The force hierarchy and geodynamo regimes of Earth's core

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Thanks to collaborators:
Emmanuel Dormy,
Ecole Normale Supérieure, Paris

University of Glasgow

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The Geomagnetic Field

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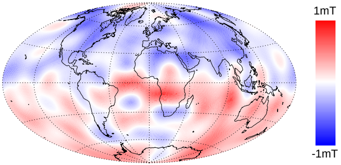

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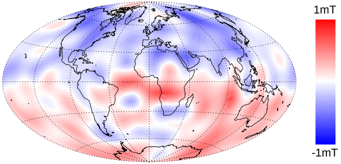
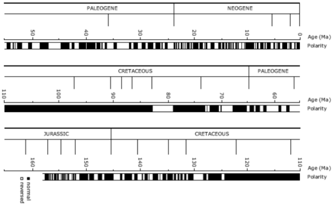
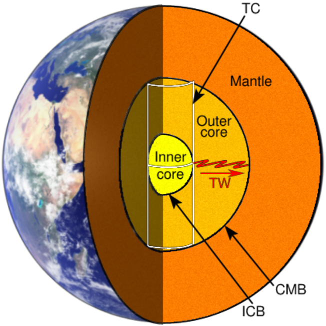




- Field is predominantly dipolar (but also note the patches of reversed ux found at high latitudes).
- Reversals of the eld dipolarity occur (seemingly at random intervals and a reversal takes thousands of years to complete).

Structure of Earth



ICB = Inner core boundary
CMB = Core-mantle boundary
TC = Tangent cylinder

- Fluid outer core is seat of dynamo giving rise to geomagnetic eld.
- Convection arises from heat and light material released at inner core boundary.
- Magnetic eld is continually replenished through induction (combining Faraday's law, Ampere's law, and Ohm's law)
- Twisting and stretching of eld lines by chaotic convection generates electric current, in turn re-generating magnetic eld.

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- Flow and magnetic field can be analysed directly
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Aim to match simulations to observations of the changing geomagnetic eld thereby understanding dynamics in the core.

Geodynamo simulations - physical setup

- Spherical polar coordinate system, $(r; \ ; \ )$.
- Spherical shell radially bounded above at $r = r_o$ by an electrically insulating mantle and below at $r = r_i$ by an electrically insulating (or conducting) inner core.
- Rotates about the vertical (z-axis) with rotation rate and gravity acts radially inward, $\mathbf{g} = g\mathbf{r}$.
- Boussinesq approximation used - density, , treated as a constant except for the source of buoyancy
- Fluid is assumed to have constant values of , , and , the outer core density, kinematic viscosity, thermal di usivity and magnetic di usivity respectively.

Geodynamo equations

Evolution equations for velocity, $\mathbf{u}$, temperature T, and magnetic field, $\mathbf{B}$:

$$\underbrace{E\mathrm{m}\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right)}_{\text{inertia (I)}} = \underbrace{-\nabla p}_{\text{pressure (P)}} \underbrace{-2\hat{\mathbf{z}}\times\mathbf{u}}_{\text{Coriolis (C)}} + \underbrace{(\nabla\times\mathbf{B})\times\mathbf{B}}_{\text{Lorentz (M)}} + \underbrace{Ra\,T\,\mathbf{r}}_{\text{buoyancy (A)}} + \underbrace{E\nabla^2\mathbf{u}}_{\text{viscous (V)}};$$

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- Often use $\widetilde{Ra}' = Ra/Ra_c = \widetilde{Ra}/qRa_c$, as a measure of supercriticality. Ra_c is the critical Rayleigh number for the onset of (non-magnetic) convection.

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2. Correct solution space: aim to nd solutions with the expected *balance of forces* within the momentum equation by performing parameter sweeps

! Allows for the identi cation of suitable parameter regimes despite input parameters not close to Earth-like values.
Then preserve the force balance by moving all parameters towards Earth-like values in a systematic way.

Force balances

- Forces acting in the non-dimensionalised system are:

$$\mathbf{F}_I = E_m\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) \qquad \mathbf{F}_P = -\nabla p \qquad \mathbf{F}_C = -2\hat{\mathbf{z}}\times\mathbf{u}$$

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  - CIA balance, $\mathbf{F}_C \approx \mathbf{F}_I \approx \mathbf{F}_A$. At high convective driving and weak magnetic field.

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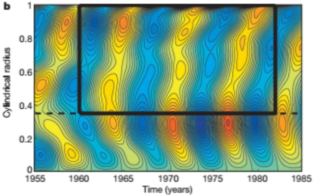

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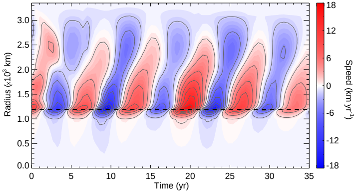

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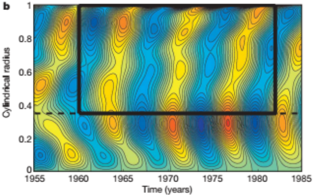

