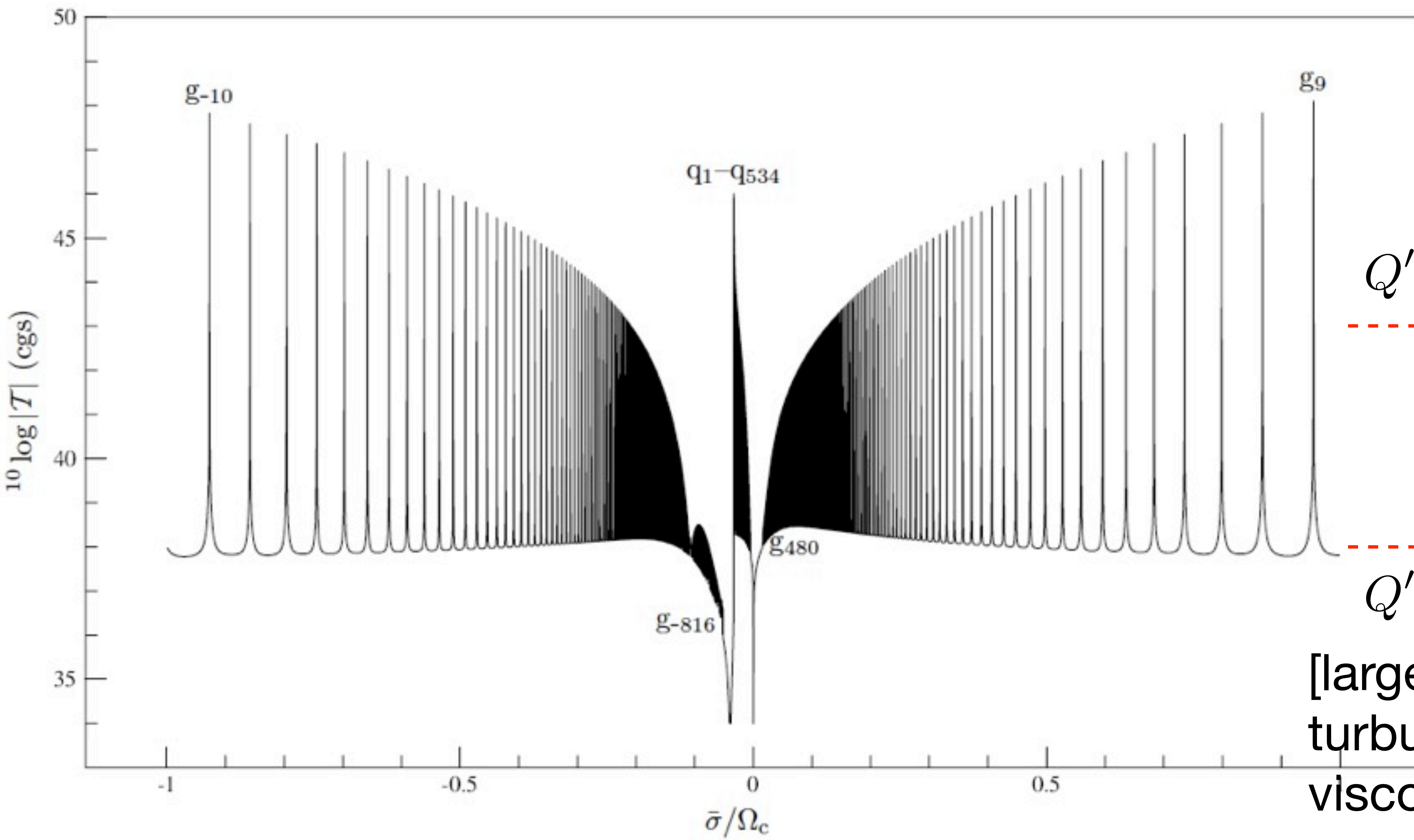
- linear tidal response of 1-solar mass star
- realistic stellar model and evolution
- Coriolis force (traditional approximation)
- radiative diffusion
- turbulent viscosity [large?]

Inertia-gravity waves in radiative regions (star)

Savonije & Witte 2002 (cf. Terquem et al. 1998)

- resonant excitation of normal modes

log (tidal torque)



tidal frequency

$Q' \approx 10^2$

$Q' \approx 10^7$

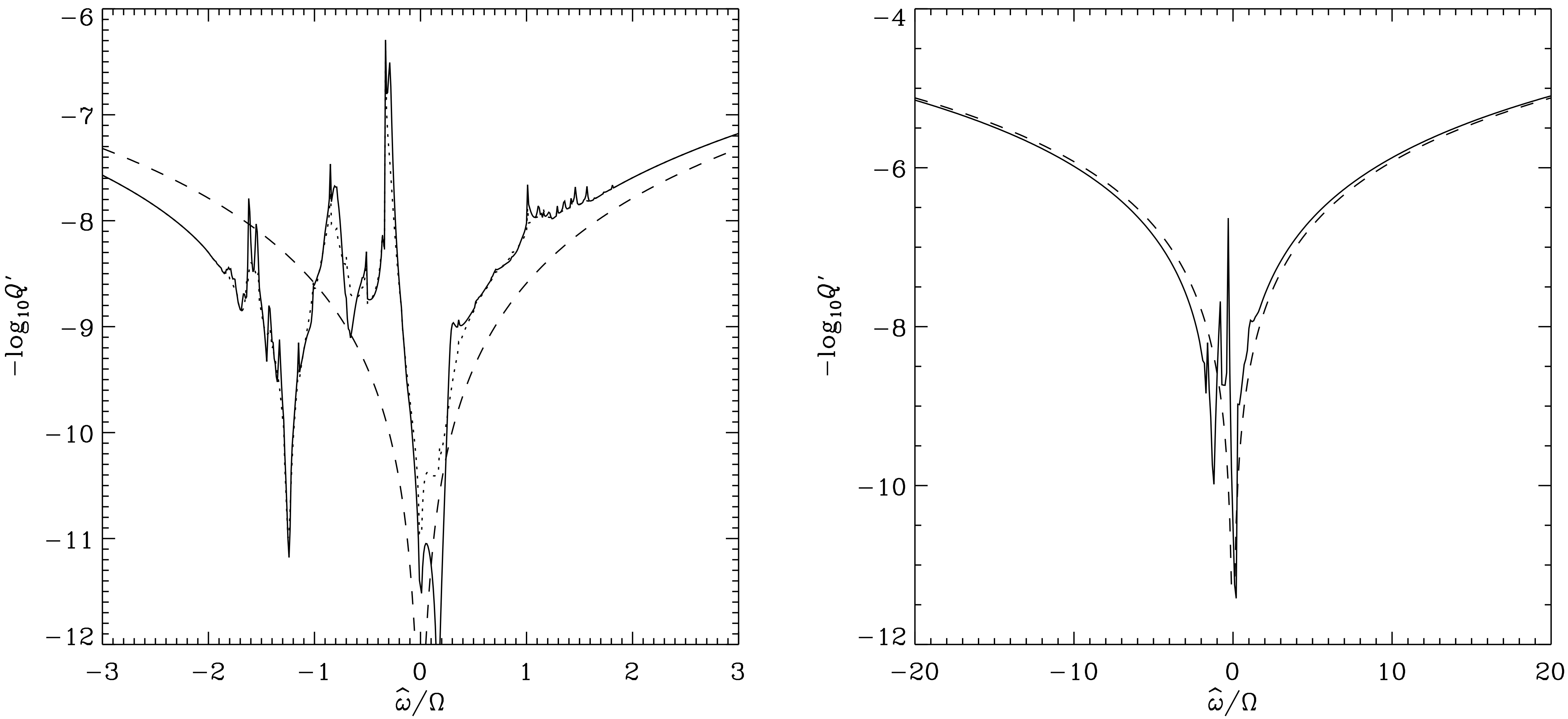
[large turbulent viscosity?]

Inertia-gravity waves in radiative regions (star)

Ogilvie & Lin 2007 (cf. Goodman & Dickson 1998)

- assumes waves do not reflect from stellar centre

-log Q'



tidal frequency

BREAKING GRAVITY WAVES

Near stellar centre :

$$\rho = \rho_0 + \rho_2 r^2 + \cdots$$

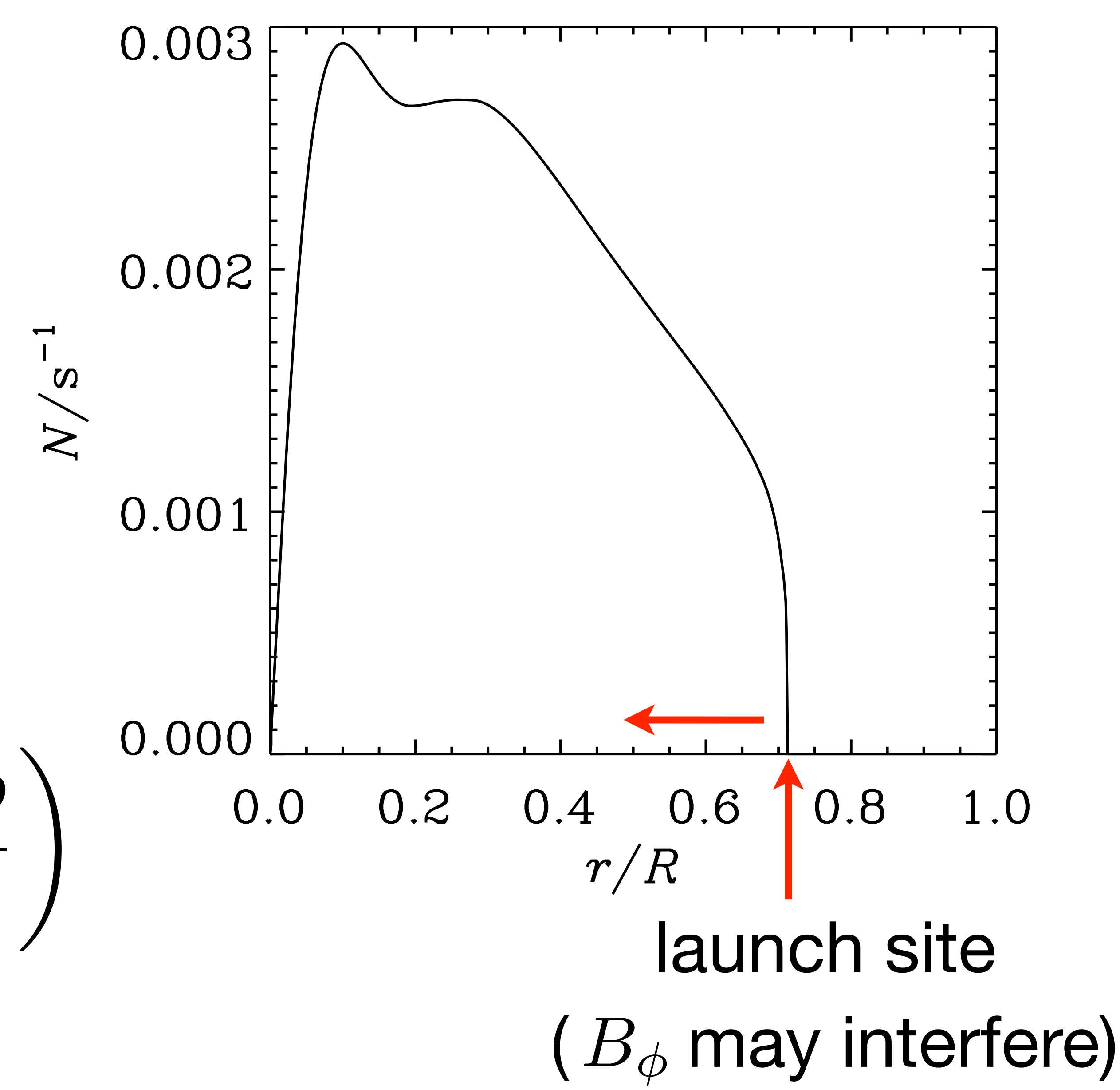
$$p = p_0 + p_2 r^2 + \cdots$$

$$g = g_1 r + g_3 r^3 + \cdots$$

Brunt-Väisälä frequency :

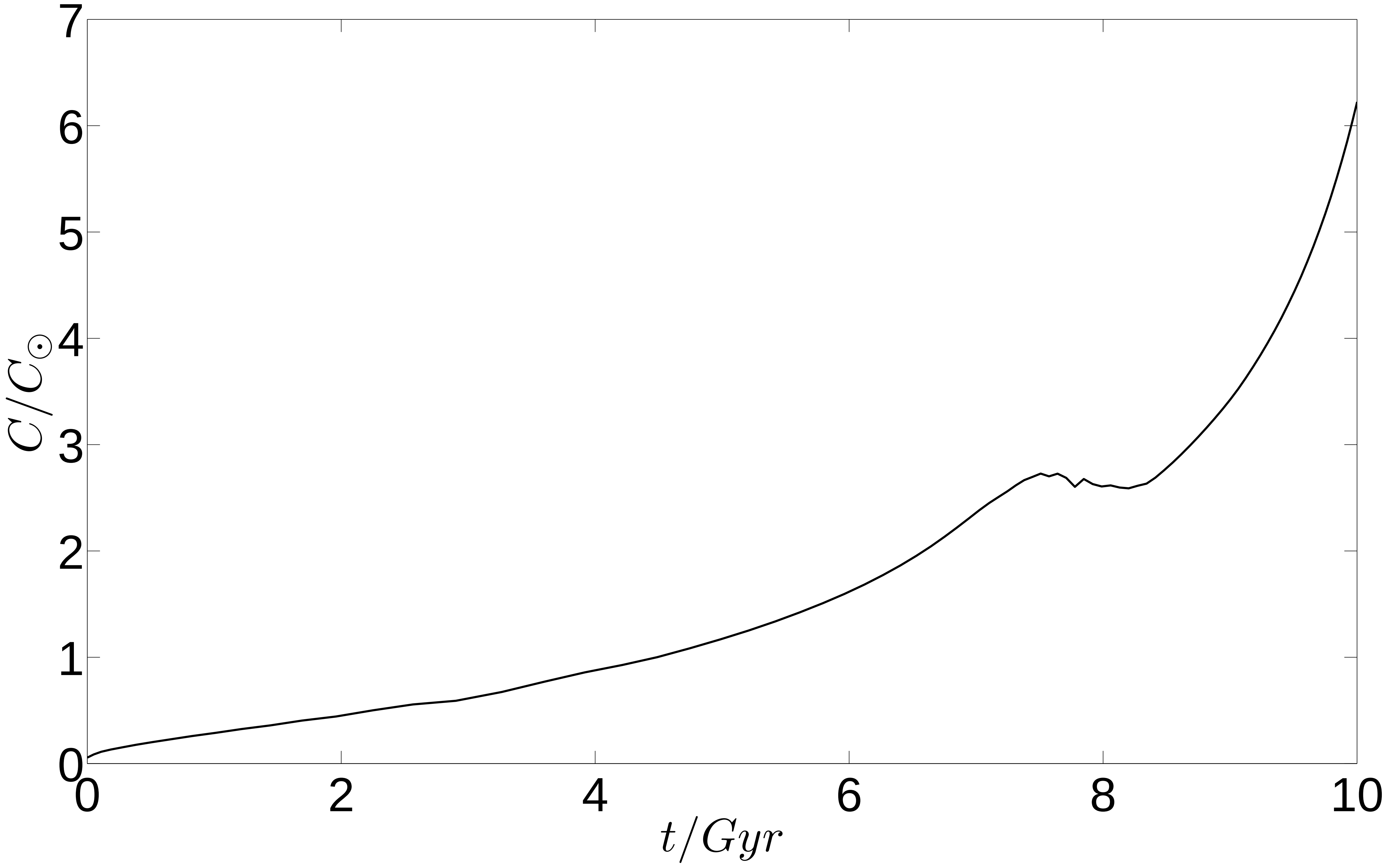
$$N^2 = g \left(\frac{1}{\Gamma_1} \frac{\mathrm{d} \ln p}{\mathrm{d}r} - \frac{\mathrm{d} \ln \rho}{\mathrm{d}r} \right)$$

$$N = N_1 r + N_3 r^3 + \cdots$$



N_1 **generally increases with stellar mass and age**

N_1 versus age for the Sun



BREAKING GRAVITY WAVES

Quasi-Boussinesq system :

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = -\boldsymbol{\nabla} p + b\,\boldsymbol{r}$$

$$\frac{\partial b}{\partial t} + \boldsymbol{u} \cdot \boldsymbol{\nabla} b + N_1^2 \boldsymbol{u} \cdot \boldsymbol{r} = 0$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0$$

Exact solution in cylindrical geometry (2D "star") :

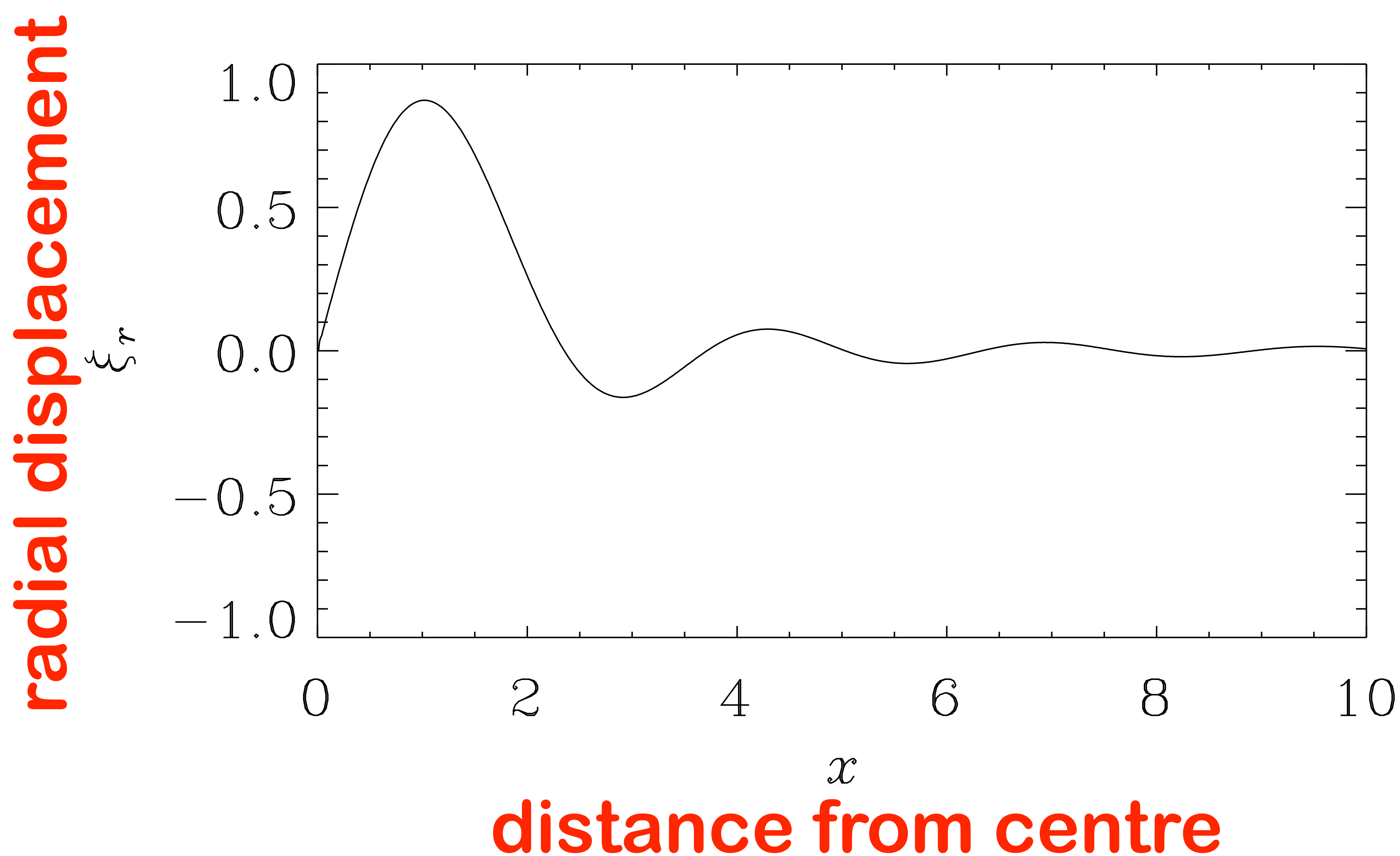
$$\psi \propto b \propto J_m(kr) \exp[\mathrm{i}m(\phi - \Omega_\mathrm{p} t)] \qquad k = N_1/\Omega_\mathrm{p}$$

