

1 微量成分凝結系, 主成分凝結系の両方で使用可能な方程式系

ある 1 種類の分子が大気主成分である場合について考える. また乾燥成分は 1 種類であり, 凝結成分が複数種類存在しうるものとする. 即ち全密度 ρ が

$$\rho = \rho_d + \sum_j \rho_v^{(j)} + \sum_j \rho_s^{(j)} \quad (1)$$

と表される場合について考える.

凝結成分のみから成る大気の場合, 混合比は常に $+\infty$ となり, 計算上の取扱いが厄介である. そこで以下に示す分母に全密度 ρ を用いた物理量 (本文書では以後「全密混合比」と呼ぶこととする) を導入する.

$$\beta_d \equiv \frac{\rho_d}{\rho}, \quad (2)$$

$$\beta_v^{(i)} \equiv \frac{\rho_v^{(i)}}{\rho}, \quad (3)$$

$$\beta_s^{(i)} \equiv \frac{\rho_s^{(i)}}{\rho}. \quad (4)$$

β_v は地球の気象用語の「比湿」に相当するものである.

また以下の仮定を行なう.

- 定圧比熱, 定積比熱は温度・圧力に依存せず一定である.
- 定圧比熱, 定積比熱は大気主成分の比熱で近似できる.

1 線形化前

新たな方程式系での予報変数は水平流速 u , 鉛直流速 w , 温位 θ , エクスナー関数 Π , 全密混合比 β_v, β_s である.

$$\frac{du}{dt} = -c_{pmj}\theta_v \frac{\partial \Pi}{\partial x} + D(u), \quad (5)$$

$$\frac{dw}{dt} = -c_{pmj}\theta_v \frac{\partial \Pi}{\partial z} - g + D(w), \quad (6)$$

$$\frac{d\theta}{dt} = \frac{1}{\Pi} \dot{Q} + D(\theta), \quad (7)$$

$$\begin{aligned} \frac{d\Pi}{dt} = & \frac{c_s^2}{c_{pmj}\theta_v} \left[-\frac{\partial u}{\partial x} - \frac{\partial w}{\partial z} + \frac{1}{\rho} \sum_j M_{fall}^{(j)} + \frac{1}{\theta} \frac{d\theta}{dt} \right. \\ & \left. + \frac{1}{\beta_{mj} + \sum_j \beta_{mn}^{(j)} M_{mj}/M_{mn}^{(j)}} \left(\frac{d\beta_{mj}}{dt} + \sum_j \frac{M_{mj}}{M_{mn}^{(j)}} \frac{d\beta_{mn}^{(j)}}{dt} \right) \right], \end{aligned} \quad (8)$$

$$\frac{d\beta_v^{(i)}}{dt} = -\frac{\beta_v^{(i)}}{\rho} \sum_j M_{fall}^{(j)} - \frac{1}{\rho} M_{src}^{(i)} + D(\beta_v^{(i)}), \quad (9)$$

$$\frac{d\beta_s^{(i)}}{dt} = -\frac{\beta_s^{(i)}}{\rho} \sum_j M_{fall}^{(j)} + \frac{1}{\rho} M_{fall}^{(i)} + \frac{1}{\rho} M_{src}^{(i)} + D(\beta_s^{(i)}). \quad (10)$$

ここで下付き添字の mj, mn はそれぞれ大気主成分, 微量成分であることを表す.
また

$$\theta = T \left(\frac{p}{p_0} \right)^{R_{mj}/c_{pmj}}, \quad (11)$$

$$\Pi = \left(\frac{p_0}{p} \right)^{R_{mj}/c_{pmj}}, \quad (12)$$

$$\theta_v = \frac{1}{1 - \sum_j \beta_s^{(j)}} \left(\beta_{mj} + \sum_j \beta_{mn}^{(j)} \frac{R_{mn}^{(j)}}{R_{mj}} \right) \theta, \quad (13)$$

$$c_s^2 = \frac{c_{pmj}}{c_{vmj}} R_{mj} \Pi \theta_v. \quad (14)$$

である.