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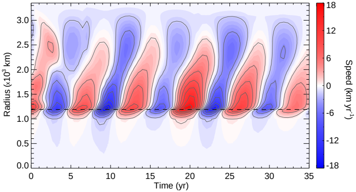

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- Seen geostrophic, MAC, VAC, etc.
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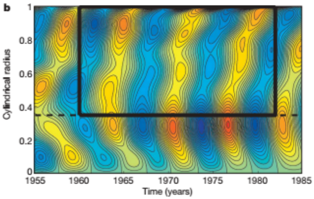
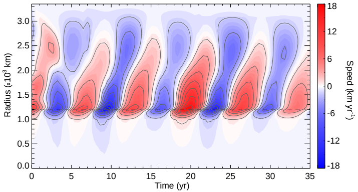


Observation (Gillet+, 2010)



Simulation (Teed+, 2014)

- Observations of waves suggest Earth in MAC regime
- Some previous investigations of force balances:
  - Rotvig & Jones, Phys Rev E, 2002
  - Soderlund+, PEPS, 2015
  - Yadav+, PNAS, 2016
  - Schae er+, GJI, 2017
  - Schwaiger+, GJI, 2019, 2021
  - Teed & Dormy, JFM, 2023

Lengthscale dependent forces

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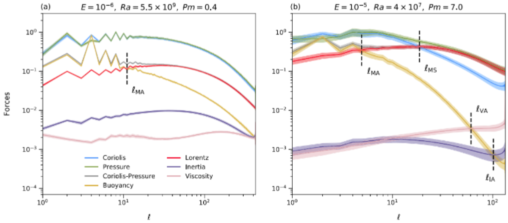

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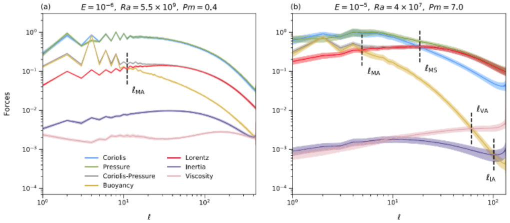

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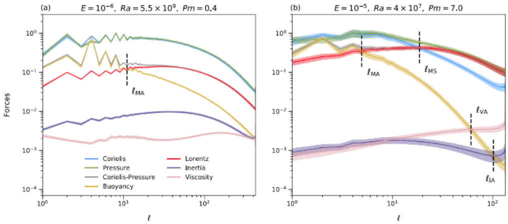

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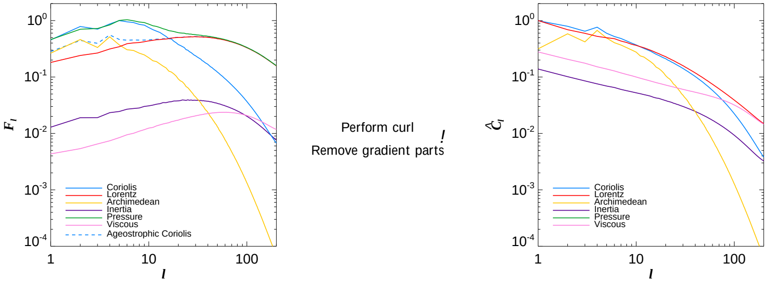
- **F** = *r* **A** + *r* ′; eliminate *r* ′ by:
  - *curling* **F**. (Note: Taylor-Proudman constraint is formed this way!)
  - *projecting of forces* onto their solenoidal part: *r* **A**



Teed & Dormy, 2023

Weak and strong field branches - theory

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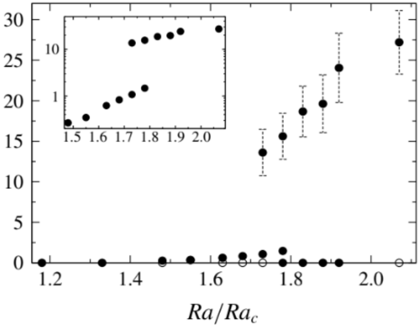
- Potential bistability - di erent regimes at the same input parameters

Weak and strong field branches - simulations

- Dormy, 2016; identi cation of strong eld branch and bistability in DNS at $E = 3 \quad 10^{4}$ and $E_m = 1{:}7 \quad 10^{5}$



- Requires E_m to be chosen within a `sweet-spot' range of values (dependent on E)...

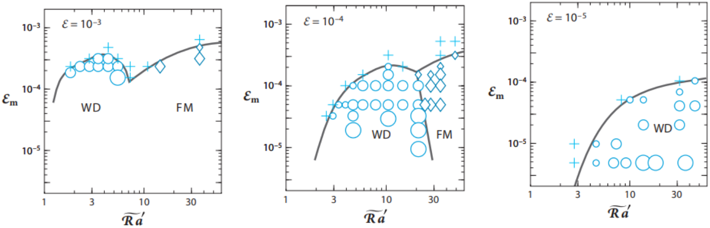
Regime diagrams

Each plot is decreasing $E_!$

Decreasing E_m

Solutions become non-magnetic

Solutions become non-dipolar



Solutions become non-dipolar with increasing $\widetilde{Ra}'$

Bifurcation diagrams

- Bifurcation diagrams for $E = 3 \quad 10^{4}$

Dormy, 2025

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Diagrams differ (isola, supercritical, subcritical bifurcations) as E_m is varied

Weak-strong branching is found at low enough E_m

Dormy, 2025

Bifurcation diagrams

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Tentative 3D bifurcation diagram for xed$E$

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Bifurcation diagrams

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- Weak-strong branching occurs via a cusp singularity
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Dormy, 2025

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Need to study dependence of such solutions on E and E_m to help determine constants

Some questions to address

Now we'll look at some results on (solenoidal) forces (Teed & Dormy, 2023) and upcoming work on geodynamo branches as E is lowered (Teed & Dormy, 2025).