Wave overturns if $\dfrac{u_\phi}{r} > \Omega_\mathrm{p}$

BREAKING GRAVITY WAVES

Barker & Ogilvie (2010)

cf. Goodman & Dickson (1998)

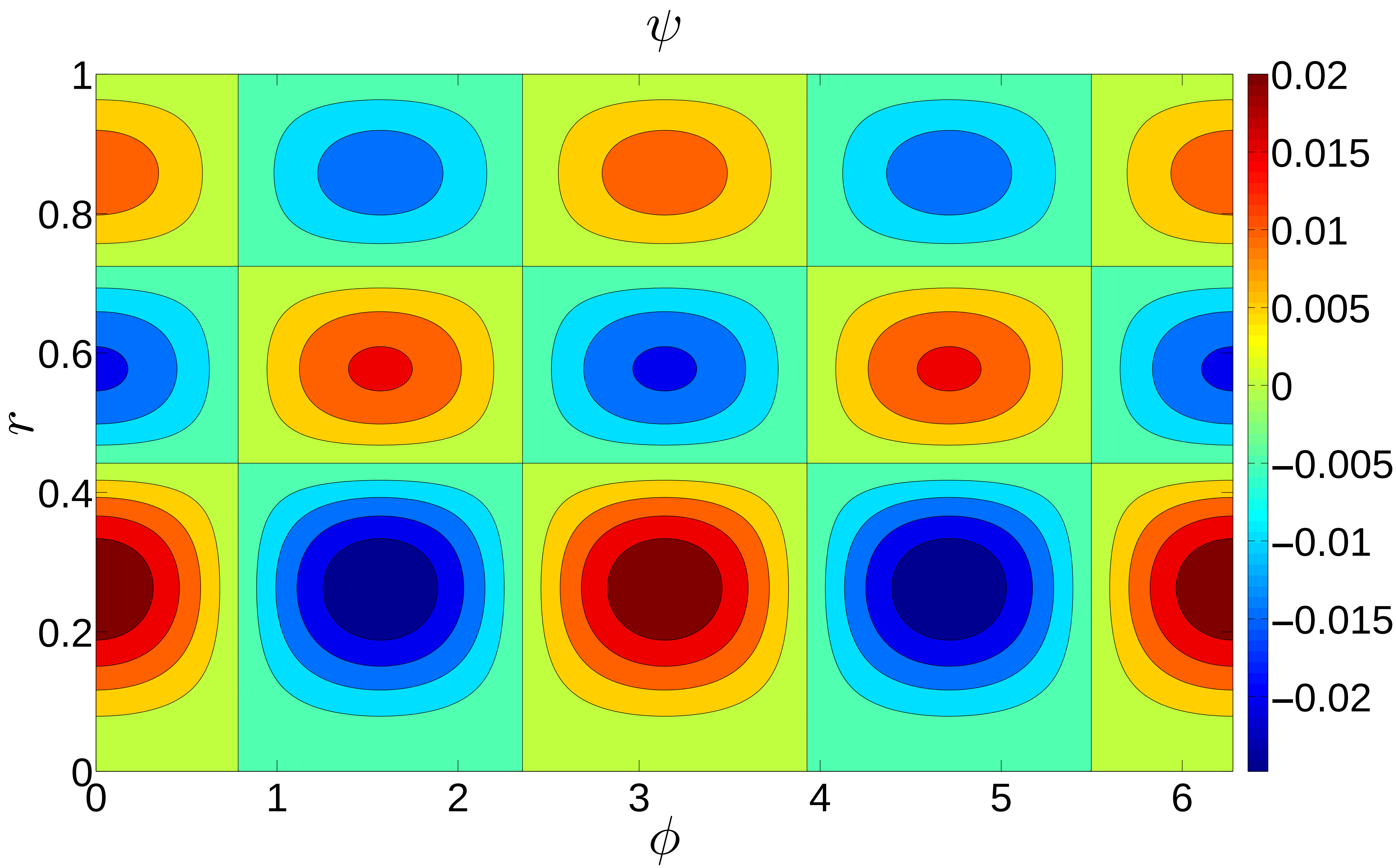


typical wavelength 0.001-0.01 R_{Sun}

Stability analysis of gravity waves

Barker & Ogilvie (2011)

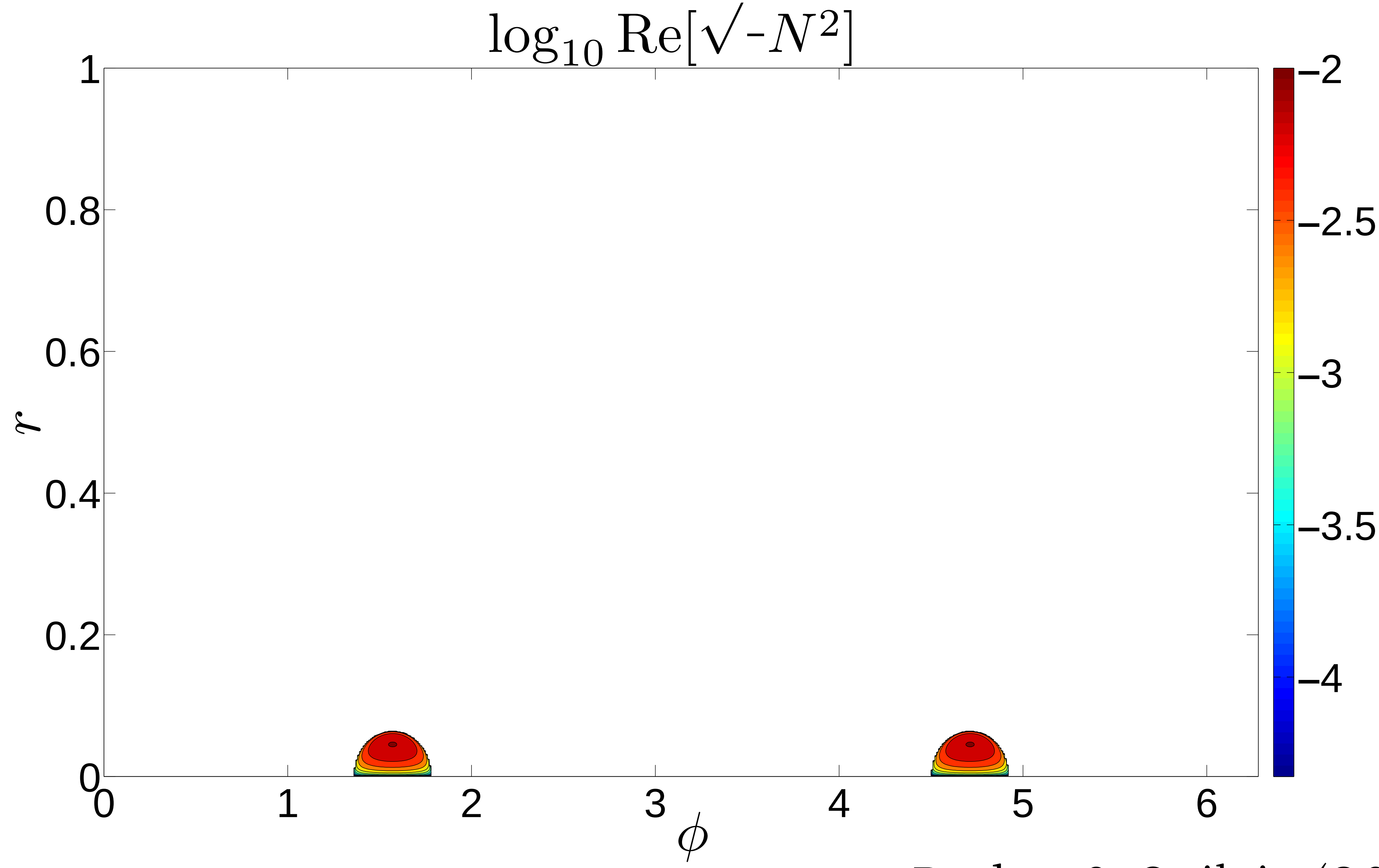
Primary wave is a steady non-axisymmetric flow in a rotating frame



Stability analysis of gravity waves

Contains convectively unstable regions if $A > 1$

$A = 1.1$

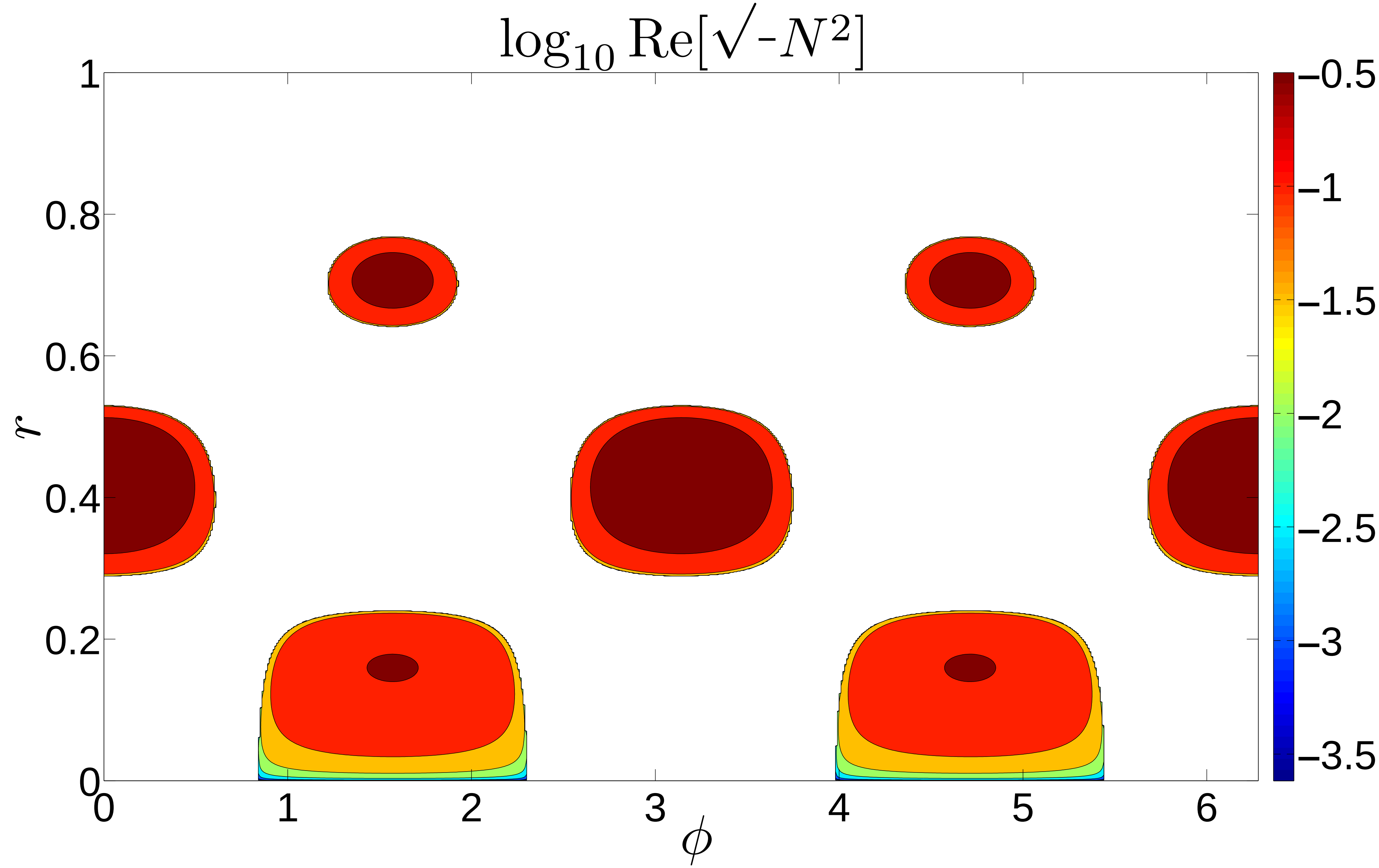


Barker & Ogilvie (2011)

Stability analysis of gravity waves

Contains convectively unstable regions if $A > 1$

$A = 10$



Barker & Ogilvie (2011)

Stability analysis of gravity waves

Stability analysis by spectral (Galerkin) method

$$\psi = \sum_{m=-\infty}^{\infty} \sum_{n=0}^{\infty} \psi_{mn} J_m(k_{kn} r)\, \mathrm{e}^{\mathrm{i}m\phi - \mathrm{i}\omega t}$$

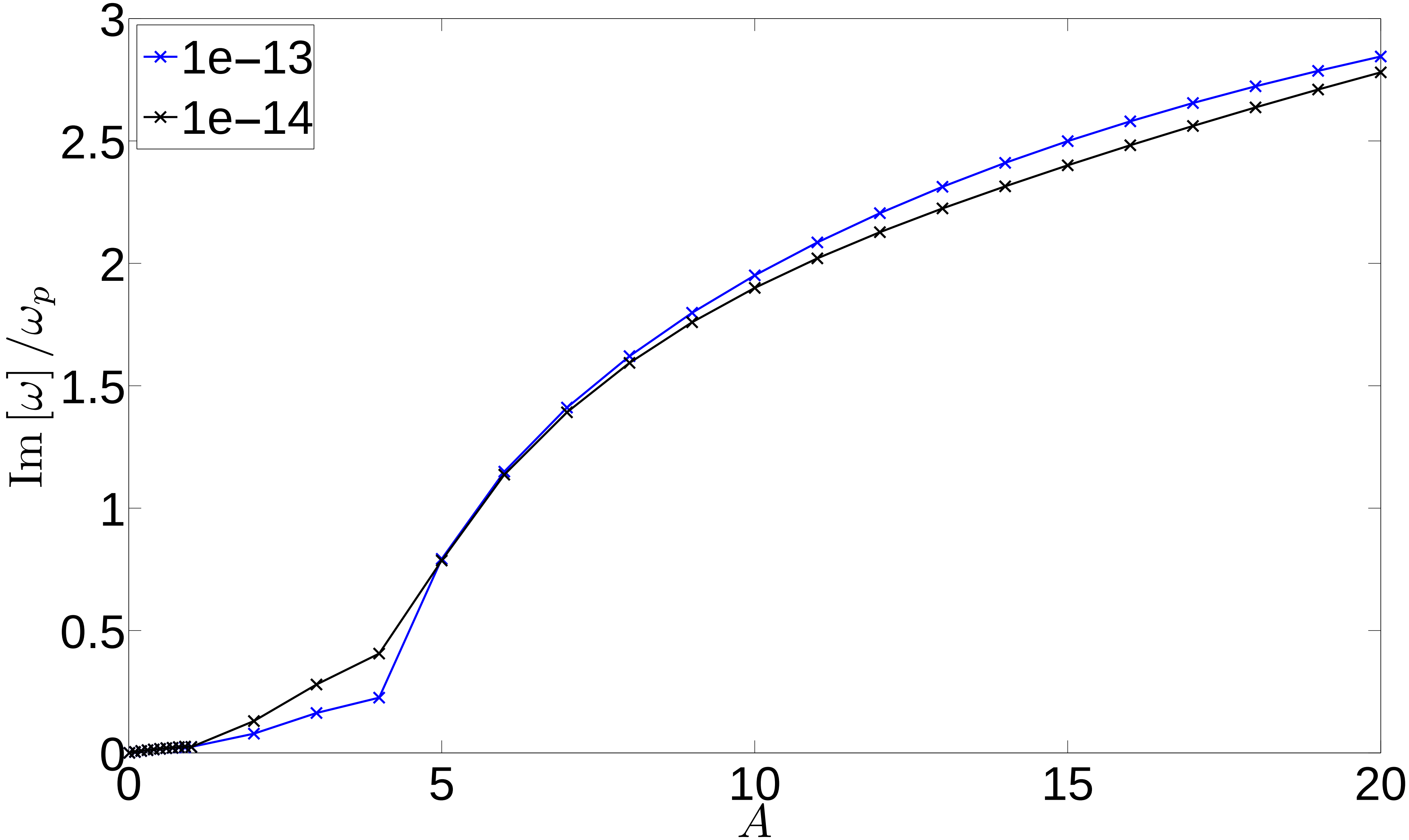
$$b = \sum_{m=-\infty}^{\infty} \sum_{n=0}^{\infty} b_{mn} J_m(k_{kn} r)\, \mathrm{e}^{\mathrm{i}m\phi - \mathrm{i}\omega t}$$