2 線形化後

各変数を以下のように基本場成分と擾乱成分に分ける.

$$u(x, z, t) = u'(x, z, t), \quad (15)$$

$$w(x, z, t) = w'(x, z, t), \quad (16)$$

$$\theta(x, z, t) = \bar{\theta}(z) + \theta'(x, z, t), \quad (17)$$

$$\Pi(x, z, t) = \bar{\Pi}(z) + \Pi'(x, z, t), \quad (18)$$

$$\beta_v^{(i)}(x, z, t) = \bar{\beta}_v(z) + \beta_v^{(i)'}(x, z, t), \quad (19)$$

$$\beta_s^{(i)}(x, z, t) = \beta_s^{(i)'}(x, z, t). \quad (20)$$

移流項以外の 2 次の微小量を無視すると, 以下ようになる.

$$\frac{\partial u'}{\partial t} = -u' \frac{\partial u'}{\partial x} - w' \frac{\partial u'}{\partial z} - c_{pmj} \bar{\theta}_v \frac{\partial \Pi'}{\partial x} + D(u'), \quad (21)$$

$$\begin{aligned} \frac{\partial w'}{\partial t} = & -u' \frac{\partial w'}{\partial x} - w' \frac{\partial w'}{\partial z} - c_{pmj} \bar{\theta}_v \frac{\partial \Pi'}{\partial z} + D(w') \\ & + \left[\frac{\theta'}{\bar{\theta}} + \frac{\beta'_{mj} + \sum_j \beta_{mn}^{(j)'} M_{mj}/M_{mn}^{(j)} + \sum_j \beta_s^{(j)'} (\bar{\beta}_{mj} + \sum_j \bar{\beta}_{mn}^{(j)} M_{mj}/M_{mn}^{(j)})}{\bar{\beta}_{mj} + \sum_j \bar{\beta}_{mn}^{(j)} M_{mj}/M_{mn}^{(j)}} \right] g, \end{aligned} \quad (22)$$

$$\frac{\partial \theta'}{\partial t} = -u' \frac{\partial \theta'}{\partial x} - w' \frac{\partial \theta'}{\partial z} + \frac{\bar{\theta}}{\bar{T}} \dot{Q} + D(\theta), \quad (23)$$

$$\begin{aligned} \frac{\partial \Pi'}{\partial t} = & -\frac{\bar{c}_s^2}{c_{pmj} \bar{\rho} \bar{\theta}_v} \left[\frac{\partial (\bar{\rho} \bar{\theta}_v u')}{\partial x} + \frac{\partial (\bar{\rho} \bar{\theta}_v w')}{\partial z} \right] + \frac{\bar{c}_s^2}{c_{pmj} \bar{\rho} \bar{\theta}_v} \sum_j M_{fall}^{(j)} \\ & + \frac{\bar{c}_s^2}{c_{pmj} \bar{\theta}_v} \left\{ \frac{1}{\bar{\rho}} \left(\frac{d\theta}{dt} \right)' + \frac{1}{\bar{\beta}_{mj} + \sum_j \bar{\beta}_{mn}^{(j)} M_{mj}/M_{mn}^{(j)}} \right. \\ & \left. \left[\left(\frac{d\beta_{mj}}{dt} \right)' + \sum_j \frac{M_{mj}}{M_{mn}^{(j)}} \left(\frac{d\beta_{mn}^{(j)}}{dt} \right)' \right] \right\}, \end{aligned} \quad (24)$$

$$\frac{\partial \beta_v^{(i)'}}{\partial t} = -u' \frac{\partial \beta_v^{(i)'}}{\partial x} - w' \frac{\partial \beta_v^{(i)'}}{\partial z} - w' \frac{\partial \bar{\beta}_v^{(i)}}{\partial z} - \frac{\bar{\beta}_v^{(i)}}{\bar{\rho}} \sum_j M_{fall}^j - \frac{1}{\bar{\rho}} M_{src}^{(i)} + D(\beta_v^{(i)}), \quad (25)$$

$$\frac{\partial \beta_s^{(i)'}}{\partial t} = -u' \frac{\partial \beta_v^{(i)'}}{\partial x} - w' \frac{\partial \beta_v^{(i)'}}{\partial z} + \frac{1}{\bar{\rho}} M_{fall}^{(i)} + \frac{1}{\bar{\rho}} M_{src}^{(i)} + D(\beta_s^{(i)}). \quad (26)$$