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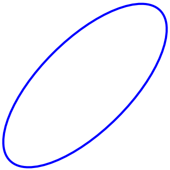
2. What regime produces the desired MAC balance relevant to Earth's core dynamics and where is it located in parameter space?
3. How does the branching between weak and strong regimes persist/scale as parameters are moved towards Earth-like values? I.e. lowering E and E_m.

Branches of dynamo action ($E = 10^{4}$)

Branches of dynamo action ($E = 10^{-4}$)

At higher E_m:

Branches of dynamo action ($E = 10^{4}$)



At higher E_m:

Dipolar regime...

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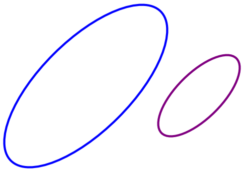

At higher E_m:

- Dipolar regime...
- ...transitions to multipolar regime at large enough Ra

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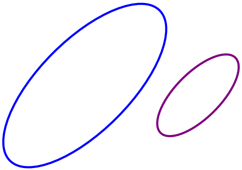

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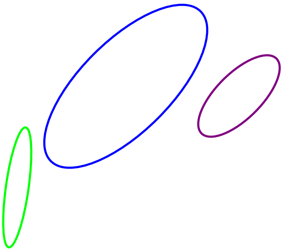

At higher E_m:

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At lower E_m:

- Weak eld dipolar regime (^{0} 1)...

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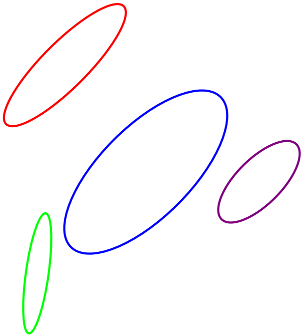

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At lower E_m:

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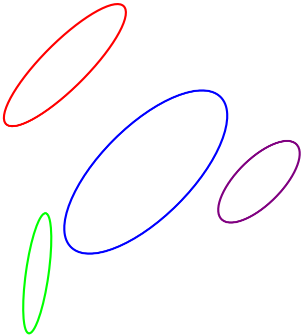

At higher E_m:

- ~~Dipolar~~ `Strongish' dipolar regime...
- ...transitions to multipolar regime at large enough Ra

At lower E_m:

- Weak field dipolar regime (^{0} 1)...
- ...transitions to strong field dipolar regime (^{0} 1)
- Bistability between weak and strong branches in a region of Ra-space

Typical regimes (aE = 10^{4})

Strongish dipolar

Typical regimes ($E = 10^{-4}$)

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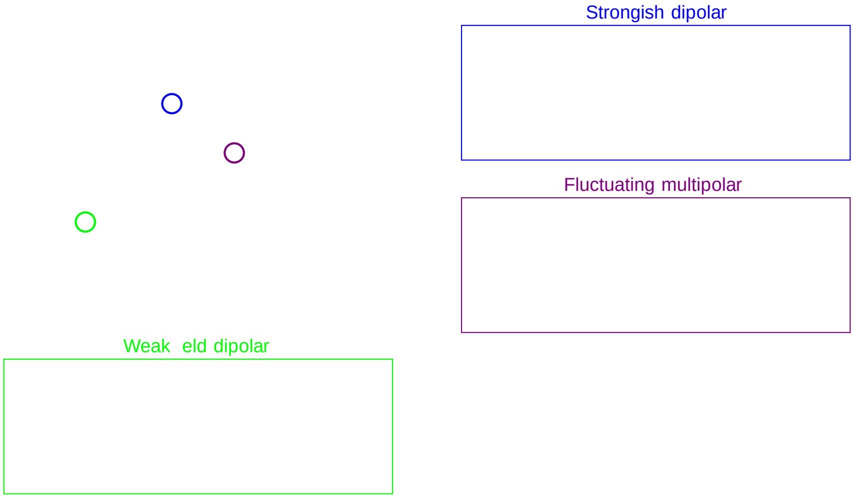
Fluctuating multipolar

Typical regimes (aE = 10^{4})



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Strong eld dipolar

Comparing forces and solenoidal forces

Perform curl
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Remove gradient parts

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- Forces show inertia entering zeroth order balance
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- Solenoidal forces reveal clear leading order MAC balance at large scale for strong eld regime
- Usefulness of ageostrophic Coriolis force lost at lengthscales where balance is not QG

Solenoidal forces

Strongish

Multipolar

Weak eld

Strong eld