Include hyperdiffusion to smooth smallest scales

Stability analysis of gravity waves

Results for $A > 1$: initial stages of wave breaking

Growth rate vs primary amplitude

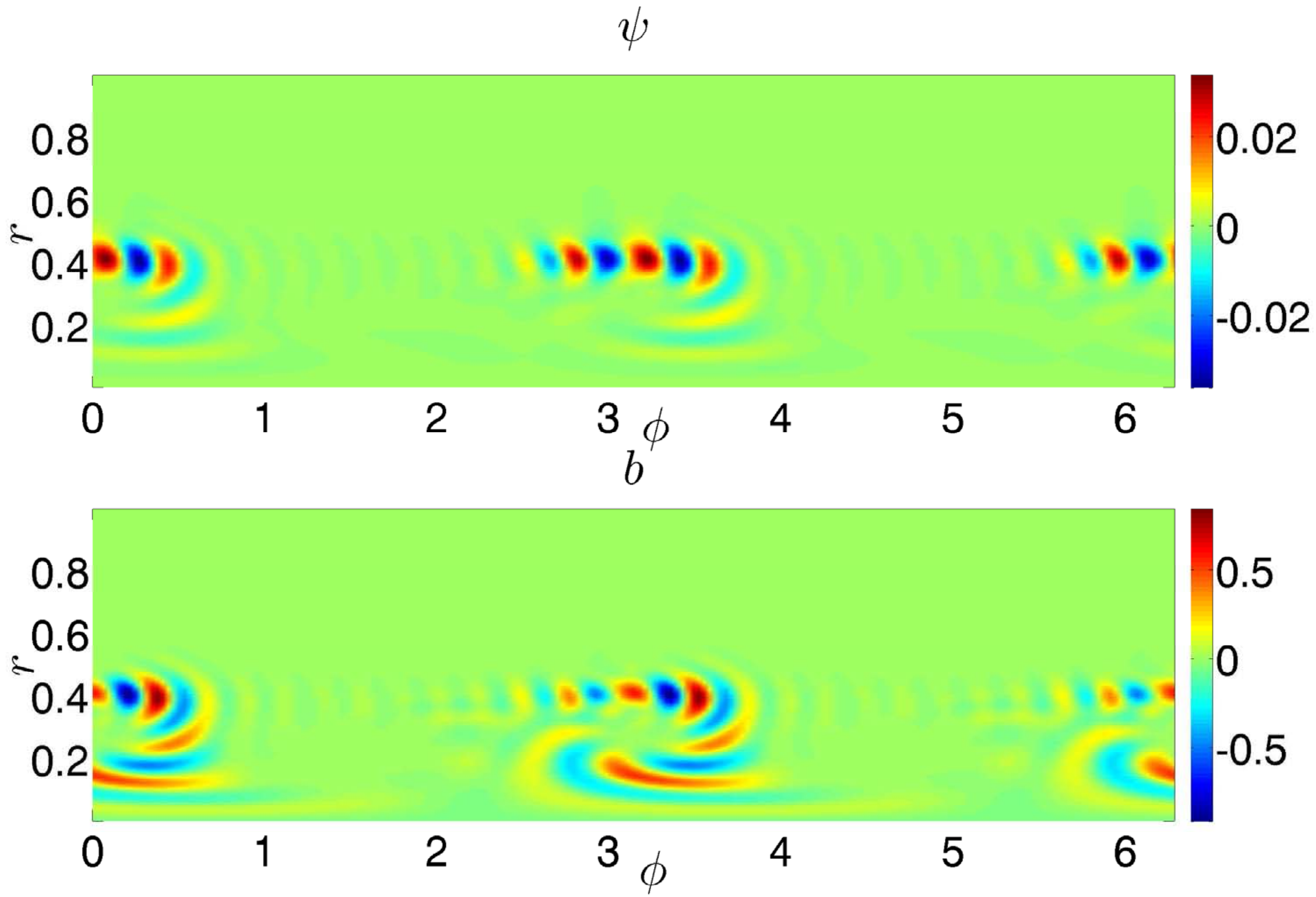


Barker & Ogilvie (2011)

Stability analysis of gravity waves

Results for $A > 1$: initial stages of wave breaking

Unstable mode for $A = 10$

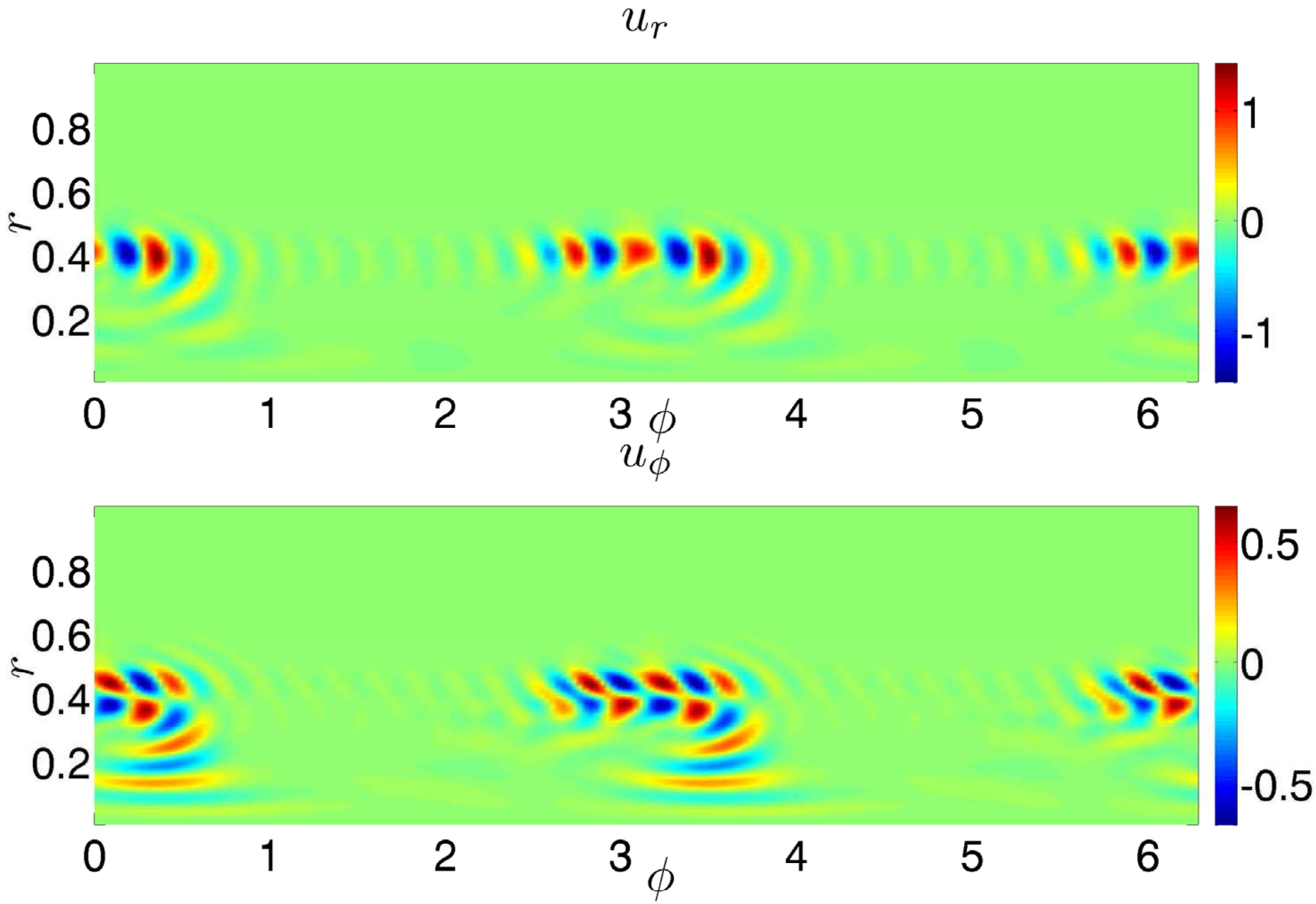


Barker & Ogilvie (2011)

Stability analysis of gravity waves

Results for $A > 1$: initial stages of wave breaking

Unstable mode for $A = 10$

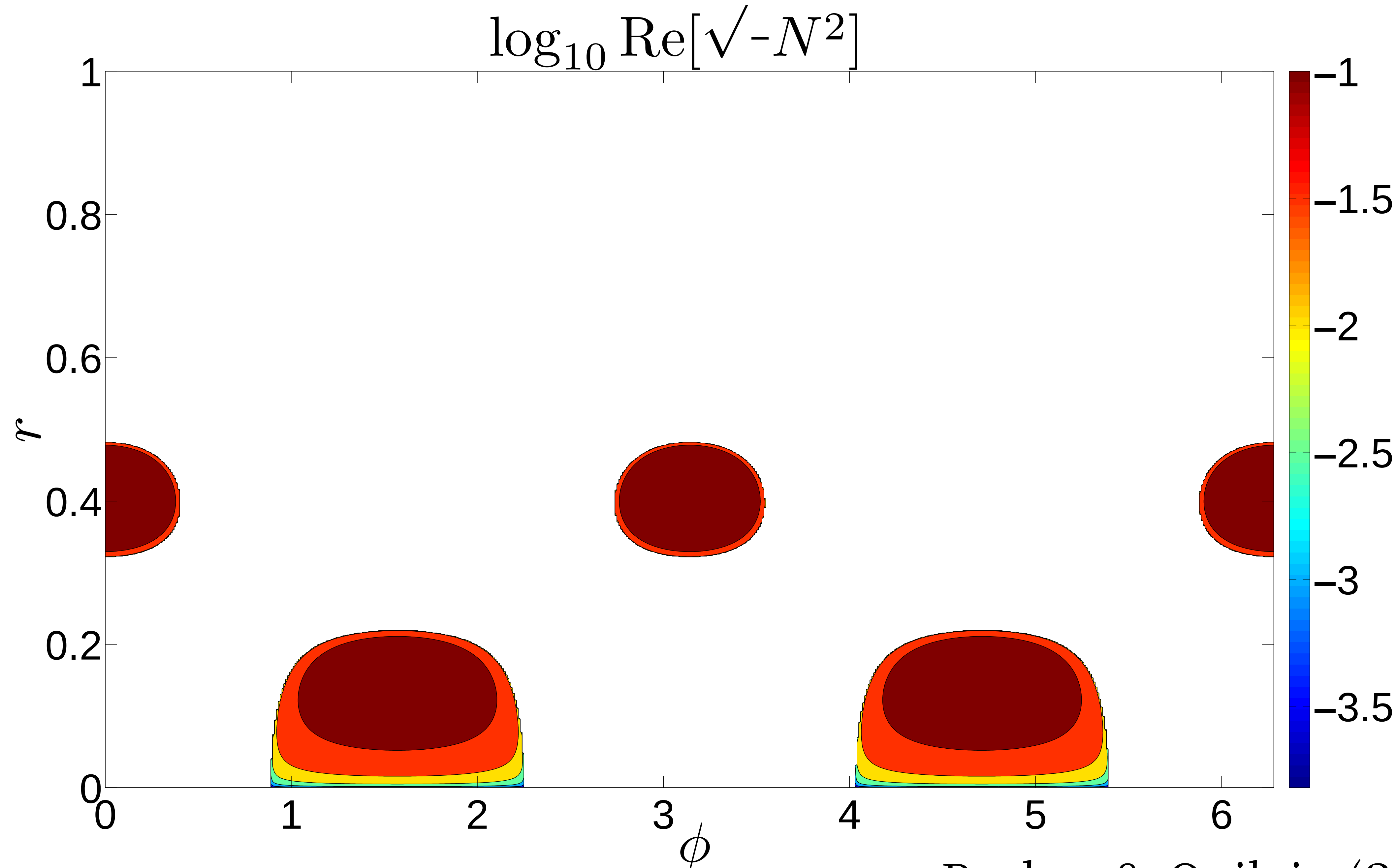


Barker & Ogilvie (2011)

Stability analysis of gravity waves

Results for $A > 1$: initial stages of wave breaking

Unstable mode for $A = 10$

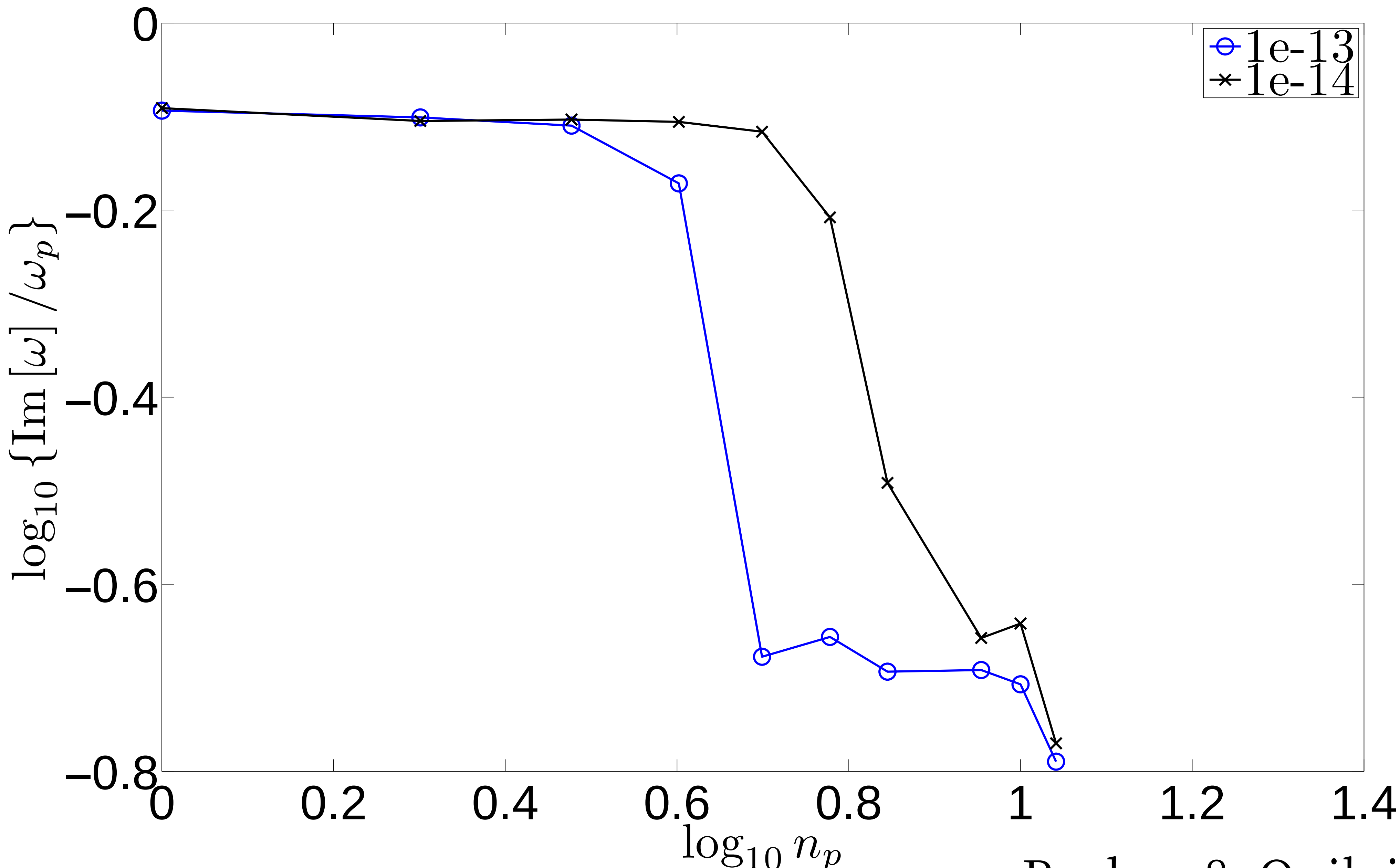


Barker & Ogilvie (2011)

Stability analysis of gravity waves

Results for $A > 1$: initial stages of wave breaking

Growth rate independent of size of domain

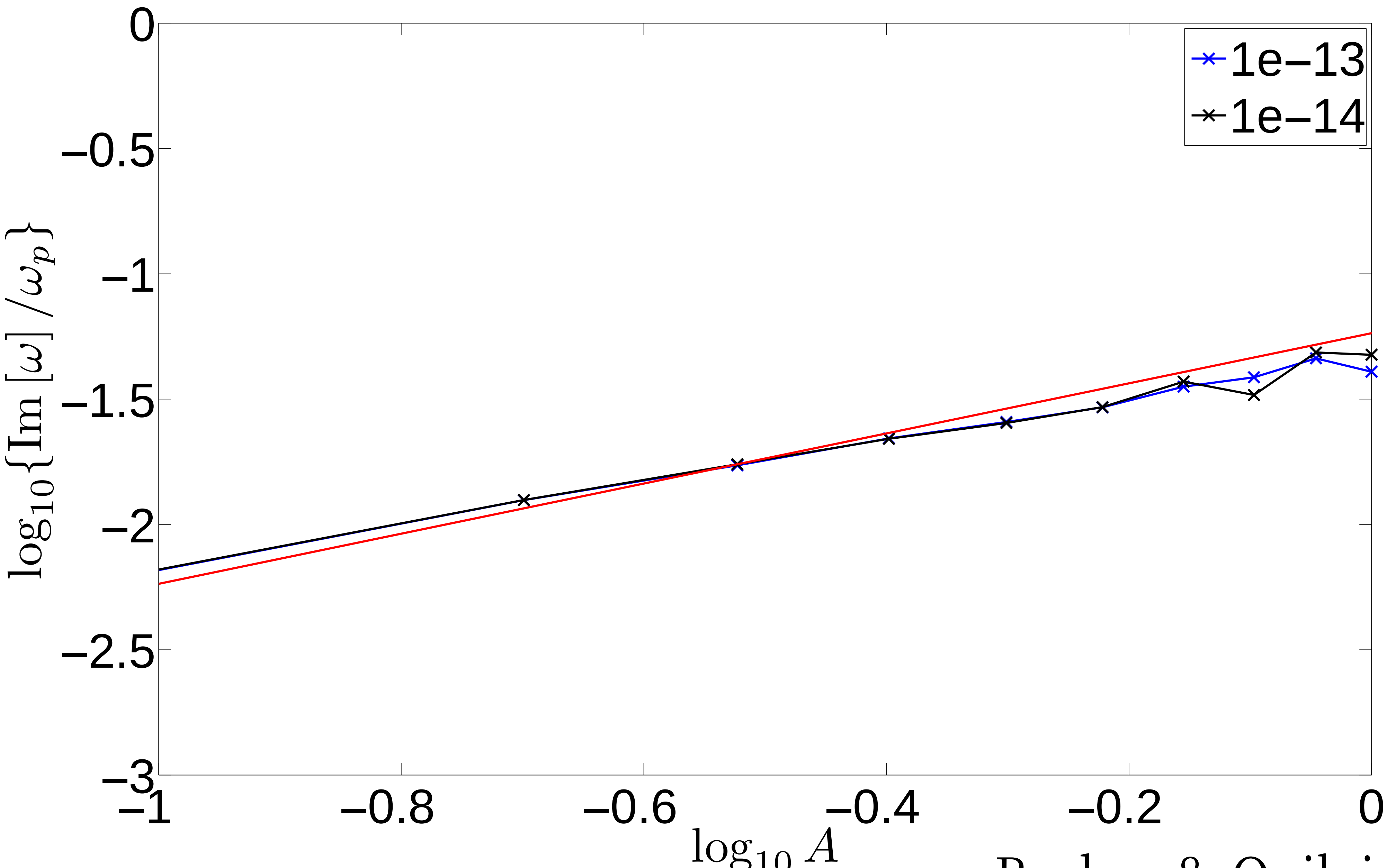


Barker & Ogilvie (2011)