2 付録

1 全密混合比の時間発展方程式の導出

$\rho_d, \rho_v^{(i)}, \rho_s^{(i)}$ の連続の式はそれぞれ

$$\frac{\partial \rho_d}{\partial t} + \frac{\partial}{\partial x_j}(\rho_d u_j) = 0, \quad (27)$$

$$\frac{\partial \rho_v^{(i)}}{\partial t} + \frac{\partial}{\partial x_j}(\rho_v^{(i)} u_j) = -M_{src}^{(i)}, \quad (28)$$

$$\frac{\partial \rho_s^{(i)}}{\partial t} + \frac{\partial}{\partial x_j}(\rho_s^{(i)} u_j) = M_{src}^{(i)} + M_{fall}^{(i)} \quad (29)$$

と表される. (27), (28), (29) より, ρ に関する連続の式は

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho u_j) = \sum_j M_{fall}^{(i)} \quad (30)$$

となる. (27), (28), (29), (30) より

$$\begin{aligned} \frac{\partial}{\partial t} \left(\frac{\rho_d}{\rho} \right) &= -\frac{\rho_d}{\rho^2} \frac{\partial \rho}{\partial t} + \frac{1}{\rho} \frac{\partial \rho_d}{\partial t} \\ &= -\frac{\rho_d}{\rho^2} \left[-\frac{\partial}{\partial x_j}(\rho u_j) + \sum_j M_{fall}^{(j)} \right] + \frac{1}{\rho} \left[-\frac{\partial}{\partial x_j}(\rho_d u_j) \right] \\ &= -u_j \frac{\partial}{\partial x_j} \left(\frac{\rho_d}{\rho} \right) - \frac{\rho_d}{\rho^2} \sum_j M_{fall}^{(j)}, \end{aligned} \quad (31)$$

$$\begin{aligned} \frac{\partial}{\partial t} \left(\frac{\rho_v^{(i)}}{\rho} \right) &= -\frac{\rho_v^{(i)}}{\rho^2} \frac{\partial \rho}{\partial t} + \frac{1}{\rho} \frac{\partial \rho_v^{(i)}}{\partial t} \\ &= -\frac{\rho_v^{(i)}}{\rho^2} \left[-\frac{\partial}{\partial x_j}(\rho u_j) + \sum_j M_{fall}^{(j)} \right] + \frac{1}{\rho} \left[-\frac{\partial}{\partial x_j}(\rho_v^{(i)} u_j) - M_{src}^{(i)} \right] \\ &= -u_j \frac{\partial}{\partial x_j} \left(\frac{\rho_v^{(i)}}{\rho} \right) - \frac{\rho_v^{(i)}}{\rho^2} \sum_j M_{fall}^{(j)} - \frac{1}{\rho} M_{cond}^{(i)}, \end{aligned} \quad (32)$$

$$\begin{aligned} \frac{\partial}{\partial t} \left(\frac{\rho_s^{(i)}}{\rho} \right) &= -\frac{\rho_s^{(i)}}{\rho^2} \frac{\partial \rho}{\partial t} + \frac{1}{\rho} \frac{\partial \rho_s^{(i)}}{\partial t} \\ &= -\frac{\rho_s^{(i)}}{\rho^2} \left[-\frac{\partial}{\partial x_j}(\rho u_j) + \sum_j M_{fall}^{(j)} \right] + \frac{1}{\rho} \left[-\frac{\partial}{\partial x_j}(\rho_s^{(i)} u_j) + M_{src}^{(i)} + M_{fall}^{(i)} \right] \\ &= -u_j \frac{\partial}{\partial x_j} \left(\frac{\rho_s^{(i)}}{\rho} \right) - \frac{\rho_s^{(i)}}{\rho^2} \sum_j M_{fall}^{(j)} + \frac{1}{\rho} M_{src}^{(i)} + \frac{1}{\rho} M_{fall}^{(i)}. \end{aligned} \quad (33)$$