Stability analysis of gravity waves

Results for $A < 1$: weak parametric instabilities

Growth rate vs primary amplitude

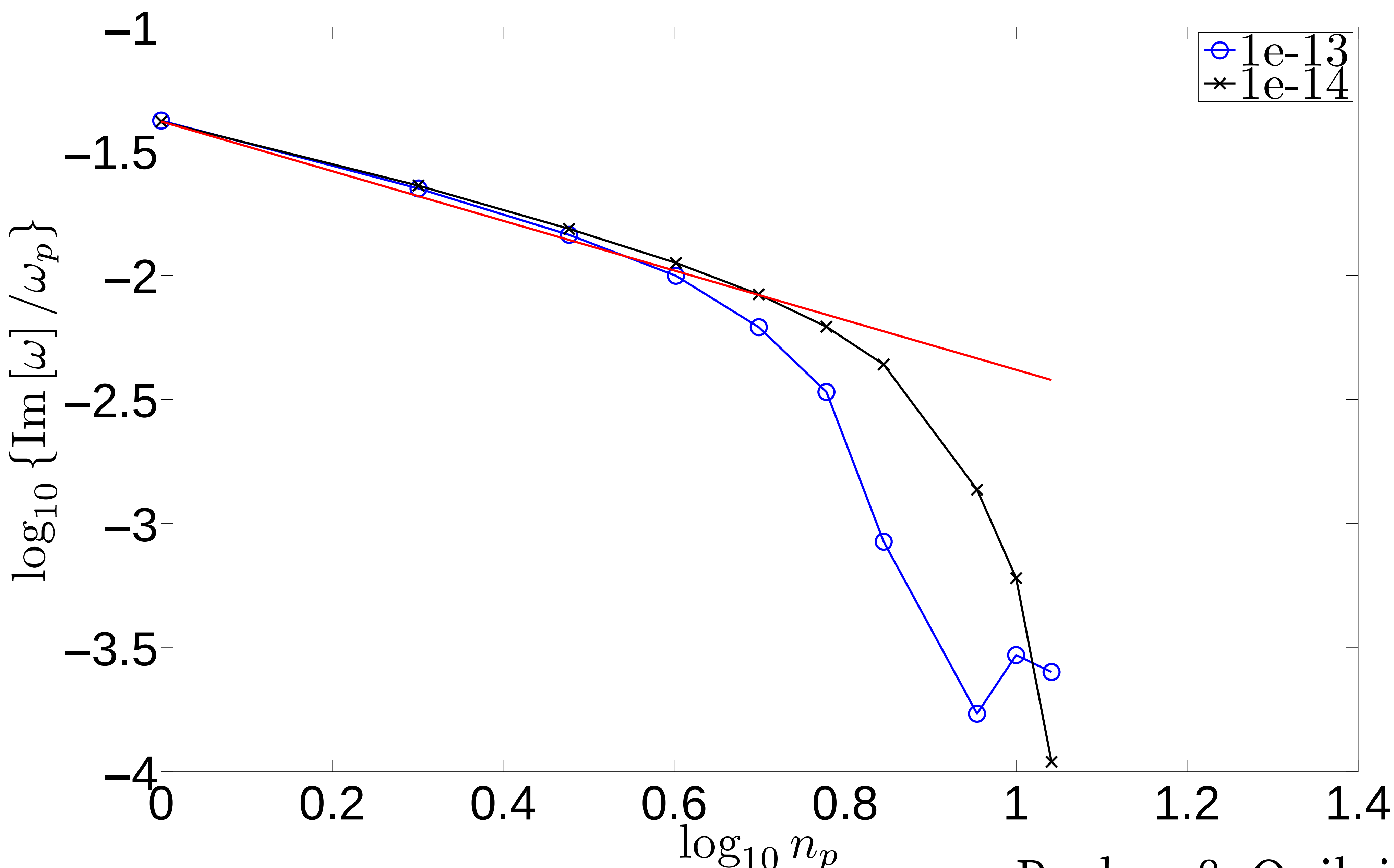


Barker & Ogilvie (2011)

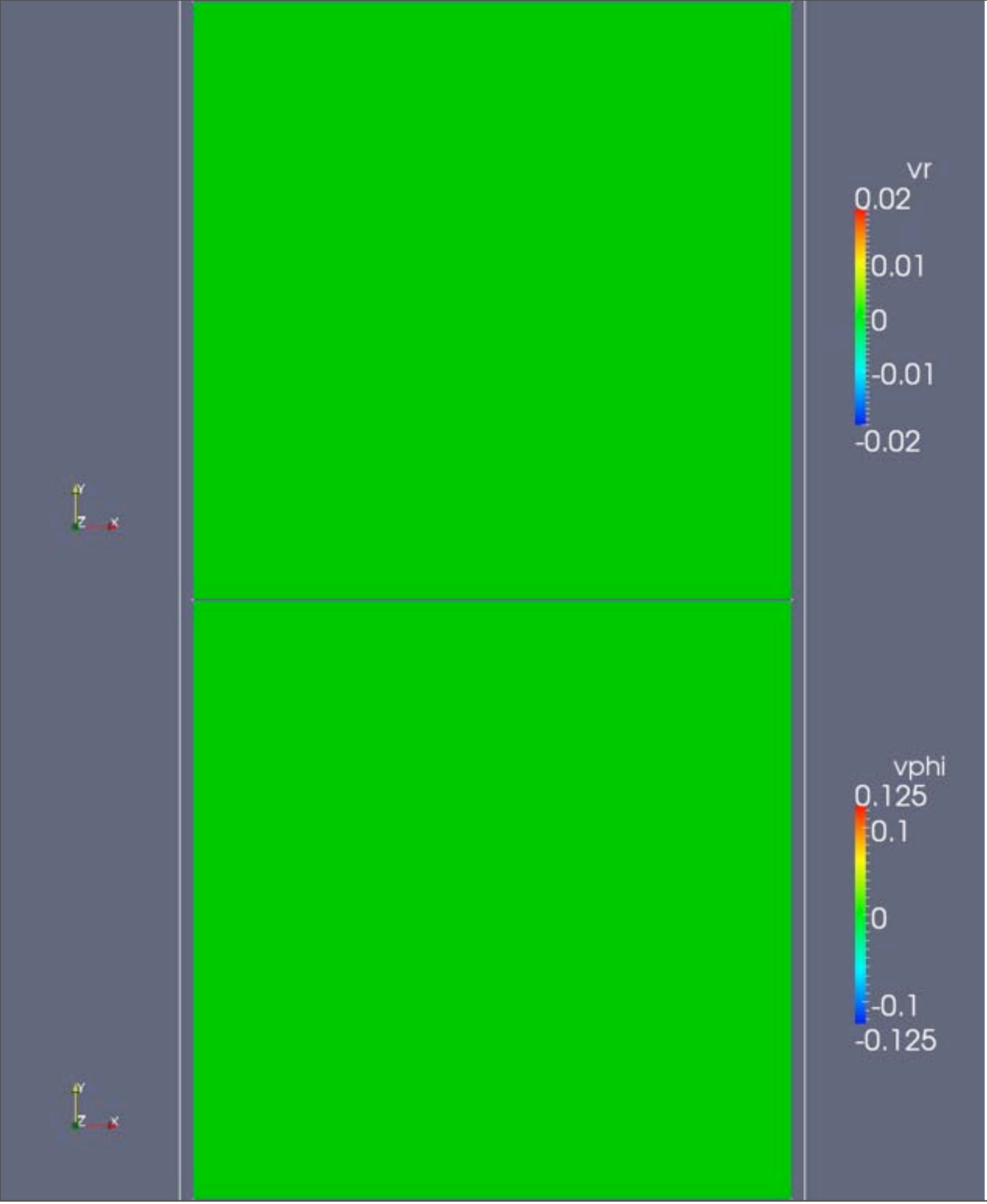
Stability analysis of gravity waves

Results for $A < 1$: weak parametric instabilities

Growth rate $\propto$ (size of domain)$^{-1}$



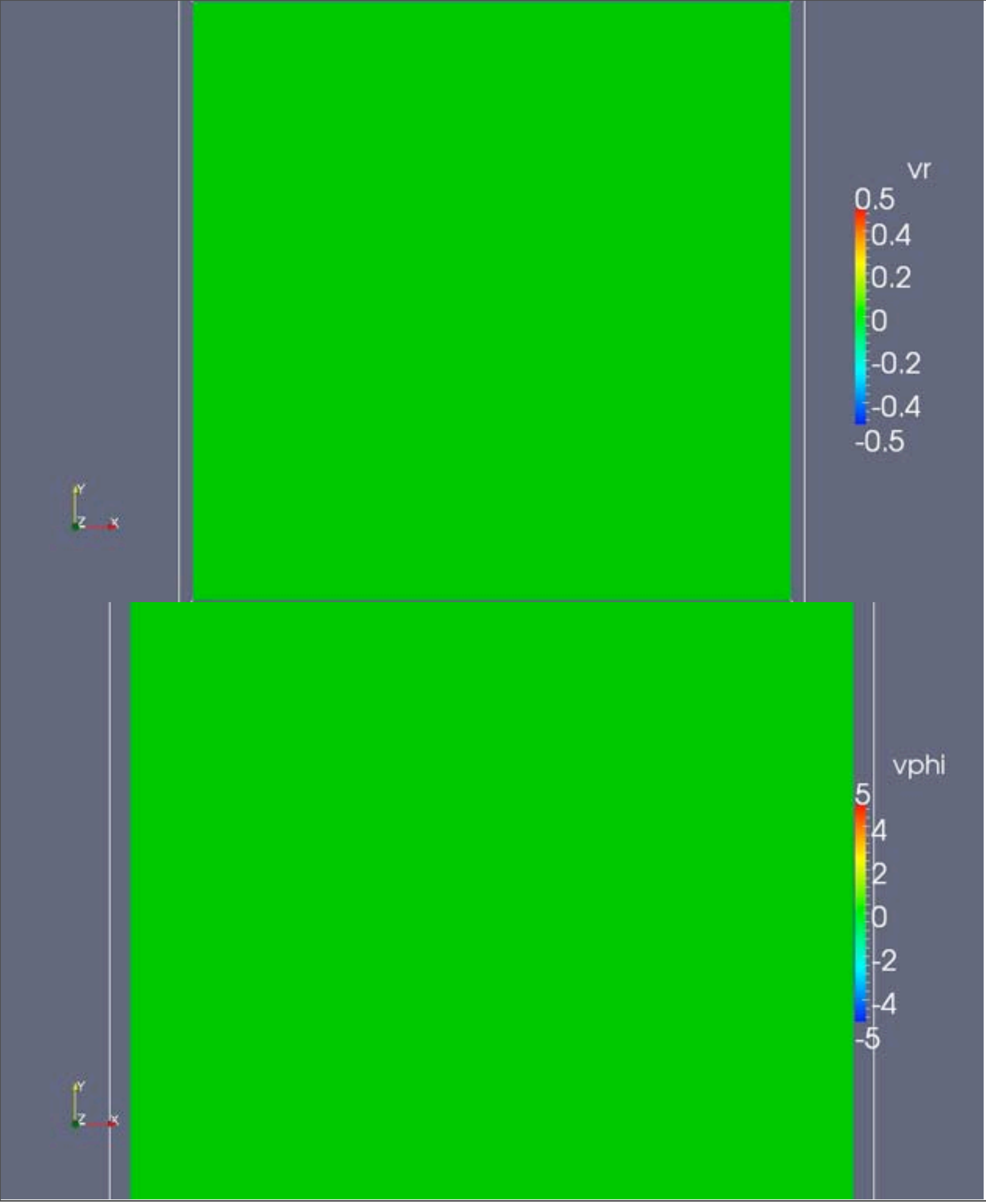
Barker & Ogilvie (2011)



2D numerical simulations

Barker & Ogilvie 2010

Standing wave



2D numerical simulations

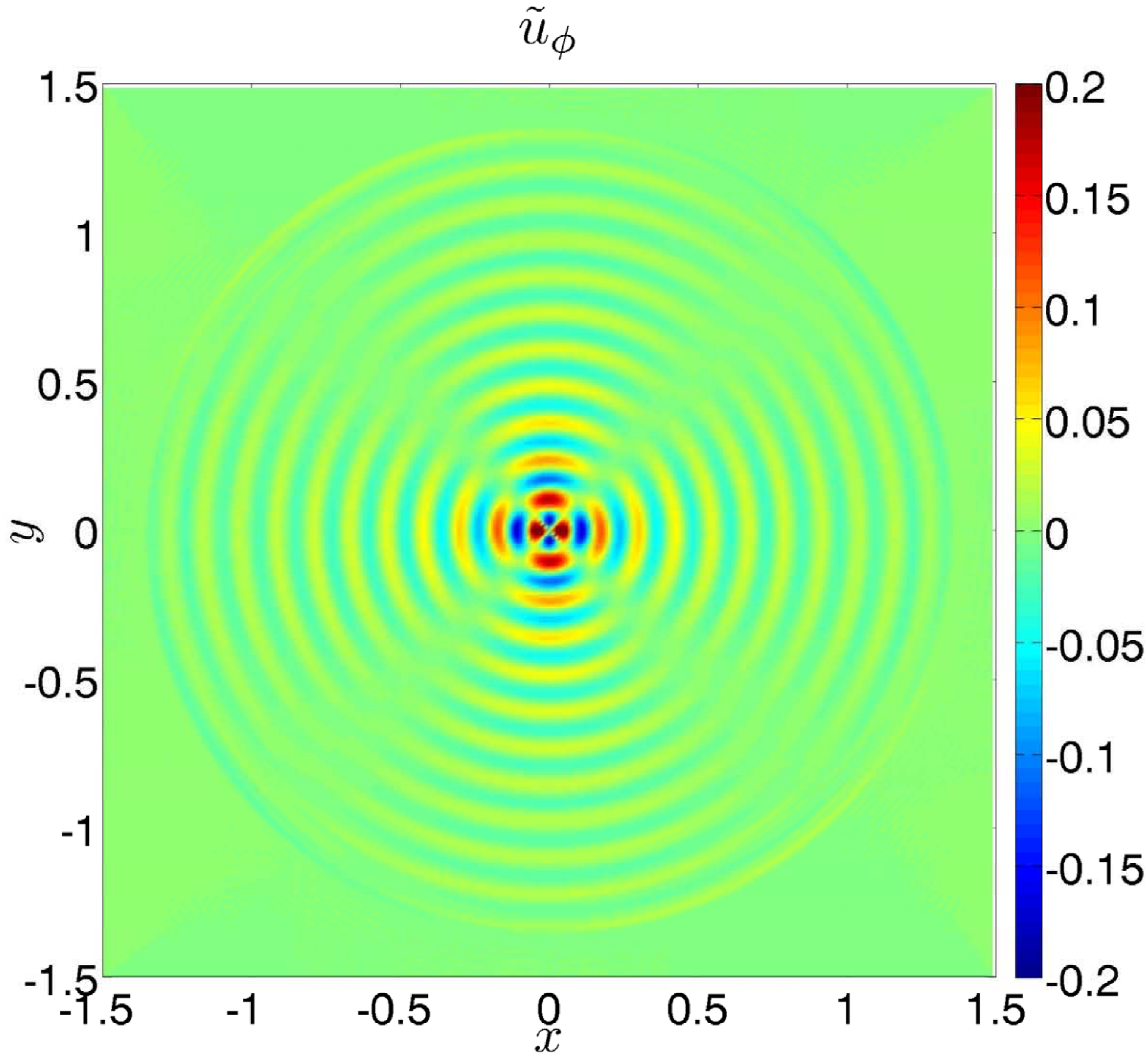
Barker & Ogilvie 2010

Breaking wave

3D numerical simulations

Barker & Ogilvie 2011

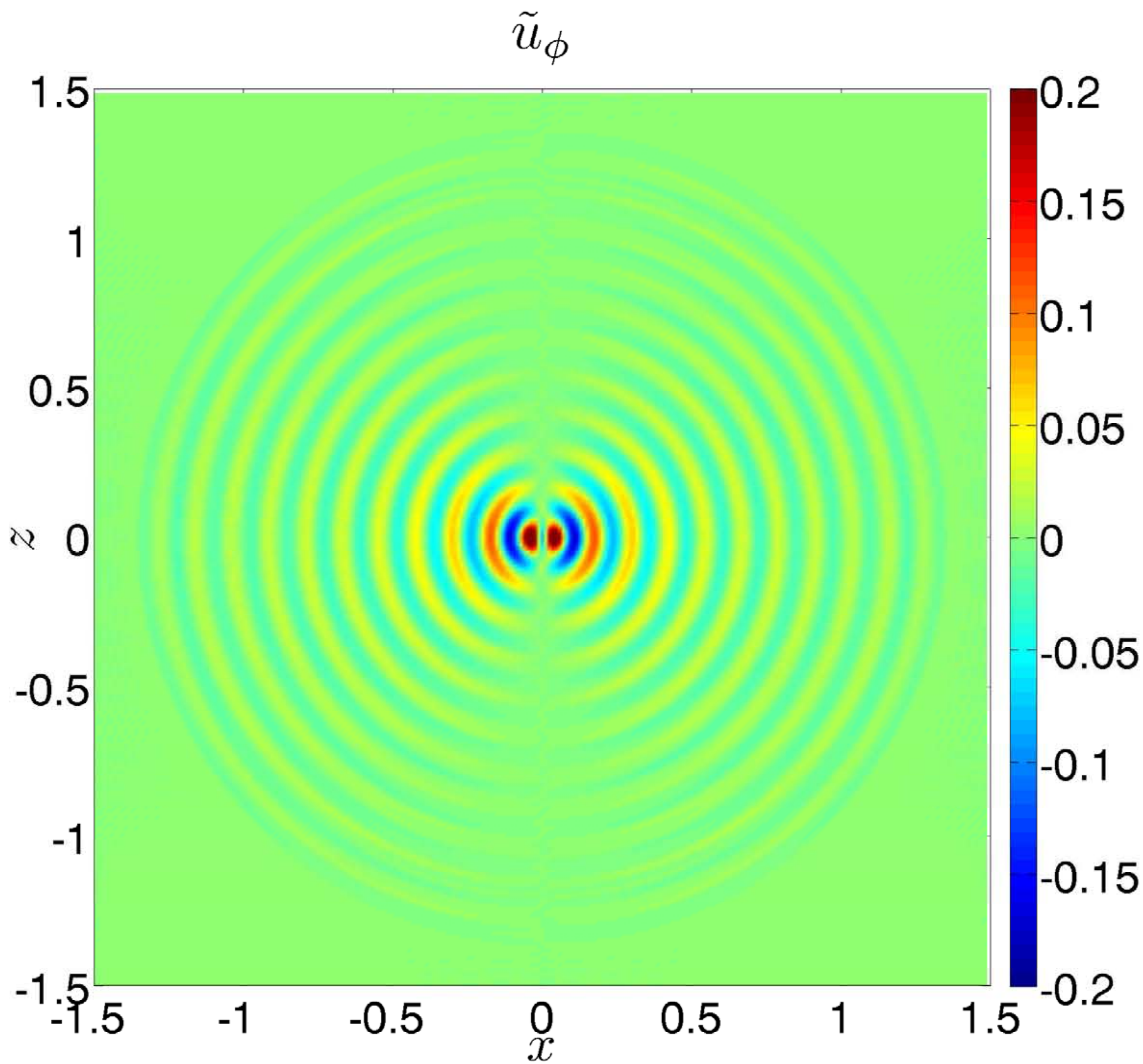
Standing wave



3D numerical simulations

Barker & Ogilvie 2011

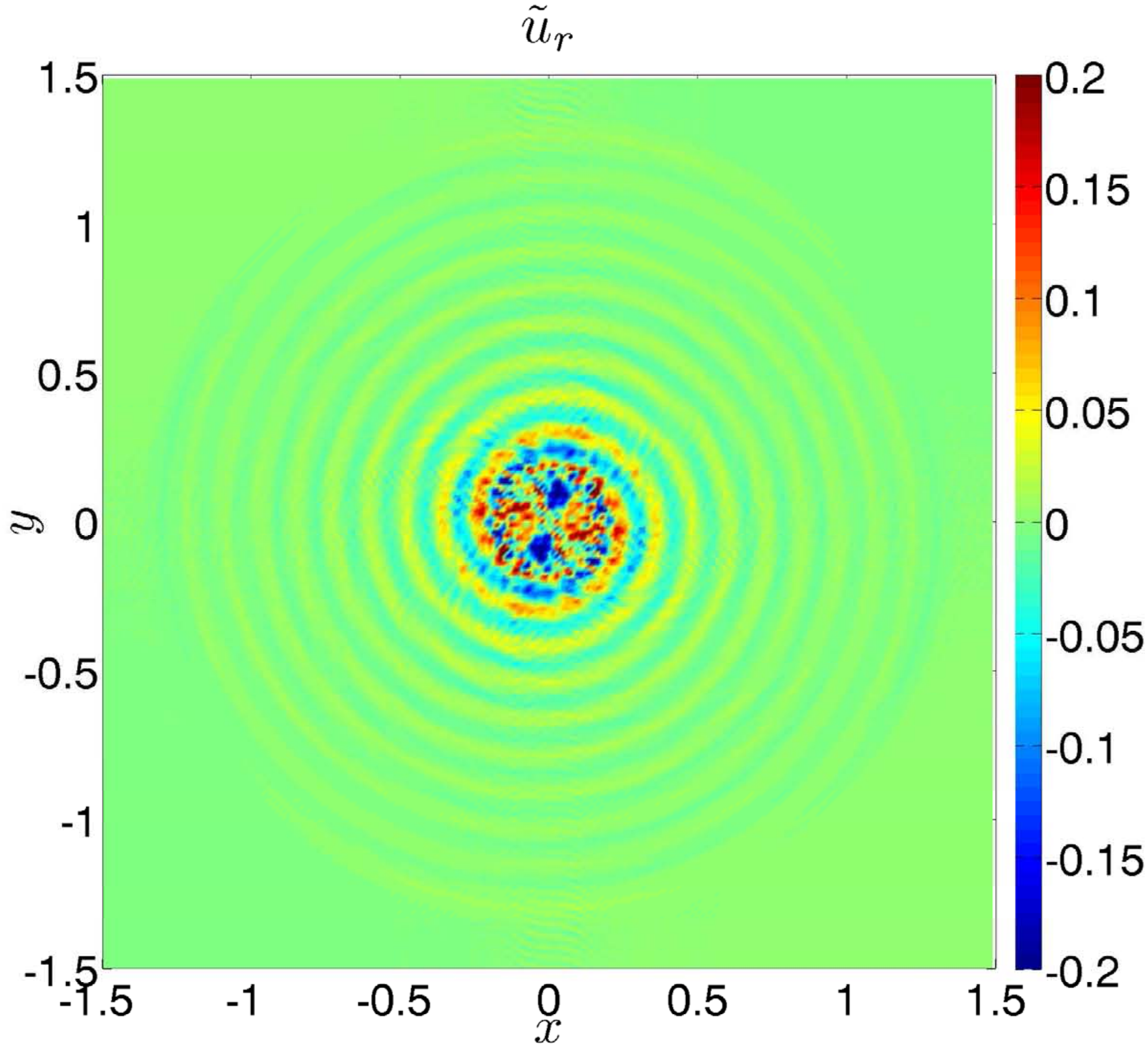
Standing wave



3D numerical simulations

Barker & Ogilvie 2011

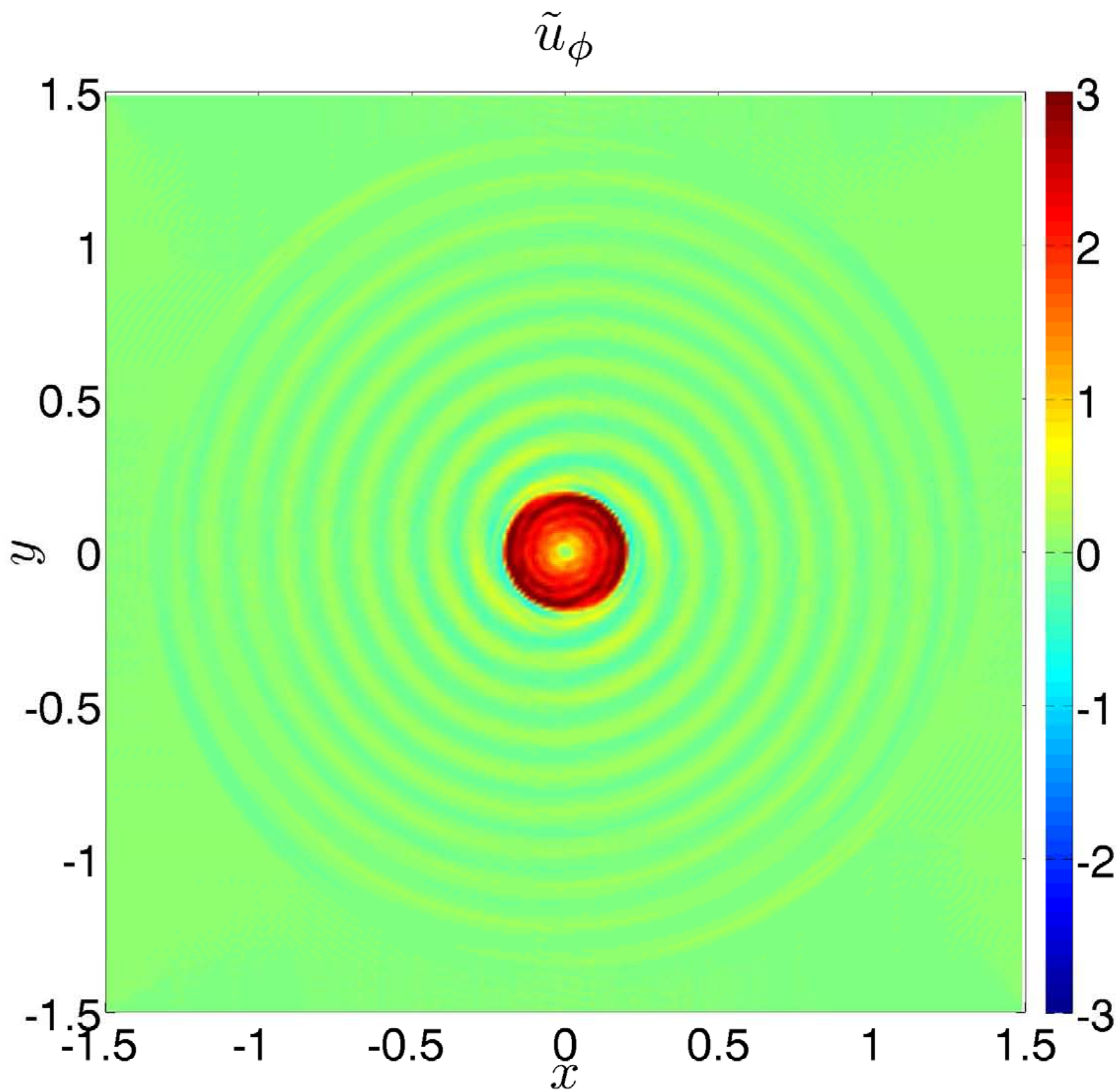
Breaking wave



3D numerical simulations

Barker & Ogilvie 2011

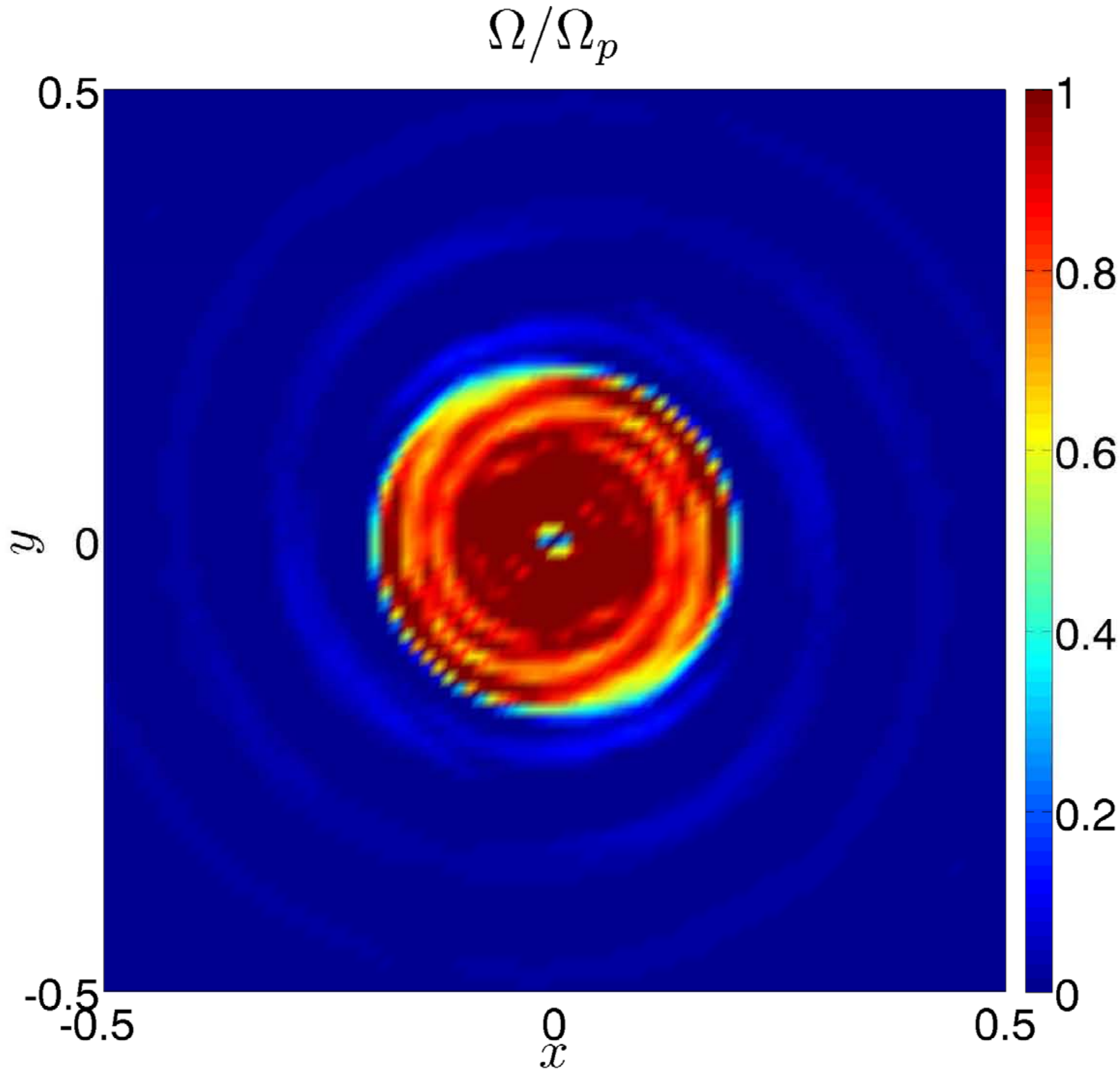
Breaking wave



3D numerical simulations

Barker & Ogilvie 2011

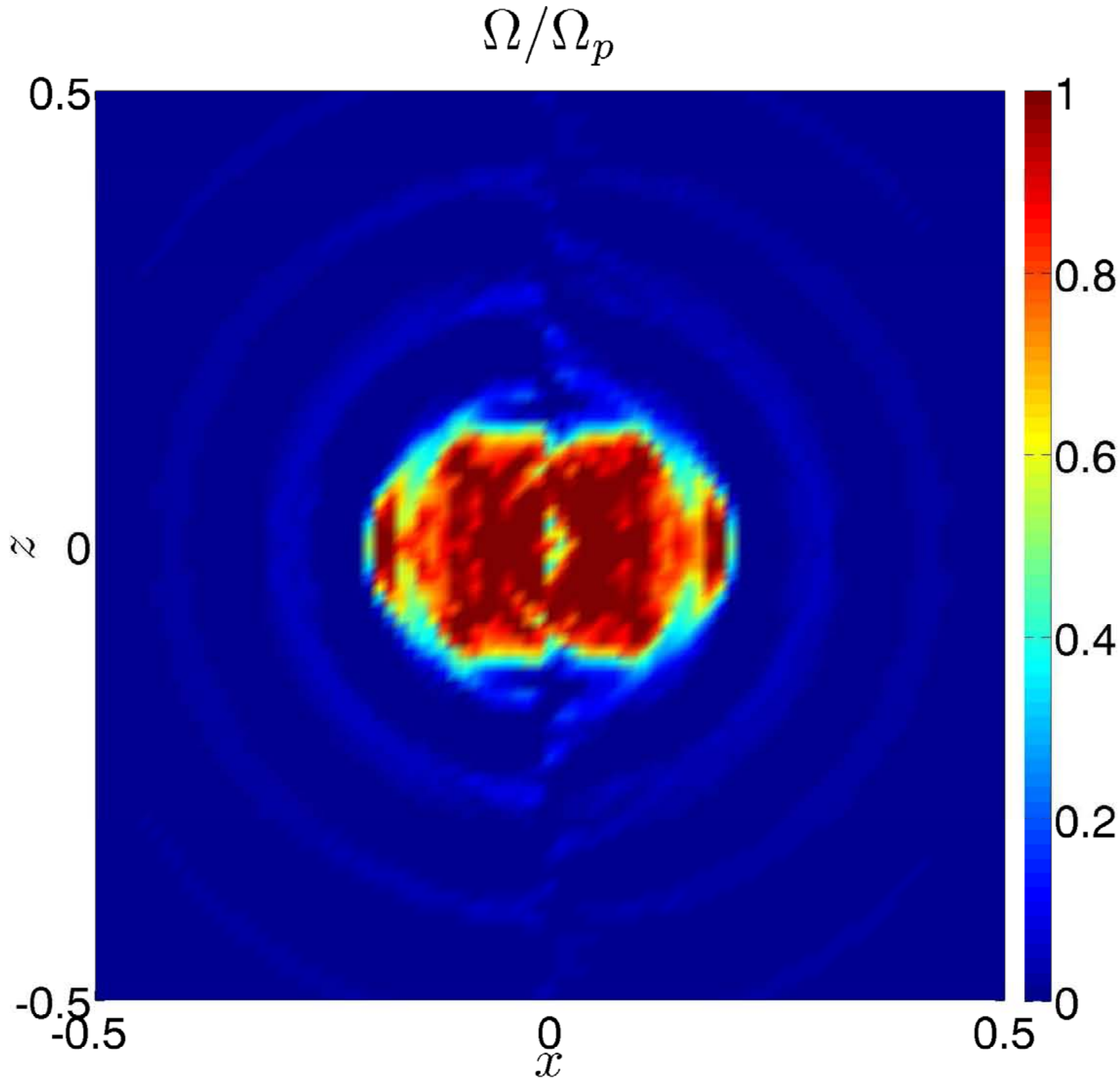
Breaking wave



3D numerical simulations

Barker & Ogilvie 2011

Breaking wave



Implications

- Waves break at centre if

$$\frac{M_\mathrm{p}}{M_\mathrm{J}} > 3.6 \left(\frac{P_\mathrm{orb}}{\mathrm{day}}\right)^{-1/6}$$

  or more easily in older or slightly more massive stars

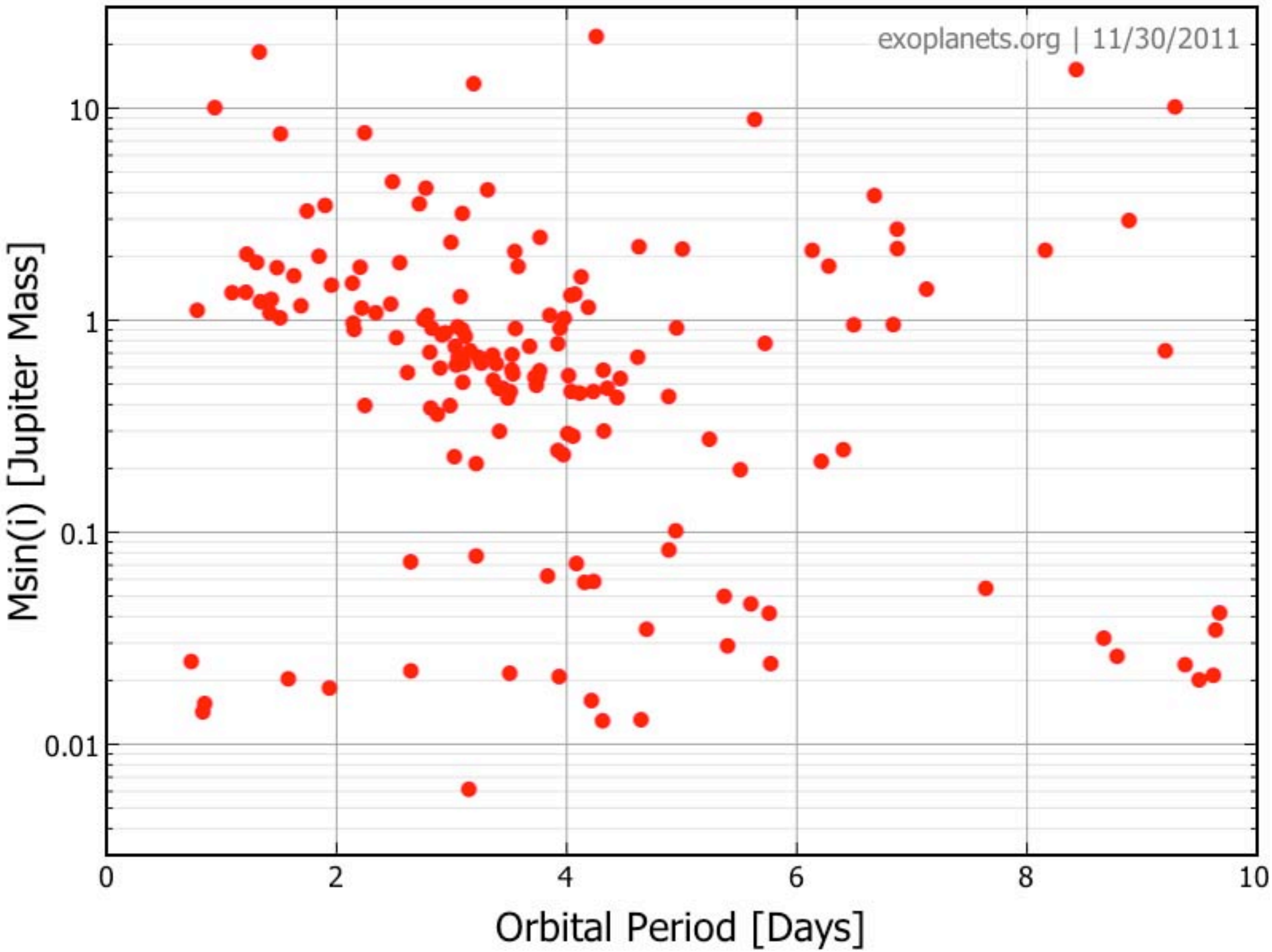
- If this occurs, then $Q'_* \approx 9 \times 10^4 \left(\frac{P_\mathrm{orb}}{\mathrm{day}}\right)^{14/5}$

  and planet is devoured within $1.4\,\mathrm{Myr}\left(\frac{M_\mathrm{p}}{M_\mathrm{J}}\right)^{-1}\left(\frac{P_\mathrm{orb}}{\mathrm{day}}\right)^{7.1}$