即ち

$$\frac{d\beta_d}{dt} = -\frac{\beta_d}{\rho} \sum_j M_{fall}^{(j)} + D(\beta_d^{(i)}), \quad (34)$$

$$\frac{d\beta_v^{(i)}}{dt} = -\frac{\beta_v^{(i)}}{\rho} \sum_j M_{fall}^{(j)} - \frac{1}{\rho} M_{src}^{(i)} + D(\beta_v^{(i)}), \quad (35)$$

$$\frac{d\beta_s^{(i)}}{dt} = -\frac{\beta_s^{(i)}}{\rho} \sum_j M_{fall}^{(j)} + \frac{1}{\rho} M_{fall}^{(i)} + \frac{1}{\rho} M_{src}^{(i)} + D(\beta_s^{(i)}). \quad (36)$$

を得る.

2 仮温度の導出

状態方程式より

$$\begin{aligned} p &= p_{mj} + \sum_j p_{mn}^j \\ &= (\rho_{mj} R_{mj} + \sum_j \rho_{mn}^{(j)} R_{mn}^{(j)}) T \\ &= \rho_g \frac{\rho}{\rho_g} \left(\frac{\rho_{mj}}{\rho} R_{mj} + \sum_j \frac{\rho_{mn}^{(j)}}{\rho} R_{mn}^{(j)} \right) T \\ &= \rho_g \frac{1}{1 - \sum_j \rho_s^{(j)} / \rho} \left(\frac{\rho_{mj}}{\rho} R_{mj} + \sum_j \frac{\rho_{mn}^{(j)}}{\rho} R_{mn}^{(j)} \right) T \\ &= \rho_g R_{mj} \frac{1}{1 - \sum_j \beta_s^{(j)}} \left(\beta_{mj} + \sum_j \beta_{mn}^{(j)} \frac{R_{mn}^{(j)}}{R_{mj}} \right) T. \end{aligned} \quad (37)$$

但し ρ_g は気相密度である. ここで仮温度 T_v を

$$T_v \equiv \frac{1}{1 - \sum_j \beta_s^{(j)}} \left(\beta_{mj} + \sum_j \beta_{mn}^{(j)} \frac{R_{mn}^{(j)}}{R_{mj}} \right) T \quad (38)$$

と定義する. 更に仮温度 θ_v を

$$\theta_v \equiv \frac{T_v}{\Pi} \quad (39)$$

と定義すると,

$$\theta_v = \frac{1}{1 - \sum_j \beta_s^{(j)}} \left(\beta_{mj} + \sum_j \beta_{mn}^{(j)} \frac{R_{mn}^{(j)}}{R_{mj}} \right) \theta \quad (40)$$

となる.

3 圧力傾度力の書き換え

運動方程式

$$\frac{du}{dt} = -\frac{1}{\rho_g} \frac{\partial p}{\partial x} + D(u), \quad (41)$$

$$\frac{dw}{dt} = -\frac{1}{\rho_g} \frac{\partial p}{\partial z} - g + D(w), \quad (42)$$

の圧力傾度力を書き換える. (37), (38), (39) より

$$\rho_g = \frac{p}{R_{mj}\Pi\theta_v} = \frac{p_0\Pi^{c_{vmj}/R_{mj}}}{R_{mj}\theta_v} \quad (43)$$

となるので,

$$\frac{1}{\rho_g} \frac{\partial p}{\partial x_i} = c_{pmj}\theta_v \frac{\partial \Pi}{\partial x_i} \quad (44)$$

となる.