- For solar-type binary stars, eccentricity tides are likely to break for any observable eccentricity

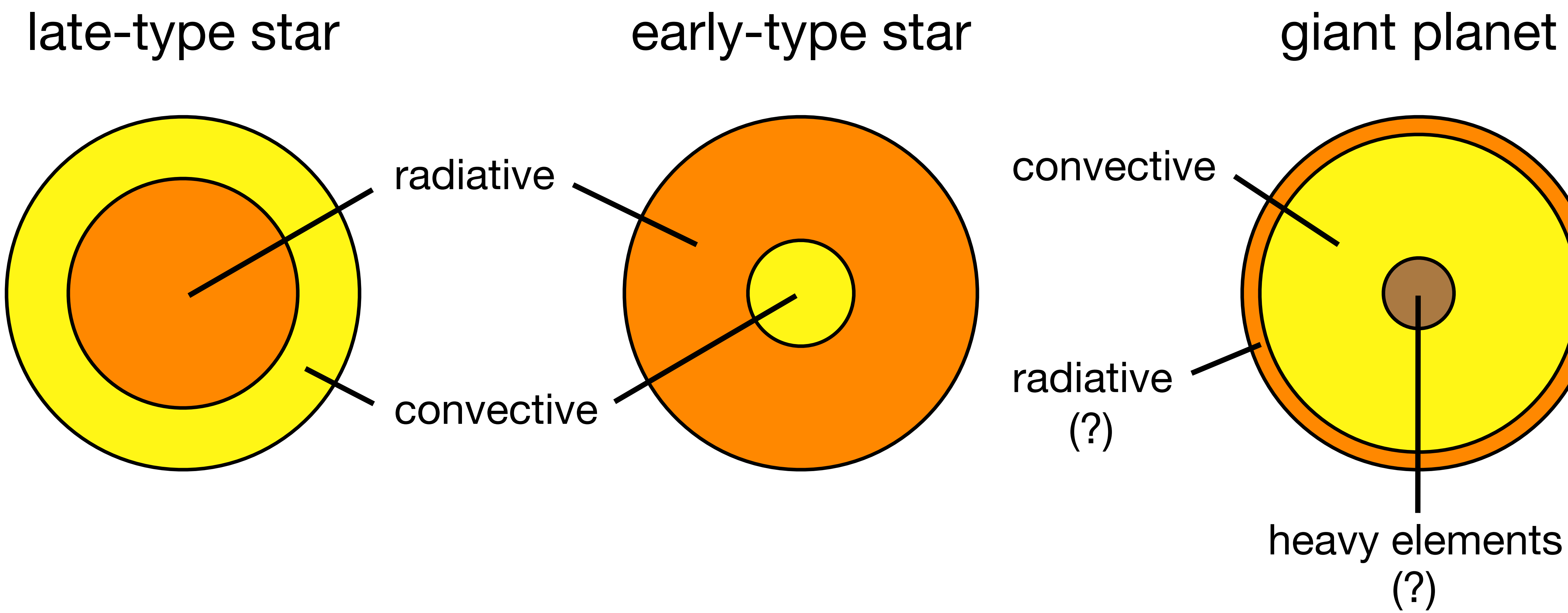
For smaller forcing amplitudes, resonant locking (Savonije & Witte) may need to be reexamined allowing for wave breaking

Short-period extrasolar planets



Effective viscosity of turbulent convection and other flows

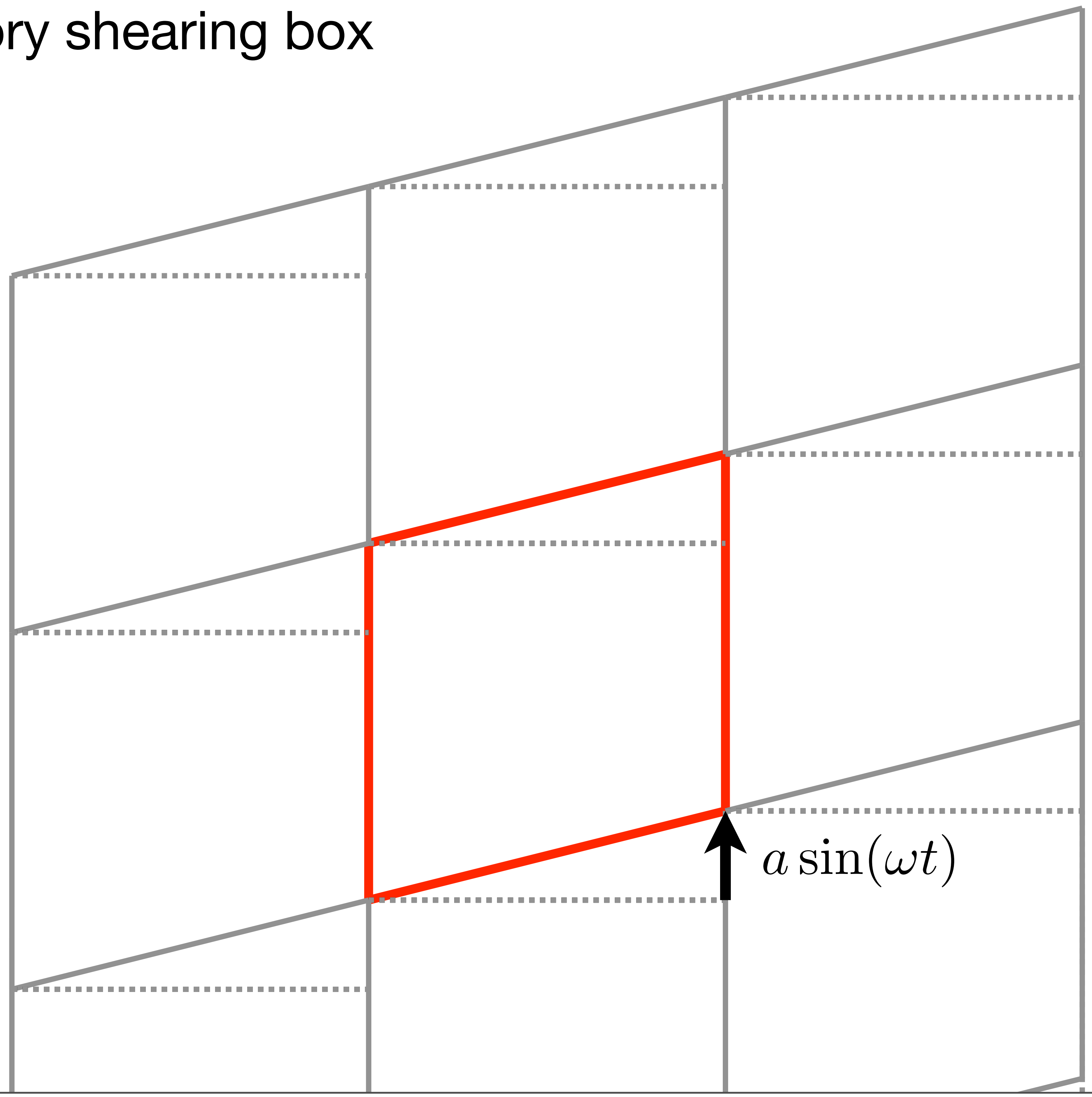
Tides in convective regions of planets and stars



- Turbulent viscosity acting on tidal bulge?
- Excitation and dissipation of inertial waves?

Effective viscosity of turbulent convection

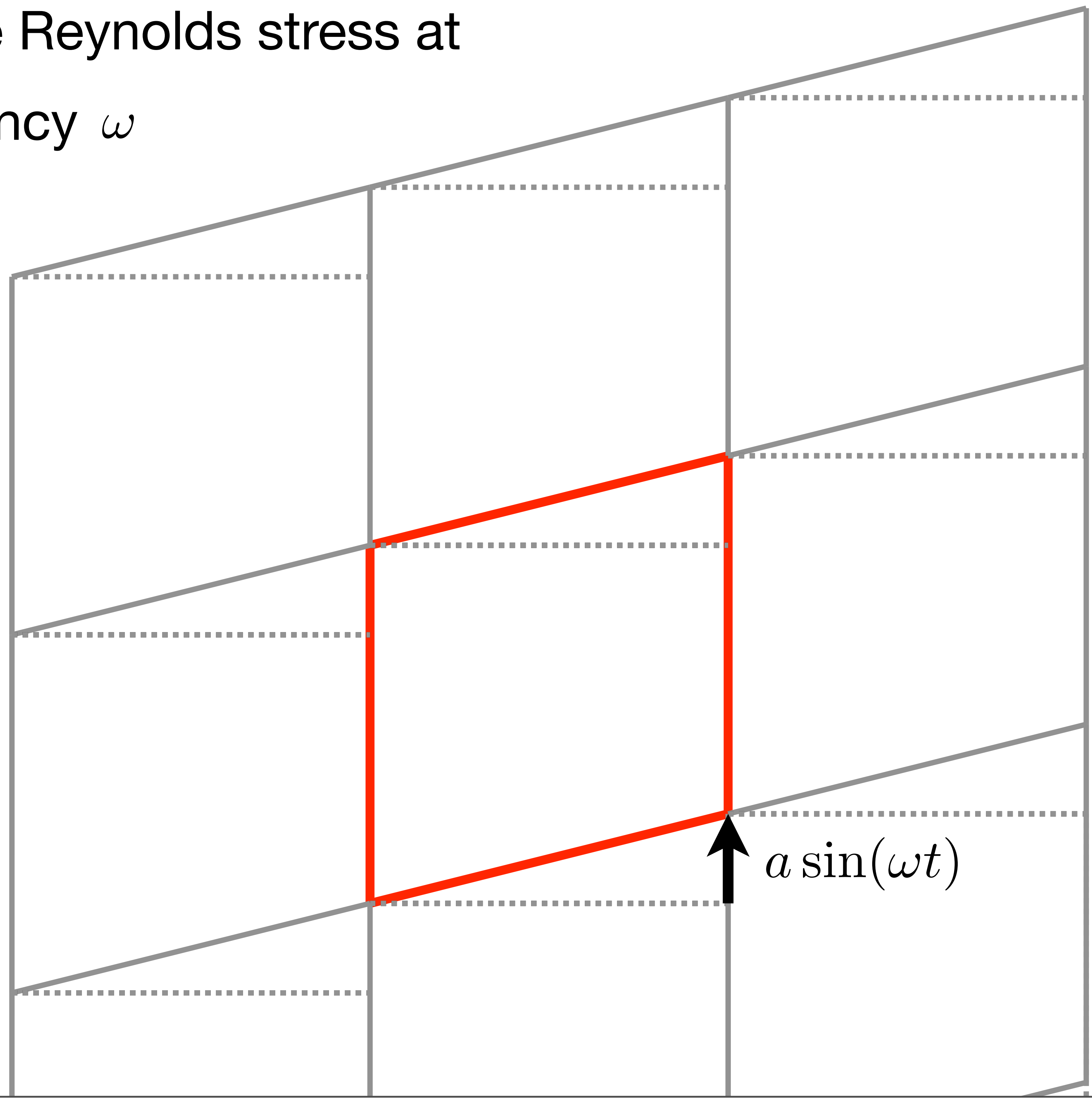
- How does a convecting fluid respond to periodic distortion?
- Oscillatory shearing box



Effective viscosity of turbulent convection

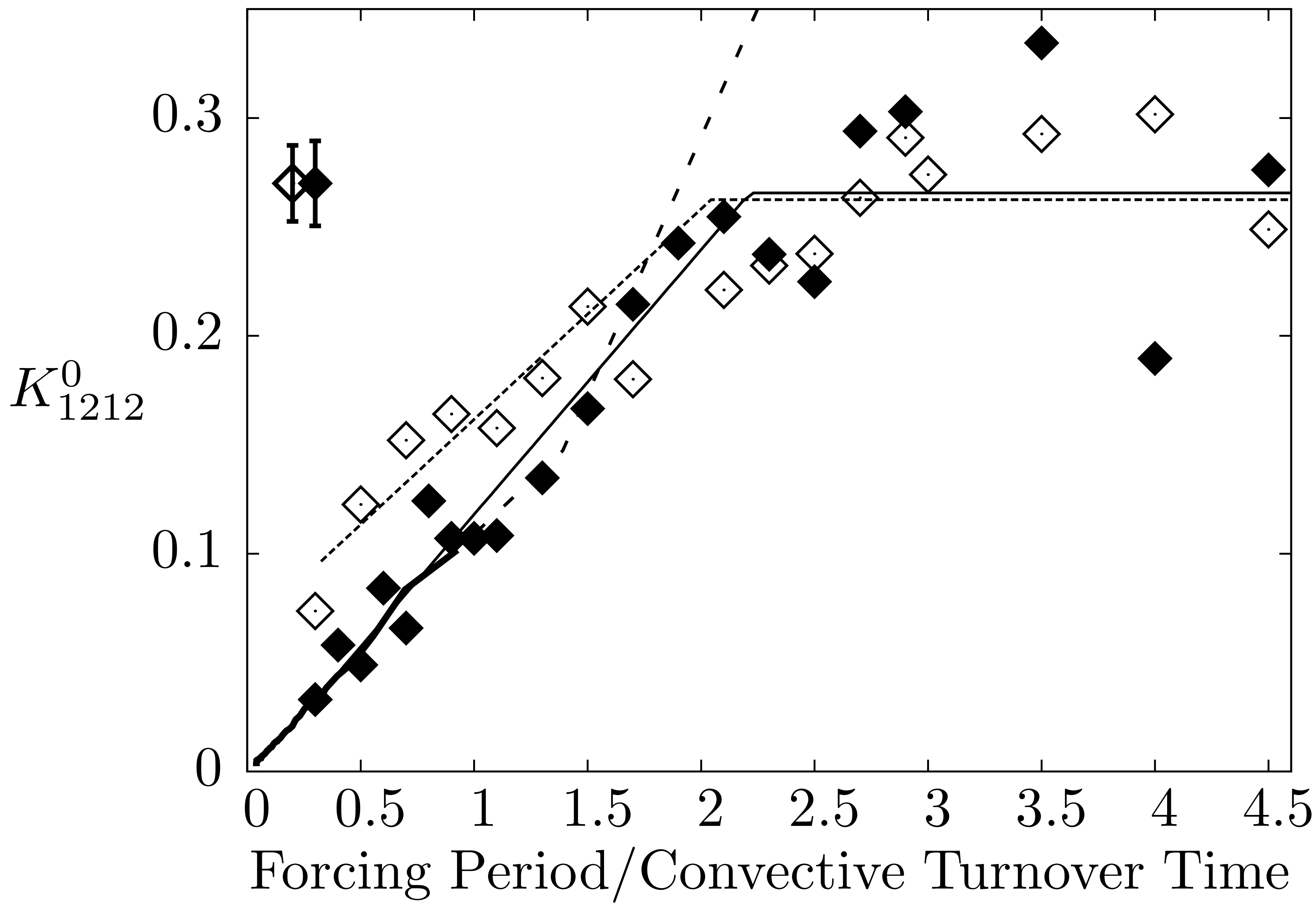
- Compute convective or other flow in OSB
- Measure Reynolds stress at

  frequency ω

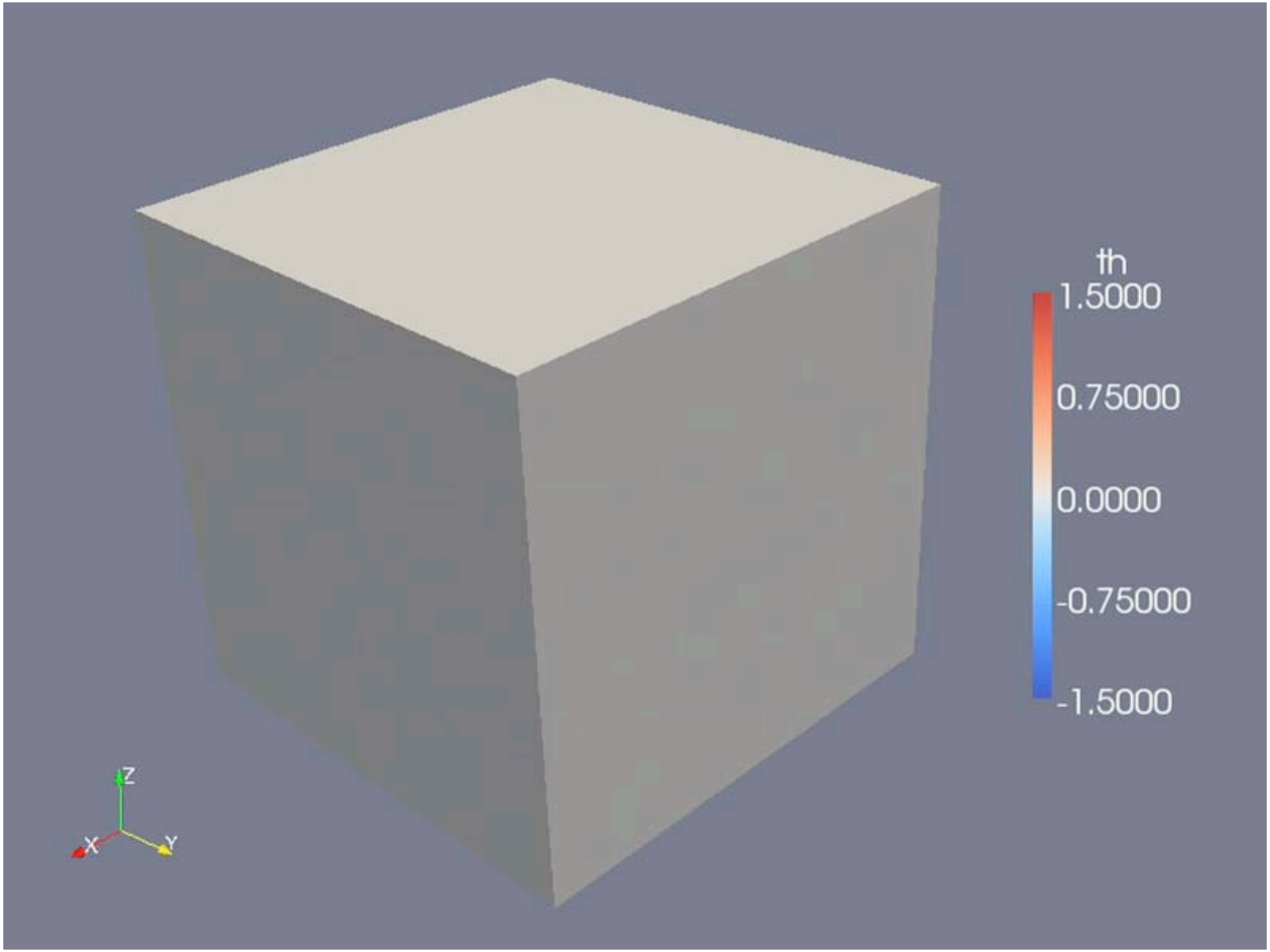


Previous hypotheses and results

- Zahn (1966) : viscosity $\propto \omega^{-1}$ (large eddies)
- Goldreich & Nicholson (1977) : viscosity $\propto \omega^{-2}$ (small eddies)
- Goodman & Oh (1997) : viscosity $\propto \omega^{-5/3}$ (small eddies)
- Penev et al. (2009) : viscosity $\propto \omega^{-1}$

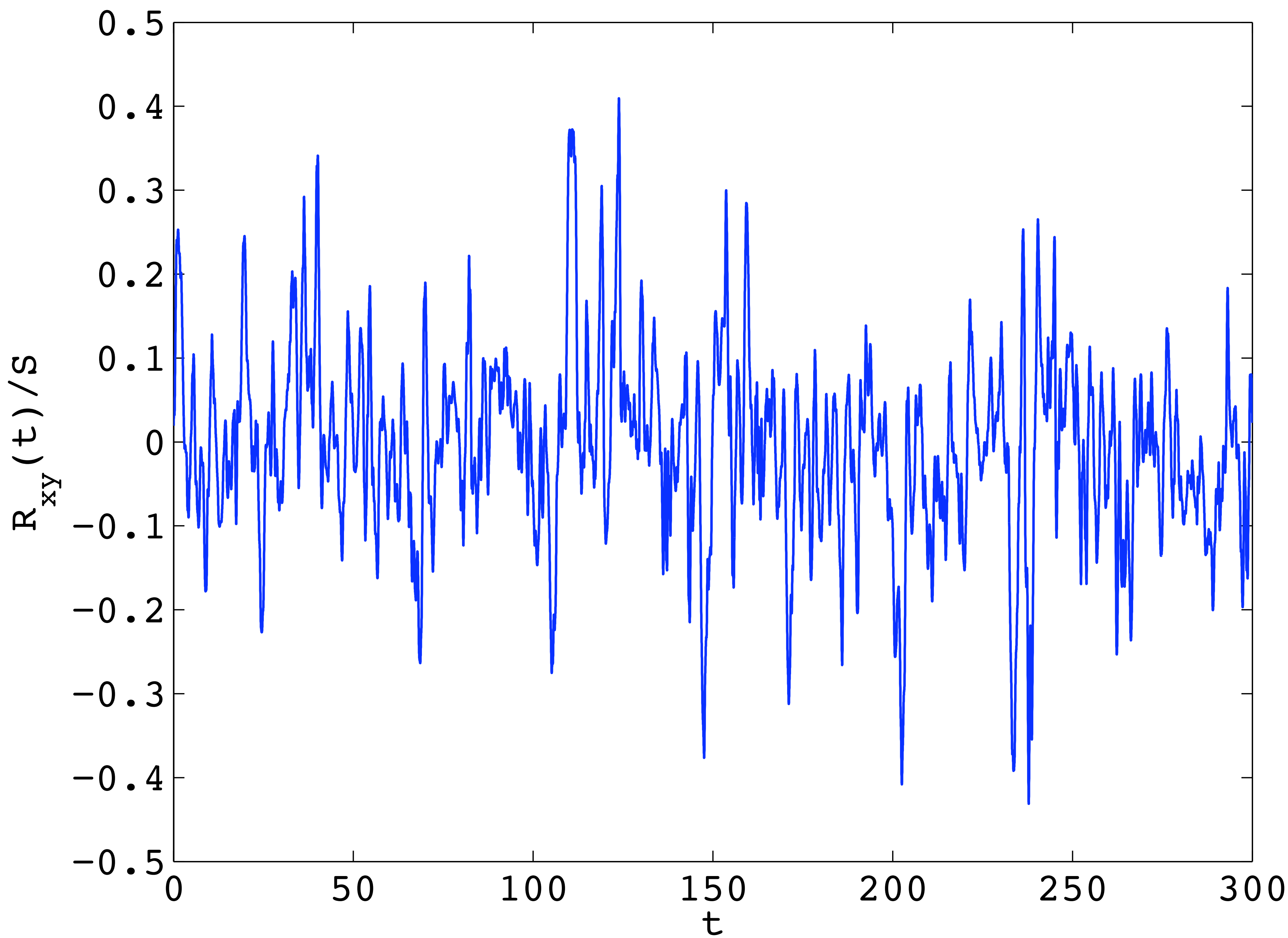


Convection in an oscillatory shearing box (Geoffroy Lesur)



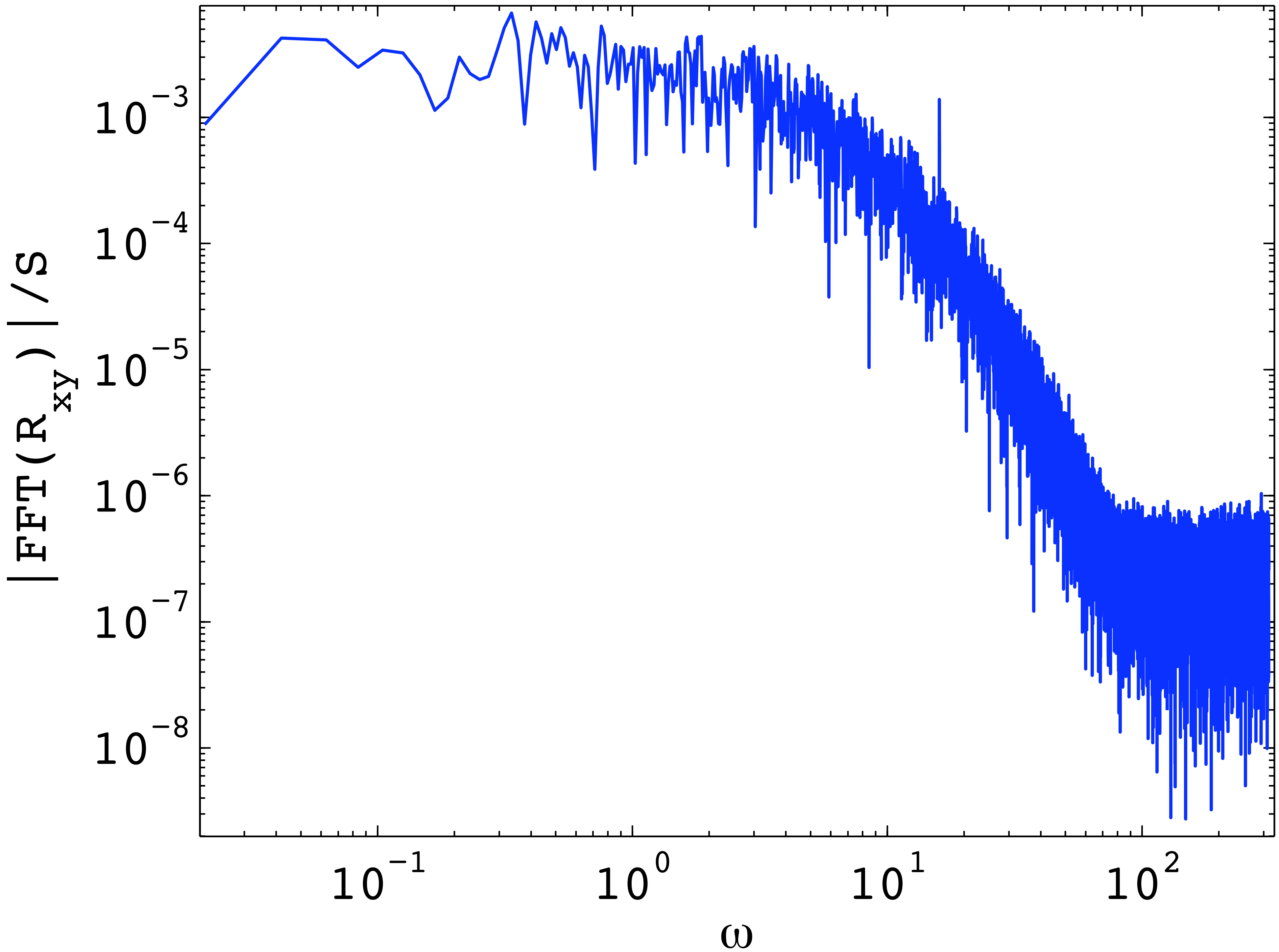
Convection in an oscillatory shearing box (Geoffroy Lesur)

- Time series of Reynolds stress (shear stress)



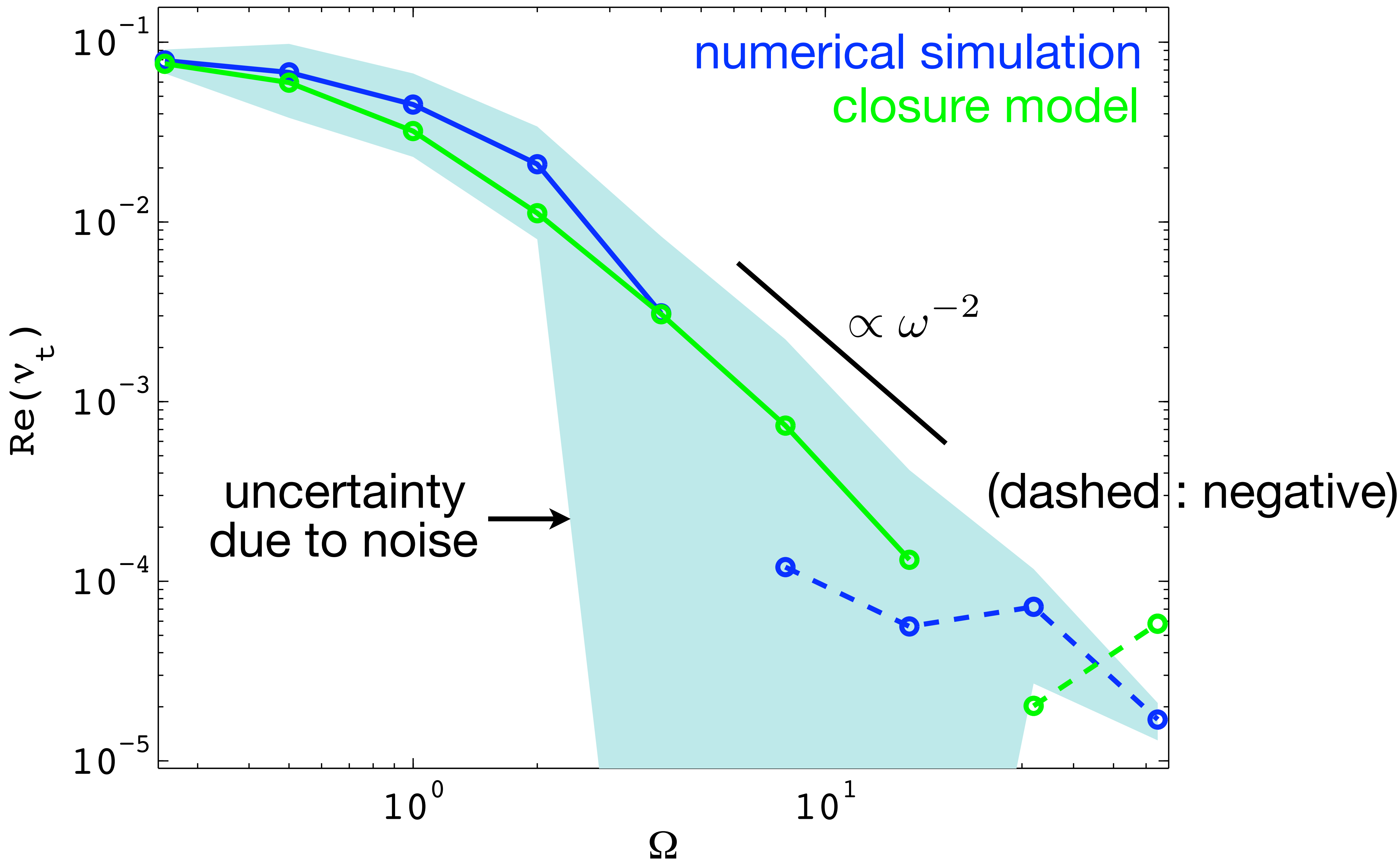
Convection in an oscillatory shearing box (Geoffroy Lesur)

- Fourier transform of Reynolds stress (shear stress)



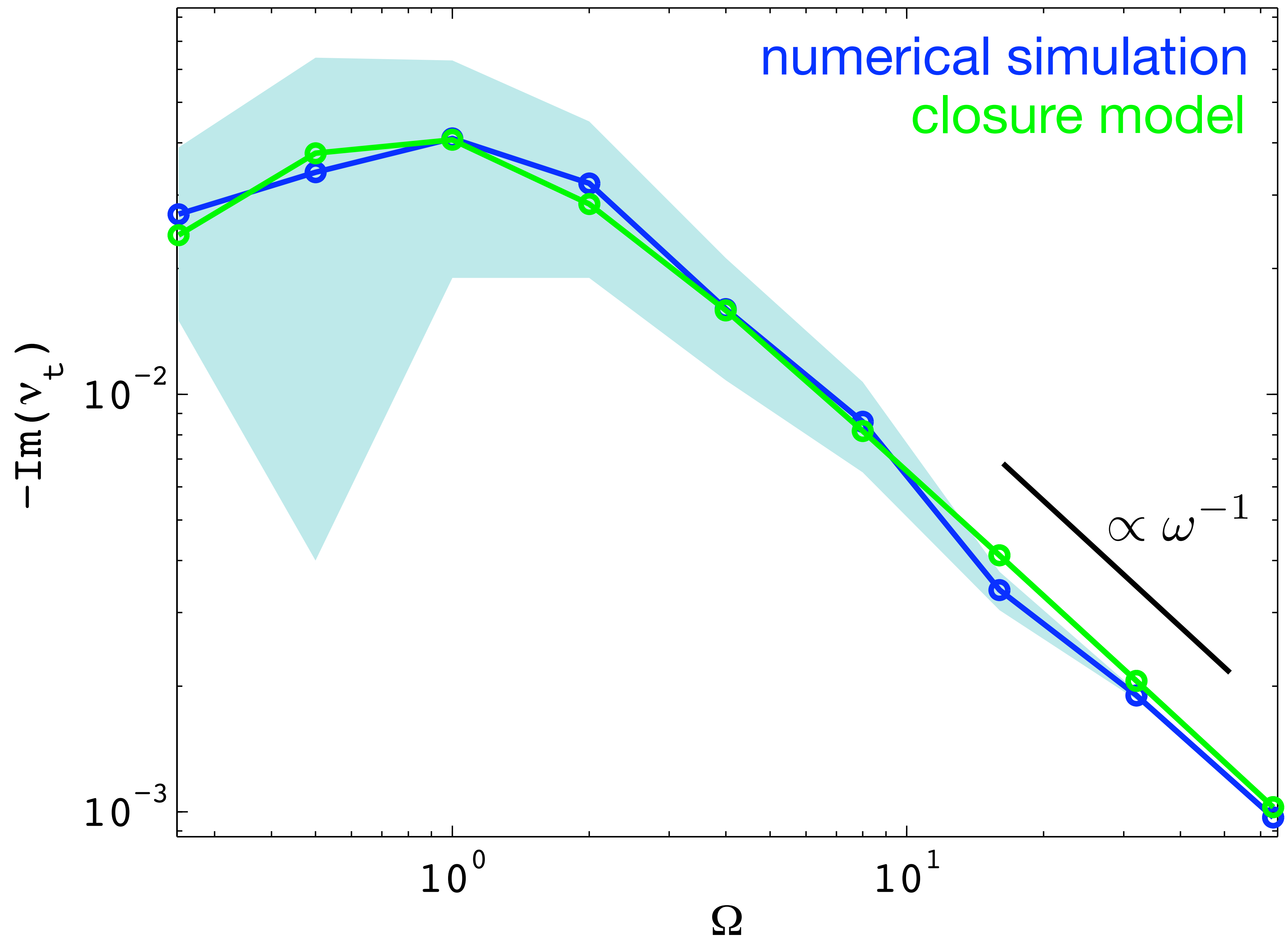
Convection in an oscillatory shearing box (Geoffroy Lesur)

- Real part of effective viscosity versus tidal frequency



Convection in an oscillatory shearing box (Geoffroy Lesur)

- Imaginary part of effective viscosity versus tidal frequency



Analytical approach

- Shearing coordinates

$$x' = x, \qquad y' = y - a(t)x, \qquad z' = z, \qquad t' = t$$

- Shear $a(t)$, shear rate $\dot{a}(t)$
- Derivatives

$$\partial_x = \partial'_x - a\partial'_y, \qquad \partial_y = \partial'_y, \qquad \partial_z = \partial'_z, \qquad \partial_t = \partial'_t - \dot{a}x\partial'_y$$

- Absolute and relative velocities

$$u_x = v_x, \qquad u_y = v_y + \dot{a}x, \qquad u_z = v_z$$

- Navier–Stokes equations in sheared coordinates

$$[\partial'_t + v_j(\partial'_j - a\delta_{j1}\partial'_y)]v_i + \dot{a}v_x\delta_{i2} = -(\partial'_i - a\delta_{i1}\partial'_y)p$$
$$+\nu(\partial'_j - a\delta_{j1}\partial'_y)(\partial'_j - a\delta_{j1}\partial'_y)v_i + f_i$$

$$(\partial'_i - a\delta_{i1}\partial'_y)v_i = 0$$

- Additional body force $\ddot{a}x'\delta_{i2}$ is required to maintain shear
- Remaining equations are spatially homogeneous

Analytical approach

- Linearize in the shear amplitude, assuming $|a| \ll 1$
- Zeroth order: basic flow satisfying

$$(\partial_t' + v_j \partial_j')v_i = -\partial_i' p + \nu \Delta' v_i + f_i$$

$$\partial_i' v_i = 0$$

(may be decaying or (quasi-)stationary, laminar or turbulent

- First order:

$$(\partial_t' + v_j \partial_j')\delta v_i + (\delta v_j \partial_j' - a v_x \partial_y')v_i + \dot{a} v_x \delta_{i2}$$
$$= -\partial_i' \delta p + a \delta_{i1} \partial_y' p + \nu(\Delta' \delta v_i - 2a \partial_x' \partial_y' v_i)$$

$$\partial_i' \delta v_i - a \partial_y' v_x = 0$$

- Aim to calculate linearized shear stress $-\delta R_{xy} = -\langle v_x \delta v_y + v_y \delta v_x \rangle$
- Could solve numerically (but not useful for chaotic flows)
- Asymptotic approach for high-frequency shear

Analytical approach

- Asymptotic approach for high-frequency shear
- Method of multiple scales
- Fast time variable for shear: $T' = t'/\epsilon, \epsilon \ll 1$

$$a \mapsto a(T'), \quad \dot{a} \mapsto \epsilon^{-1}\dot{a}$$
($\dot{a}$ now means $\mathrm{d}a/\mathrm{d}T'$)

- Expand

$$\delta v_i = \delta v_{i0} + \epsilon\,\delta v_{i1} + \cdots$$
$$\delta p = \epsilon^{-1}(\delta p_0 + \epsilon\,\delta p_1 + \cdots)$$

quantities depending on $(\boldsymbol{x}', t', T')$

- Leading order

$$\partial'_T \delta v_{i0} + \dot{a} v_x \delta_{i2} = -\partial'_i \delta p_0$$
$$\partial'_i \delta v_{i0} - a\partial'_y v_x = 0$$

- Rough argument:

$$\delta v_{y0} \approx -a v_x \;\Rightarrow\; -\langle v_x \delta v_{y0}\rangle \approx a\langle v_x^2\rangle$$
elastic stress

Analytical approach

- Leading order

$$\partial'_T \delta v_{i0} + \dot{a} v_x \delta_{i2} = -\partial'_i \delta p_0$$

$$\partial'_i \delta v_{i0} - a \partial'_y v_x = 0$$

- More precise argument

$$\Delta' \delta p_0 = -2\dot{a}\partial'_y v_x$$

$$\partial'_T \delta v_{i0} = 2\dot{a}\partial'_i \partial'_y \Delta'^{-1} v_x - \dot{a} v_x \delta_{i2}$$

- Linearized shear stress $-\delta R_{xy0} = -\langle v_x \delta v_{y0} + v_y \delta v_{x0} \rangle$ satisfies

$$\partial'_T(-\delta R_{xy0}) = \dot{a}\langle v_x^2 - 2(v_x \partial'_y + v_y \partial'_x)\partial'_y \Delta'^{-1} v_x \rangle$$
$$= \dot{a}(A_{1jj1} - 2A_{1221} - 2A_{2121})$$

in terms of the tensor

$$A_{ijkl} = \langle v_i \partial'_j \partial'_k \Delta'^{-1} v_l \rangle$$

Analytical approach

- Next order can be treated in a similar way

$$\partial_T'^2(-\delta R_{xy1}) = -\dot{a}(B_{1jj1} - B_{1221} - B_{1122} - C_{1221} + C_{1jj1} + 3C_{1122}$$
$$-2D_{1jj221} - 2D_{2jj121} - 3D_{1jj221} - 3D_{1jj212}$$
$$-D_{ijij1221} - D_{ijij1212} + D_{ijij22} + 4E_{2121} + 4E_{2112})$$

in terms of the tensors

$$B_{ijkl} = \langle(\partial_t' v_i)\partial_j'\partial_k'\Delta'^{-1} v_l\rangle$$

$$C_{ijkl} = -\nu\langle v_i \partial_j'\partial_k' v_l\rangle$$

$$D_{ijkl} = \langle v_i v_j \partial_k' v_l\rangle$$

$$D_{ijklmn} = \langle v_i v_j \partial_k'\partial_l'\partial_m'\Delta'^{-1} v_n\rangle$$

$$D_{ijklmnpq} = \langle v_i v_j \partial_k'\partial_l'\partial_m'\partial_n'\partial_p'\Delta'^{-2} v_q\rangle$$

$$E_{ijkl} = \langle v_m(\partial_m'\partial_n'\Delta'^{-1}\partial_i' v_j)\partial_n'\Delta'^{-1}\partial_k' v_l\rangle$$

Analytical approach

- Interpretation

$$\partial_T'(-\delta R_{xy0}) = \dot{a}\mathcal{G}_0$$

$$\partial_T'^2(-\delta R_{xy1}) = -\dot{a}\mathcal{G}_1$$

- For a shear $a \propto \exp(-\mathrm{i}\omega t)$ with $\omega = O(\epsilon^{-1})$, deduce that

$$-\delta R_{xy} = a\left[\mathcal{G}_0 - \frac{\mathrm{i}\mathcal{G}_1}{\omega} + O(\epsilon^2)\right]$$

$\mathcal{G}_0$: ideal elastic response

$\frac{\mathrm{i}\mathcal{G}_1}{\omega}$: imperfection associated with dissipation

- Compare with elastic stress $\mathcal{G}a$ or viscous stress $\nu\dot{a} = -\mathrm{i}\omega\nu a$
- Effective elastic (shear) modulus $\mathcal{G}_0$ (+, – or 0)
- Effective viscosity at high frequencies $\mathcal{G}_1/\omega^2$ (+, – or 0)

Analytical approach

- Evaluation in special cases
  - Statistically isotropic flows in d dimensions

$$\mathcal{G}_0 = \frac{2(d-2)(d+1)}{d(d-1)(d+2)} K \qquad K = \langle \tfrac{1}{2} v_i v_i \rangle$$

$$\mathcal{G}_1 = \frac{(d^2-2)\dot{K} + (d^2-6)D}{d(d-1)(d+2)} \qquad \dot{K} = \langle (\partial_t' v_i) v_i \rangle \qquad D = -\nu \langle v_i \Delta' v_i \rangle$$

  - Thus effective elasticity > 0 in 3D but $= 0$ in 2D
  - Effective viscosity > 0 in 3D but < 0 in 2D if flow maintained but < 0 in 3D or 2D if flow decays freely

Analytical approach

- Evaluation in special cases
  - ABC flows (Arnol'd–Beltrami–Childress)

$$\boldsymbol{v} = \begin{pmatrix} A\sin kz' + C\cos ky' \\ B\sin kx' + A\cos kz' \\ C\sin ky' + B\cos kx' \end{pmatrix}$$

    in a period cube of length $2\pi/k$
  - Nonlinearity absent because of Beltrami property $\boldsymbol{\nabla}' \times \boldsymbol{v} = k\boldsymbol{v}$
  - If unforced, $A, B, C \propto \exp(-\nu k^2 t)$
  - Or maintain flow with body force $\boldsymbol{f} = \nu k^2 \boldsymbol{v}$
  - Find

$$\mathcal{G}_0 = \tfrac{1}{2}(A^2 - C^2) \qquad \text{(depends on anisotropy)}$$

$$\mathcal{G}_1 = \tfrac{1}{2}A(\dot{A} + \nu k^2 A) \qquad \text{(vanishes if freely decaying)}$$

- These analytical examples lack genuine nonlinearity / irreversibility

Analytical results for high-frequency shear

General flow (laminar, turbulent, convective, ...)

Tidal period $\ll$ flow timescales (relevant for large eddies)

- Dominant response is elastic
- Next effect is viscosity $\propto \omega^{-2}$
- Coefficients may be positive, negative or zero depending on flow statistics, anisotropy, etc.
- Incompatible with Zahn (1966)
- Different from Goldreich & Nicholson (1977), Goodman & Oh (1997)
- Different from Penev et al. (2009)
- Raises the possibility of tidal anti-dissipation

Conclusions

Conclusions

- Tidal evolution probably determines the fate of short-period extrasolar planets
- Idealized linear inertial waves give an intricate frequency dependence of Q', still only partly understood
- Frequency-averaged dissipation is robust and readily calculated
- For $l = m = 2$ dissipation is most efficient for :
  - larger, more rigid or denser cores
  - greater density stratification (larger polytropic index)
- Other (tesseral) harmonics excite richer response and may be important even though intrinsically weaker
- Better models of planetary (and stellar) interiors are needed and more understanding of the interaction of tides with convection, magnetic fields, etc.

Conclusions

- Nonlinear aspects (wave breaking, mode coupling, etc.) can be important even for “weak” tides. Extrasolar planets may be in a different regime from solar-system planets
- Wave breaking can lead to the destruction of sufficiently massive planets orbiting close to solar-type stars at a critical age
- Effective viscosity of convection is strongly suppressed at tidal frequencies higher than the convective turnover rate
- Thermal and magnetic tides also require further investigation as well as waves in extrasolar planetary atmospheres
- Extrasolar systems are diverse and can reveal much when examined on an individual basis

References

- Arras, P. and Socrates, A., 2010: Thermal Tides in Fluid Extrasolar Planets, ApJ, 714, 1-12.
- Barker, A. J. and Ogilvie, G. I., 2009: On the tidal evolution of Hot Jupiters on inclined orbits, MNRAS, 395, 2268-2287.
- Barker, A. J. and Ogilvie, G. I., 2010: On internal wave breaking and tidal dissipation near the centre of a solar-type star, MNRAS, 404, 1849-1868.
- Barker, A. J. and Ogilvie, G. I., 2011: Stability analysis of a tidally excited internal gravity wave near the centre of a solar-type star, MNRAS, 417, 745-761.
- Bretherton, F. P., 1964: Low frequency oscillations trapped near the equator", Tellus 16, 181–185.
- Bryan, G., 1889: The waves on a rotating liquid spheroid of finite ellipticity. Phil. Trans. R. Soc. Lond. 180, 187–219.

References

- Butler, R. P., Steven S. V., Geoffrey, W. M., Debra, A. F., Jason, T. W., Gregory, W. H., Laughlin, G. and Lissauer, J. J., 2004: A Neptune-Mass Planet Orbiting the Nearby M Dwarf GJ 436, ApJ, 617, 580.

- Chaisson, E. and McMillan, S., 2005: Astronomy Today.

- Christensen-Dalsgaard, J., 2003: Lecture Notes on Stellar Oscillations, 5th ed. (Aarhus: Aarhus Univ.), http://www.astro.caltech.edu/~jspineda/ay123/notes/christensen-dalsgaard.pdf

- Cowling, T. G., 1941: The non-radial oscillations of polytropic stars, MNRAS, 101, 367.

- Goldreich, P., 1963: On the eccentricity of satellite orbits in the solar system, MNRAS, 126, 257.

- Goldreich, P. and Nicholson, P. D., 1977: Turbulent viscosity and Jupiter's tidal Q, Icarus, 30, 301-314.

References

- Goldreich, P. and Nicholson, P. D., 1989: Tidal friction in early-type stars, ApJ, 342, 1079-1084.
- Goodman, J., and Dickson, E. S., 1998: Dynamical Tide in Solar-Type Binaries, ApJ, 507, 938-944.
- Goodman, J. and Lackner, C., 2009: Dynamical Tides in Rotating Planets and Stars, ApJ, 696, 2054.
- Goodman, J. and Oh, S. P., 1997: Fast Tides in Slow Stars: The Efficiency of Eddy Viscosity, ApJ, 486, 403-412.
- Gostiaux, L. and Dauxois, T., 2007: Laboratory experiments on the generation of internal tidal beams over steep slopes, Phys. Fluids 19, 028102.
- Gu, P.-G. and Ogilvie, G. I., 2009: Diurnal thermal tides in a non-synchronized hot Jupiter, MNRAS, 395, 422-435.

References

- Hebb, L., Cameron, A. C., Triaud, A.H.M.J., Lister, T. A., Smalley, B., Maxted, P. F. L., Hellier, C., Anderson, D. R., Pollacco, D., Gillon, M., Queloz, D., West, R. G., Bentley, S., Enoch, B., Haswell, C. A., Horne, K., Mayor, M., Pepe, F., Segransan, D., Skillen, I., Udry, S. and Wheatley, P.J., 2010: WASP-19b: The Shortest Period Transiting Exoplanet Yet Discovered, ApJ, 708, 224.

- Hebb, L., Collier-Cameron, A., Loeillet, B., Pollacco, D., Hebrard, G., Street, R. A., Bouchy, F., Stempels,H. C., Moutou, C., Simpson, E., Udry, S., Joshi, Y. C., West, R. G., Skillen, I., Wilson, D. M., McDonald, I., Gibson, N. P., Aigrain, S., Anderson, D. R., Benn, C. R., Christian, D. J., Enoch, B., Haswell, C. A., Hellier, C., Horne, K., Irwin, J., Lister, T. A., Maxted, P., Mayor, M., Norton, A. J., Parley, N., Pont, F., Queloz, D., Smalley, B. and Wheatley, P. J., 2009: WASP-12b: The Hottest Transiting Extrasolar Planet Yet Discovered, ApJ, 693, 1920.

References

- Hellier, C., Anderson, D. R., Gillon, M., Lister, T. A., Maxted, P. F. L., Queloz, D., Smalley, B., Triaud, A. H. M. J., West, R. G., Wilson, D. M., Alsubai, K., Bentley, S. J., Cameron, A. C., Hebb, L., Horne,K., Irwin, J., Kane, S. R., Mayor, M., Pepe, F., Pollacco, D., Skillen, I., Udry, S., Wheatley, P. J., Christian, D. J., Enoch, R., Haswell, C. A., Joshi, Y. C., Norton, A. J., Parley, N., Ryans, R., Street, R. A. and Todd, I., 2009: WASP-7: A Bright Transiting-Exoplanet System in the Southern Hemisphere, The ApJL, 690, L89.

- Ioannou, P. J. and Lindzen, R. S., 1993a: Gravitational tides in the outer planets. I - Implications of classical tidal theory. II - Interior calculations and estimation of the tidal dissipation factor, ApJ, 406, 252-278.

- Ioannou, P. J. and Lindzen, R. S. 1993b: Gravitational Tides in the Outer Planets. II. Interior Calculations and Estimation of the Tidal Dissipation Factor, ApJ, 406, 266-278.

References

- Ivanov, P. B. and Papaloizou, J. C. B., 2007: Dynamic tides in rotating objects: orbital circularization of extrasolar planets for realistic planet models, MNRAS, 376, 682-704.

- Johns-Krull, C. M., McCullough, P. R., Burke, C. J., Valenti, J. A., Janes, K. A., Heasley, J. N., Prato, L., Bissinger, R., Fleenor, M., Foote, C. N., Garcia-Melendo, E., Gary, B. L., Howell, P. J., Mallia, F., Masi, G. and Vanmunster, T., 2008: XO-3b: A Massive Planet in an Eccentric Orbit Transiting an F5 V Star, ApJ, 677, 657.

- Konacki, M., Torres, G., Jha, S. and Sasselov, D, D., 2003: An extrasolar planet that transits the disk of its parent star, Nature, 421, 507-509.

- Lacaze, L., Gal, P. L. and Dizes, S. L., 2004: Elliptical instability in a rotating spheroid, Journal of Fluid Mechanics, 505, 1-22.

References

- Lainey, V., Arlot, J. E., Karatekin, O. and Hoolst, T. V., 2009: Strong tidal dissipation in Io and Jupiter from astrometric observations, Nature, 459, 957-959.
- Llewellyn Smith Stefan G. and W. R. Young, 2002: Conversion of the Barotropic Tide, J. Phys. Oceanogr., 32, 1554–1566.
- Lubow S. H., Tout C. A. and Livio M., 1997: Long-term tidal evolution of short-period planets with companions, ApJ, 484, 866-870.
- Meibom, S. and Mathieu, R. D., 2005: A Robust Measure of Tidal Circularization in Coeval Binary Populations: The Solar-Type Spectroscopic Binary Population in the Open Cluster M35, ApJ, 620, 970.
- Ogilvie, G. I., 2009: Tidal dissipation in rotating fluid bodies: a simplified model, MNRAS, 396, 794-806.

References

- Ogilvie, G. I. and Lin, D. N. C., 2004: Tidal Dissipation in Rotating Giant Planets, ApJ, 610, 477-509. (Paper I)
- Ogilvie, G. I. and Lin, D. N. C., 2007: Tidal Dissipation in Rotating Solar-Type Stars, ApJ, 661, 1180-1191.
- Papaloizou, J. C. B. and Ivanov, P. B., 2005: Oscillations of rotating bodies: a self-adjoint formalism applied to dynamic tides and tidal capture, MNRASL, 364, L66-L70.
- Penev, K., Sasselov, D., Robinson, F. and Demarque, P., 2009: Dissipation Efficiency in Turbulent Convective Zones in Low-Mass Stars, ApJ, 704, 930-936.
- Rieutord, M., Georgeot, B., and Valdettaro, L., 2001: Inertial waves in a rotating spherical shell: attractors and asymptotic spectrum, J. Fluid Mech., 435, 103-144.

References

- Rieutord, M., and Valdettaro, L., 2010: Viscous dissipation by tidally forced inertial modes in a rotating spherical shell, J. Fluid Mech, 643, 363-394.
- Savonije, G. J. and Witte, M. G., 2002: Tidal interaction of a rotating 1 vec {M_sun} star with a binary companion, A&A, 386, 211-221.
- Stewartson, K., 1972: On trapped oscillations of a rotating fluid in a thin spherical shell, Tellus 24, 283–287.
- Terquem, C., Papaloizou, J. C. B., Nelson, R. P. and Lin, D. N. C., 1998: On the Tidal Interaction of a Solar-Type Star with an Orbiting Companion: Excitation of g-Mode Oscillation and Orbital Evolution, ApJ, 502, 788-801.
- Torres, G., Winn, J. N. and Holman, M. J., 2008: Improved Parameters for Extrasolar Transiting Planets, ApJ, 677, 1324.

References

- Winn, J. N., Johnson, J. A., Albrecht, S., Howard, A. W., Marcy, G W., Crossfield, I. J. and Holman, M. J., 2009: HAT-P-7: A Retrograde or Polar Orbit, and a Third Body, ApJL, 703, L99.

- Wu, Y., 2005: Origin of Tidal Dissipation in Jupiter. II. The Value of Q, The ApJ, 635, 688.

- Zahn, J. P., 1966: Les marées dans une étoile double serrée (suite), Ann. d'Astrophys., 29, 489.

- Zucker, D. B., Kniazev, A. Y., Bell, E. F., Delgado, D. M., Grebel, E. K., Rix, H. W., Rockosi, C. M., Holtzman, J. A., Walterbos, R. A. M., Ivezi, E., Brinkmann, J., Brewington, H., Harvanek, M., Kleinman, S. J., Krzesinski, J., Lamb, D. Q., Long, D., Newman, P. R., Nitta, A. and Snedden, S. A., 2004: A New Giant Stellar Structure in the Outer Halo of M31, ApJL, 612, L117-L120.